On the Sufficiency of a Single Logical Sign for the Truth Functional Mode of Compostion

Byung Ho Han

1. The sufficiency of three logical signs, negation, conjunction and disjunction for the truth-functional mode of composition will be presented first.

Let $P_1, P_2, \cdots, P_n$ be n propositional variables. Then by assigning truth and falsehood to $P'_i s$ we get 2^n cases of truth value distribution for $P_1, P_2, \cdots, P_n$. Now take any n-ary mode of composition of $P_1, P_2, \cdots, P_n$ and denote it by X. Then for each assignment of truth value to $P'_i s$, there corresponds the truth value of the whole composition X. Let the number of X taking truth m. Now take any case of these and say falsehood is assigned to P_j, $P_k (j<k)$ and for the rest of $P'_i s$ truth is assigned. Then we take the following conjunction

$$P_1 \cdot \cdots \cdot P_{j-1} \cdot \bar{P}_j \cdot P_{j+1} \cdot \cdots \cdot P_{k-1} \cdot \bar{P}_k \cdot P_{k+1} \cdot \cdots \cdot P_n$$

Then this is true if and only if P_j and P_k are false and the other $p'_i s$ are true. In this way we take the m conjunctions corresponding for the assignment of truths to X, and let us denotl these

$$\phi_1, \phi_2, \cdots, \phi_m.$$

Then the disjunction

$$\phi_1 \vee \phi_2 \vee \cdots \vee \phi_m$$

has the same truth table as X. That is, X is expressed by this disjunction.

For the one exceptional case where X takes falsehood only, we may take for example

$$p_1 \cdot \bar{p}_1 \cdot p_2 \cdot \cdots \cdot p_n$$

for an expression of X with the three logical signs.

Now this completes the sufficiency of three loical signs, negation, conjunction and disjunction for the truth-functional mode of composition.

2. Now we take the binary mode of statement composition which is true if and only if both components of it are false and this will be denoted by "$p|q$" where "p" and "q" are component statements. Then the three logical signs are expressible by this new logical sign alone. For "$\bar{p}$" is experessible as "$p|p$", "$p \cdot q$" as "$(p|p)|(q|q)$" and "$p \vee q$" as "$(p|q)|(p|q)$". The validity of this assertion can be ascertained by construction of the truth tables. This shows the sufficiency of this single logical sign for the truth-functional mode of composition.

Soodo Engineering College