P_1-MAPPING AND PROPERTY K

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1. Introduction.

Paul McDougle [3] has given many results of the following type: Given assigned properties to X and f what further property of f is equivalent to a specified property of Y. In this paper we consider the property K [1] as the assigned property of X and Y.

2. Preliminaries.

Let $f(X)=Y$, where X, Y are Hausdorff spaces and f is a continuous mapping. f is a *quasicompact mapping* provided that the image of open inverse set in X is open in Y. The mapping f is a *P_1-mapping* provided that whenever $y \in Y$ and U is a neighbourhood of $f^{-1}(y)$ then $y \in int\ f(U)$. It is obviously observed from the definition that every P_1-mapping is quasicompact. In [2], it was shown that the necessary and sufficient condition to the mapping f to be P_1-mapping is $y \in \overline{A}-A \subset Y$ implies $f^{-1}(y) \cap Cl(f^{-1}(A)) \neq \phi$. Where ϕ is denoted the empty set. It is notable that the same concept is obtained if we substitute the word "closed" for "open" throughout the definition of quasicompatcness since, for inverse sets, the image of complement is the complement of the image. E. Helfer has presented the concept of property K in his note [1] as followings: A space X will be said to have *property* K at a point x if for each infinite subset A having x as an accumulation point, there is a compact subset of $A \cup \{x\}$ which has x as an accumulation point. To avoid repetition the fact a space has property K at each point of X is simply called by X has property K.

3. Result.

THEOREM. *Let $f(X)=Y$ be a quasicompact mapping of a Hausdorff space having property K onto a Hausdorff space Y. Then Y has property K if and only if f is a P_1-mapping.*

Proof. Suppose f is not a P_1-mapping and Y has property K. By the definition of P_1-mapping there exists a $y \in Y$ having a neighbourhood U of $f^{-1}(y)$ which satisfies $y \in int\ f(U)$. Thus y is an accumulation point of $Y-f(U)$ since for all neighbourhood V of y, $y \in f(U)$, $V \cap (Y-f(U)) \neq \phi$. Here $Y-f(U)$ is not a finite points set in Y. For if $Y-f(U)$ is a finite points set, then $Y-f(U)$ is a closed set in Y, because Y is a Hausdorff space. But y is an accumulation point of $Y-f(U)$ and $y \in f(U)$. This is a contradiction. Thus $Y-f(U)$ is not a finite points set. And y is an accumulation point of $Y-f(U)$. Since Y has property K, there is a compact subset E of $[Y-f(U)] \cup \{y\}$ and y is an accumulation point of E. E is closed in Y because Y is a Hausdorff space. So that $y \in E$. While $f^{-1}(E) \cap (X-U) = f^{-1}(E-y)$ is closed in X. For $f^{-1}(E-y) \subset f^{-1}(Y-f(U)) = X-f^{-1}f(U) \subset X-U$, and because of continuous mapping f, $f^{-1}(E)$ is a closed set in X. Since $X-U$ is a closed set in X, $f^{-1}(E-y)$ is a closed inverse set in X. Since f is a quasicompact mapping $E-y$ is closed in Y. But this is a contradiction to the fact y is an accumulation point of $E-y$. Therefore f is a P_1-mapping.

Now we prove the converse statement. Let B be an infinite points set and y be an accumulation point of B which does not belong to B. Then we can put $y \in \overline{B}-B$. Since f is a P_1-mapping we have $f^{-1}(y) \cap Cl(f^{-1}(B)) \neq \phi$. And $f^{-1}(y) \cap (Cl(f^{-1}(B)-f^{-1}(B)) \neq \phi$, because y does not belong to B. Thus there exists a point $x \in X$ such that $x \in f^{-1}(y) \cap [Cl(f^{-1}(B))-f^{-1}(B)]$ and having a net $\{x_\alpha\}$, which is a subset of $f^{-1}(B)$ and converges to x. Since X has property K there is a compact subset F of $\{x_\alpha\} \cup \{x\}$ and x is an accumulation point of F. Thus $f(F)$ is compact in Y and $y=f(x)$ is an accumulation point of $f(F)$, under a continuous mapping f. Therefore there is a compact subset $f(F)$ of $B \cup \{y\}$ which has y as an accumulation point of $f(F)$. Then Y has property K at y. In the case y is an accumulation point of B, whch is an infinite points set containinig y. We can prove the same result by using $B-y$ instesd of B in the above process.

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