이진 희소 신호의 L_0 복원 성능에 대한 상한치

Upper Bound for L_0 Recovery Performance of Binary Sparse Signals

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Abstract

In this paper, we consider a binary recovery framework of the Compressed Sensing (CS) problem. We derive an upper bound for L_0 recovery performance of a binary sparse signal in terms of the dimension N and sparsity K of signals, the number of measurements M. We show that the upper bound obtained from this work goes to the limit bound when the sensing matrix sufficiently become dense. In addition, for perfect recovery performance, if the signals are very sparse, the sensing matrices required for L_0 recovery are little more dense.

I. Introduction

The problem of the CS has been considered mainly in the field of real and complex systems. One of the key issues in CS problems is to minimize the number of measurements while unknown signals are perfectly recovered [1]. In this paper, we aim to find recovery bounds for CS problems over binary field. There are a couple of applications that this problem can be useful, including, i) problem of collecting data samples from a group of correlated sources [2], ii) minimization of file servers to contact in order to complete a download in a file sharing network [3]. We investigate the core question of CS problems again, but for a binary framework where all arithmetic operations are performed in the binary field or finite fields of size 2. To this end, we use an ideal L0 recovery routine for a goal of providing benchmark. For different sparseness of the sensing matrix whether how much dense or sparse of sensing matrices, we obtain the upper bounds with respect to a compression ratio (M/N) and a sparsity ratio (K/N). For further detail, refer to the paper [4] which deals with more general frameworks with finite fields of a lot of size. This paper shows some results of the paper [4] for a finite field of size 2.

Ⅱ. Upper Bounds for Recovery Performance

Let x be a sparse signal of length N with sparsity K. Let A be an $N\times M$ sensing matrix. The measured signal y is given by the linear equation as $y=Ax$. We assume that the elements of the sensing matrix A are independent and identically distributed, so that where γ denotes the sparse factor which is the probability of nonzero value of each element in the sensing matrix. Note that the error event of L0 minimization for decoding is the subset of the event ε defined as follows,

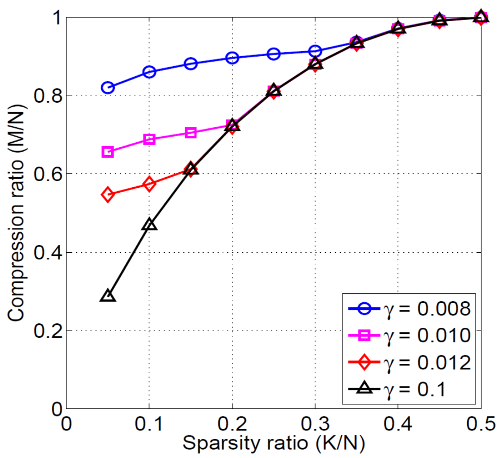
$$\varepsilon := \left\{(x,\bar{x}) : \|\bar{x}\|_0 \le \|x\|_0, \bar{x} \ne x, y = A\bar{x}\right\} \quad (1)$$

By enumerating all difference vectors into smaller sets each of which has the vectors with the same Hamming weights of h, i.e., $d_h = x - \bar{x}$, the upper bound of the error probability $\Pr\{\varepsilon\}$ can be written by

$$\Pr\{\varepsilon\} \le \binom{N}{K}^{-1} \sum_{h=1}^{2K} N_h \Pr\{Ad_h = 0\} \quad (2)$$

where N_h denotes the number of difference vectors with the same Hamming weights, $\Pr\{Ad_h=0\} = \Pr\{Ax = A\bar{x} | x\}$.

Ⅲ. Results and Conclusion



▶▶ Figure 1. Upper bounds for $\Pr\{\varepsilon\} < 10^{-1}$ of the error probability with the sparse factor γ at $N = 1000$.

Figure 1 shows the upper bounds of the error probability with respect to different sparse factors. It can be observed that in the region of the small sparsity ratio, a higher sparse factor is required to recover the sparse signals. In this case of $N = 1000$, for example, to recover the signals of the sparsity ratio (K/N) greater than 0.2, the sensing matrix having the number of nonzero entries greater than 10 in each row is needed. Once the sensing matrix is sufficiently dense, i.e., $\gamma > 0.1$, the upper bounds with different sparse factors are almost identical. Otherwise, the compression ratio grows as the sparse factor decreases.

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