Slope 보상을 가진 벅 LED 구동기의 모델링 및 해석

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Modeling and Analysis of Buck LED Driver with Slope Compensation

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ABSTRACT

A discrete time domain modeling for the current-mode-controlled buck LED driver is presented in this paper. Based on the modeling result, a root locus analysis for the buck LED driver with slope compensation is done to derive the stability boundaries of feedback gains.

1. Introduction

The luminous flux of LEDs is mostly determined by the LED forward current. The regulated constant current control is needed to achieve constant brightness of LEDs. In the current regulated converter like LED driver, the control signal v_c is not slow. Because there is no low pass filter for the output current. The instantaneous control signal v_c should be employed to describe the behavior of the duty-cycle modulator in the current regulated converter. State-of-the-art approach for designing the feedback loops cannot be used for the current regulation problem. Very little work has been done in the area of modeling and control to improve dynamic performance of the current regulated LED driver.

In this paper, the systematic discrete time domain approach is adapted to modeling and analysis for the duty-cycle-controlled buck LED driver shown in Fig. 1. Root locus analysis is employed to derive the stability boundaries.

2. Modeling of current-mode-controlled buck LED driver with slope compensation

$$\delta X_{k+1} = A \cdot \delta X_k + B \cdot \delta v_r \qquad (1)$$

where

$$\delta X_{k+1} = [\ \delta i_{k+1} \quad \delta v_{k+1}\ ]^T,\ \delta X_k = [\ \delta i_k \quad \delta v_k\ ]^T,$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},\ B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$

$$a_{11} = 1 - \frac{1}{(1-D)}\frac{1+k_p+k_{ni}D}{(1+k_p+k_{ni}D/2+S_r)}, a_{12} = \frac{1}{R_s}\frac{1}{(1-D)}\frac{1}{(1+k_p+k_{ni}D/2+S_r)},$$

$$a_{21} = R_s\frac{k_{ni}(k_{ni}D/2-S_r)}{(1+k_p+k_{ni}D/2+S_r)},\quad a_{22} = 1 - \frac{k_{ni}}{(1+k_p+k_{ni}D/2+S_r)},$$

$$b_1 = \frac{1}{R_s}\frac{1}{(1-D)}\frac{1+k_p+k_{ni}D}{(1+k_p+k_{ni}D/2+S_r)},\quad b_2 = \frac{k_{ni}(S_r-k_{ni}D/2)}{(1+k_p+k_{ni}D/2+S_r)},$$

$$D = V_o/V_i,\ k_p = \frac{R_1}{R_2},\ k_{ni} = \frac{T_s}{R_2C_1},\ S_r = \frac{M_e}{(V_oR_s/L)}\frac{D}{(1-D)} = S_{r0}\frac{D}{(1-D)}.$$

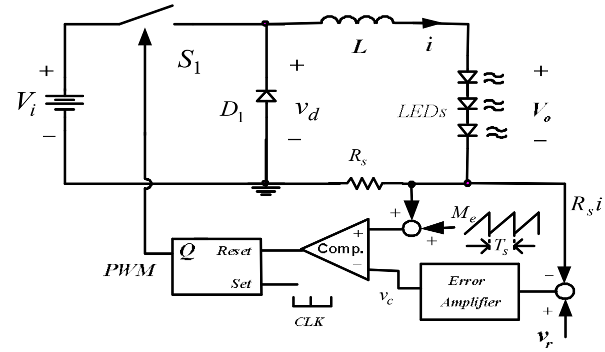

Fig. 1. Current-mode-controlled buck LED driver with slope compensation

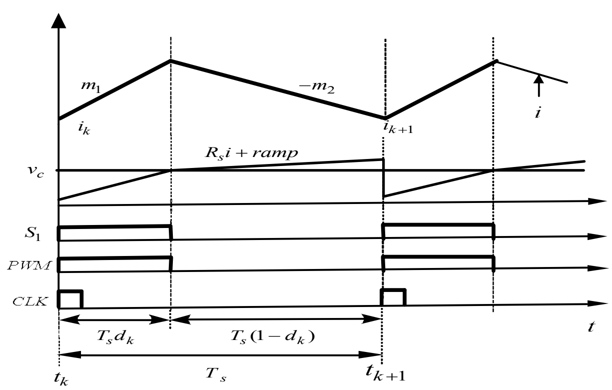

Fig. 2. Key theoretical waveforms of Fig. 1

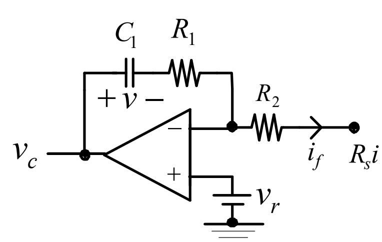

Fig. 3. Proportional-Integral error amplifier circuit

3. Analysis

Bode plots have been commonly used to assess the stability of the closed-loop system by finding the phase margin, but these plots cannot give information on the dynamic behavior of the individual state variables. On the other hand, root locus analysis can provide the engineer with the stability and the transient performance of the individual state variables related to the location of the roots of the characteristic equation.

To analyze the stability and dynamic characteristics of the closed-loop system, the eigenvalues of the system matrix is evaluated. The eigenvalues of A is the solutions of

$$|\mathbf{A} - \mathbf{zI}| = 0 \qquad (2)$$

where I is the identity matrix. The following root locus analysis is performed for $R_s = 1$.

The root locus as a function of the integral gain k_{ni} for $k_p = 0$, D=0.4, and $S_{r0} = 1.19$ is shown in Fig. 4. The relationship between the s-plane poles and the z-plane poles is $= e^{sT_s}|_{s=\sigma \pm jw} = e^{\sigma T_s}/\underline{\pm wT_s}$. When the I gain k_{ni} is increased from 0 to 0.49, the transient response is changed from overdamped to critically damped, and the overall system response becomes faster due to the slower eigenvalue λ_2 moving towards the origin of the unit circle. The transient response is underdamped when the I gain k_{ni} is greater than 0.49. Selecting k_{ni} greater than 4.43, the closed loop system becomes unstable. In the unstable mode, the oscillation frequency is equal to half of the switching frequency.

Fig. 5 shows stability boundaries of k_{ni} as a function of D with increasing S_{r0}. When k_{ni} is between zero and the stability boundary, the closed-loop system is stable. The closed-loop system is unstable for the integral gain k_{ni} greater than the stability boundary. While the duty-cycle-controlled buck LED driver is always unstable for $k_p = 0$[7] , this CMC buck LED driver with slope compensation can be stable by selecting a proper I gain for $k_p = 0$. The stability boundary of k_{ni} is increasing with increasing k_p for D <0.5. But, this stability boundary of k_{ni} is decreasing with increasing k_p for D > 0.5. The stable range of D becomes widened when S_{r0} is increased and k_p is decreased. When k_p is 0, the stable range of D between 0 and 1 can be designed if S_{r0} is greater than or equal to 0.9. Therefore, it can be said that the widest stable range of D can be obtained when k_p is 0. S_{r0} should be greater than or equal to 0.9 for the stable operation over all the operating area. However, the design engineer need to select the optimum k_{ni} for a good transient response.

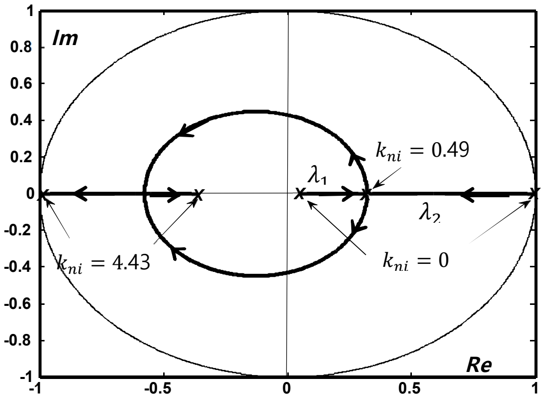

Fig. 4. Root locus as a function of k_{ni} ($k_p = 0, \mathrm{D} = 0.4, S_r = 0.79$)

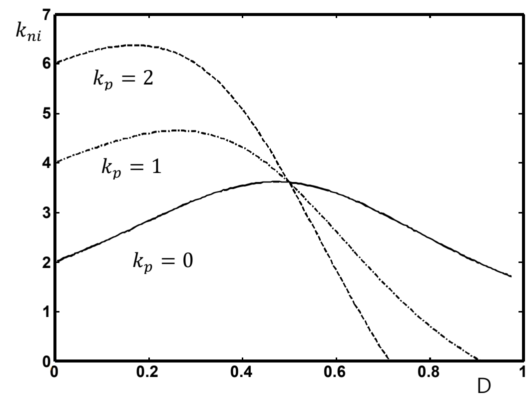

Fig. 5. Theoretical stability boundaries of k_{ni} as a function of D ($S_{r0} = 0.9$)

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