A GENERALIZATION OF A THEOREM OF WOLK

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Let $(Y, \mathscr{P})$ be a symmetric generalized proximity pace (c.f. [1]). Let $(Y, \mathscr{U})$ be a symmetric generalized uniform space (c.f. [2]). Let $\{f_n, n \in D\}$ be a net of members of Y^X.

DEFINITION 1. (Leader, S) f_n converges to f with respect to $\mathscr{P}$ (notation: $(f_n, f; \mathscr{P})$) iff for every A in $P(X)$ and B in $P(Y)$ $(f[A], B)$ not in $\mathscr{P}$ implies there exists m_0 such that if $n \geqq m_0$ then $(f_n[A], B)$ is not in $\mathscr{P}$.

DEFINITION 2. f_n converges to f with respect to $\mathscr{U}$ (notation: $(f_n, f; \mathscr{U})$) iff for every U in $\mathscr{U}$ there exists m_0 such that for every x in X if $n \geqq m_0$ then $f_n(x)$ is in $U[f(x)]$.

THEOREM 1. *Let $\mathscr{U}(\mathscr{P})^*$ be p-correct in $\Pi(\mathscr{P})$ (c.f. [3]) where $\mathscr{P}$ is a symmetric generalized proximity space on a set X. Then $(f_n, f; \mathscr{P})$ implies $(f_n, f; \mathscr{U}(\mathscr{P})^*)$.*

PROOF. Let $V = U_{A_1, B_1} \cap \cdots \cap U_{An, Bn} \in \mathscr{U}(\mathscr{P})^*$. Note that if $f(x) \notin \bigcup_{k=1}^{n} (A_k \cup B_k)$, then $V(f(x)) = X$. Suppose there exists $z \in X$ such that $z \in A_{p_1} \cap \cdots \cap A_{p_r} \cap B_{q_1} \cap \cdots \cap B_{q_s} = E_1$ where $1 \leqq r \leqq n$ and $1 \leqq s \leqq n$, but z is not in any other A_i or B_i. (We call E_1 a *residual* intersection of the A_i and B_i). Then (by lemma 1 in [3]) we have that $V(z)$ equals $-(B_{p_1} \cup \cdots \cup B_{p_r} \cup A_{q_1} \cup \cdots \cup A_{q_s}) = \sim F_1$. Clearly, $E_1 \overline{\mathscr{P}} F F_1$. Let E_1^* equal $\{f(x) | f(x) \in E_1$ but in no other A_i or $B_i\}$. Let $A_1 = f^{-1}(E_1^*)$; clearly, $f[A_1] \overline{\mathscr{P}} F_1$. Hence there exists m_{E_1} such that for every $n \geqq m_{E_1}$ $f_n[A_1] \overline{\mathscr{P}} F_1$ implies $f_n[A_1] \subseteq -F_1$. Hence for every $x \in A_1$ we have that $f_n(x) \in V(f(x))$ if

$n \geqq m_{E_1}$. Since there exists only a finite number of residual intersections of the A_i and B_i, there exists $m^* \geqq m_{E_1} \cdots, m_{El}$ such that for every $x \in X$ $f_n(x) \in V[f(x)]$ if $n \geqq m^*$.

COROLLARY (Wolk, c.f. [4]) *Let* $(Y, \mathscr{P})$ *be a proximity space with proximity class* $\Pi(\mathscr{P})$. *Let* $(Y, \mathscr{U}_0)$ *be a uniform space where* $\mathscr{U}_0$ *is the Alfsen-Fenstad uniformity in* $\Pi(\mathscr{P})$. *Then* $(f_n, f; \mathscr{P})$ *implies that* f_n *converges to* f *uniformly.*

PROOF. This is an immediate consequence of theorem 1, theorem 2 in [3] and the fact that f_n converges to f uniformly iff $(f_n, f; \mathscr{U})$.

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