A SHORT NOTE ON THE RIEMANN MAPPING THEOREM

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Let R be a simply connected region which is not the whole complex plane and let D be the open unit disk in the plane. The Riemann Mapping Theorem [1] states that if $a \epsilon R$ then there is a unique analytic homeomorphism f_a of R onto D such that $f_a(a)=0$ and $f_a'(a)>0$. Let $H(R)$ be the space of all such f_a with the compact-open topology. We showed in [2] that $H(R)$ is homeomorphic to R. Now let H be the set of all analytic homeomorphisms g of R onto D such that if $a \epsilon R$ is the zero of g, then $|g'(a)|>0$. We give the compact-open topology to the set H. Let S' be the unit circle in the plane.

THEOREM. H and $S' \times R$ are homeomorphic.

PROOF. Let $\varphi : S' \times H(R) \to H$ be a function defined by $\varphi(c, f_a)=cf_a$. Then φ is one-to-one. Let $g \epsilon H$ and $a \epsilon R$ such that $g(a)=0$ and $|g'(a)|>0$. Let $k=\overline{g'(a)}/|g'(a)|$. Then $k \epsilon S'$ and $kg \epsilon H(R)$. Since such elment in $H(R)$ is unique, we let $f_a=kg$. Then $\varphi(k, f_a)=g$. Now suppose $\{c_n\}$ is a sequence of elements in S' which converges to c_o and $\{f_{a_n}\}$ is a sequence of elements of $H(R)$ which converges to $f_{a_o} \epsilon H(R)$. Then by the inequality $|c_n f_{a_n}(z)-c_o f_{a_o}| \leqq |c_n-c_o|\ |f_{a_n}(z)|+|c_o|\ |f_{a_n}(z)-f_{a_o}|$, we see that the sequence $\{c_n f_{a_n}\}$ converges to $c_o f_{a_o}$ uniformly on each compact subset of R. Therefore φ is continuous.

Let $\{g_n\}$ be a sequence in H which converges to $g_o \epsilon H$. Let $k_n g_n=f_{a_n}$, $k_n \epsilon S'$. Then the sequence $\{a_n\}$ converges to $a_o \epsilon R$ and hence $\{f_{a_n}\}$ converges uniformly to f_{a_o} on each compact subset of R. Let $\{k_{n_i}\}$ and $\{k_{m_i}\}$ be subsequences of $\{k_n\}$ which converge k_o and k_o' respectively. Then, since $k_o f_{a_o}=k_o' f_{a_o}$, we must have $k_o=k_o'$. Therefore φ^{-1} is continuous.

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