Analysis of Generalized n-winding Coupled Inductor in dc-dc Converters

Taewon Kang and Yongsug Suh

Dept. of Electrical Engineering, Smart Grid Research Center, Chonbuk National University

Abstract

This paper investigates the design of multi-winding coupled inductor for minimum inductor current ripple in rapid traction battery charger systems. Based on the general circuit model of multi-winding coupled inductor together with the operating principles of dc-dc converter, the relationship between the ripple size of inductor current and the coupling factor is derived under the different duty ratio. The optimal coupling factor which corresponds to a minimum inductor ripple current becomes -(1/*n*-1), i.e. a complete inverse coupling without leakage inductance, as the steady-state duty ratio operating point approaches 1/*n*, 2/*n*, … or (*n*-1)/*n*. In an opposite manner, the optimal coupling factor value of zero, i.e. zero mutual inductance, is required when the steady-state duty ratio operating point approaches either zero or one.

1. Introduction

Electric vehicle in a passenger car industry has become widely accepted in general public recently. The improvement of battery and charging technique has an important contribution in this wide acceptance by general public [1],[2]. In general, power electronics plays a significant role in battery charging technique including EV charging systems. In power electronic solutions of chargers, parallel connected multiple bidirectional dc-dc converters with active or diode bridge front-end rectifier are well adopted in industry. Phase-staggering operation of multiple bidirectional dc-dc converters is considered to reduce the ripple size of summed inductor currents and filtering requirement for the battery charging current. Reduction of inductor current ripple by a coupled inductor in the interleaving structure has been also proposed. A coupled inductor can decrease the physical size of inductor itself while still complying with the peak switching current requirements from power semiconductor switches in dc-dc converters. Design and analysis on two or multi-winding coupled inductors have been reported in previous literatures as summarized in Table I.

The coupling factor of coupled inductor has a significant impact on the phase-staggering operation of multiple bidirectional dc-dc converters. The selection of the optimal magnetic structure and coupling factor is regarded to be an important task in designing a coupled inductor. However, there has been a little work focusing on the optimal coupling factor of a coupled inductor and its relationship with the operation of dc-dc converters in previous literatures. Also, the investigation on the generalized *n*-winding coupled inductor with respect to the selection of coupling factor has not been fully covered. This paper investigates the design of coupled inductor for minimum inductor current ripple in rapid traction battery charger systems. The influence of coupling factor of coupled inductor on the operation of interleaved dc-dc converters is studied. The selection of optimal coupling factor under various operating conditions is also presented. As compared to previous literatures, this paper newly proposes a generalized approach to model n-winding coupled inductor and obtain the optimized coupling factor for the minimal inductor peak current.

TABLE I Summary of Previous Research Work about Coupled Inductor

	In this paper	[3]	[4]	[5]	[6]
Topology	Bi-directional dc-dc converter	Bi-directional dc-dc converter	3-level dc-dc converter	Buck converter	Boost converter
Number of winding	*n*-winding	*n*-winding	2 and 3	*n*-winding	2
Analysis method	nductor voltage equation	Inductor voltage equation	Pole voltage equation	Magnetic circuit equation	Inductor voltage equation
Design target	Inductor current ripple	Transient response, Current ripple	Inductor current ripple	Inductor current ripple	Analysis of CCM, DCM
Coupling factor (k)	Optimal k under each duty cycle	0 622 under 2-winding	0 832	0 33	None

2. Modeling of Coupled Inductor

Figure 1 and 2 show the schematic of rapid charger system considered in this paper. As shown in Fig. 2 power can flow in both directions within a Battery Charging Unit. The large ripple in battery charging current incurs stresses to a battery and eventually shortens the life time of battery. In order to reduce the ripple of battery charging current and the size of filter inductor, coupling of output inductors in multi-phase interleaved dc-dc converters is employed. In general, interleaved bi-directional dc-dc converter with the *n*-winding coupled inductor core structure is shown in Fig. 3. The coupled inductor has the symmetric magnetic structure. The equivalent circuit of coupled inductor is illustrated in Fig. 4. Equivalent circuit of coupled inductor is based on transformer equivalent circuit. The circuit basically consists of *n* leakage inductances and one mutual inductance. The core structure has *n* legs of radial type core so that the coupling factor between *n* windings can be adjusted by the air gap distance in the center leg. The number of core leg is proportional to the number of phase for the interleaved operation.

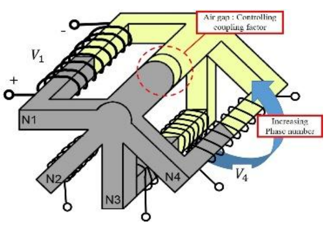

Fig 3 Magnetic structure of n-winding coupled inductor

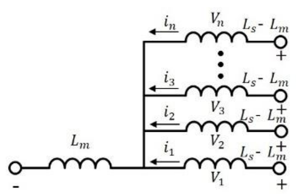

Fig 4 Equivalent circuit of n-winding coupled inductor

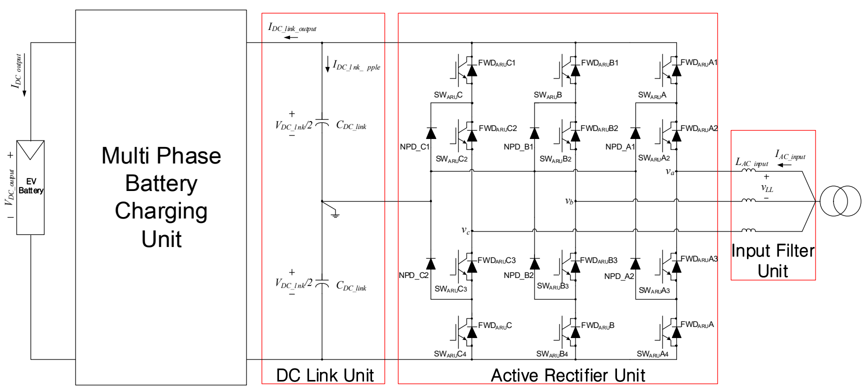

Fig 1 Multi-winding coupled inductor in rapid battery charging application

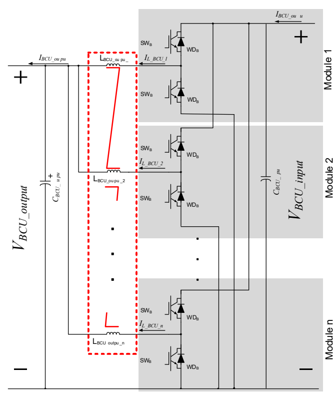

Fig 2 Bi-directional DC-DC converter

3. Design of Optimized Coupling Factor

Among many design factors related with a coupled inductor, the coupling factor between multiple windings plays an important role in determining various characteristics of coupled inductor such as a ripple size of inductor current. It is of high practical importance to find an optimized coupling factor which generates the least ripple size of inductor current under different operating conditions. Optimized coupling factor for the minimum inductor current ripple is derived based on the equivalent circuit of coupled inductor in Fig. 4 and the phase-staggering operation of paralleled multiple modules of buck converter in Fig. 2.

$$V_n = L_{nn}\frac{d}{dt}i_n + L_{n1}\frac{d}{dt}i_1 + L_{n2}\frac{d}{dt}i_2 + \ldots + L_{n(n-1)}\frac{d}{dt}i_{n-1} \quad (1)$$

$$k = \frac{L_m}{\sqrt{L_{11}L_{22}}} = \frac{L_m}{\sqrt{L_{11}L_{33}}} = \cdots = \frac{L_m}{\sqrt{L_{11}L_{nn}}} \quad (2)$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = L_s \begin{bmatrix} \frac{d}{dt}i_1 \\ \frac{d}{dt}i_2 \\ \vdots \\ \frac{d}{dt}i_n \end{bmatrix} + L_m \begin{bmatrix} 0 & \frac{d}{dt}i_2 & \cdots & \frac{d}{dt}i_n \\ \frac{d}{dt}i_1 & 0 & \cdots & \frac{d}{dt}i_n \\ \vdots & \vdots & & \vdots \\ \frac{d}{dt}i_1 & \frac{d}{dt}i_2 & \cdots & 0 \end{bmatrix} \quad (3)$$

$$\frac{(V_T - V_1) - (n-1)L_m\frac{d}{dt}i_1}{L_s + (n-2)L_m} = \frac{d}{dt}(i_2 + i_3 + \cdots + i_n) \quad (4)$$

The coupling factor (k) is the key design factor in a coupled inductor and is defined as in (3). It is noted from Fig. 3 that, under the condition of complete coupling of n-windings, the mutually coupled flux linkage becomes 1 / (n-1) of self-flux linkage due to n-legged symmetrical core structure. Therefore, the coupling factor k takes a value between –1 / (n-1) and 1 / (n-1).

$$(1 + (n-1)k)V_1 - kV_T = \ L_s(1 + (n-2)k - (n-1)k^2)\frac{d}{dt}i_1 \quad (5)$$

$$V_1 = L_s\frac{1 + (n-2)k - (n-1)k^2}{1 + (n-1-p(t))k + (n-p(t))\frac{D}{1-D}}\frac{d}{dt}i_1 \quad (6)$$

The sum of all winding voltages is defined as V_T.The sum of 2nd through nth row of (3) can be expressed as in (5) using V_T. Applying (4) into 1st row of (3) and employing (2) result in (5) after simplification. A new variable of $p(t)$ is introduced in this paper. This variable is defined to be the total number of switches being turned on while one particular pivotal switch is turned on. In this paper, SW_1 is selected to be a pivotal switch for the sake of simplicity. The relationship between switching states and the value of $p(t)$ is described in Fig. 5. Owing to this newly introduced variable of $p(t)$, the values of winding voltages in (5) can be greatly simplified into (6).

$$V_1 = L_{eq}(t)\frac{d}{dt}i_1 \quad (7)$$

$$\Delta i_L = \frac{V_{in}T_s}{Ls}(1-D)\left\{A\cdot(m-1)\left(\frac{m}{n} - D\right) + B\cdot m\cdot\left(D - \frac{m-1}{n}\right)\right\} \quad (8)$$

$$\text{A} = \frac{1 + (n-m)k + (n-m+1)\frac{n}{1-D}k}{1 + (n-2(k-(n-1)k^2} \quad (9)$$

$$\text{B} = \frac{1 + (n-1-m)k + (n-m)\frac{n}{1-D}k}{1 + (n-2(k-(n-1)k^2} \quad (10)$$

The waveform of inductor current of phase-1 under the example of 3-winding coupled inductor is described in Fig. 5. It is noted from this inductor current waveform that the total peak-to-peak ripple size of inductor current (i_1) can be represented by a sum of several incremental terms depending on the equivalent inductance and time duration of the particular switching time interval. In (10) – (12), the new parameter of m is defined. The value of m is determined by the duty ratio (D). The relationship between m and duty ratio (D) is explained in Fig. 6.

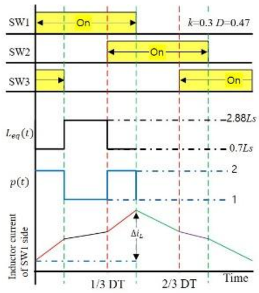

Fig 5 Switch mode and inductor current under 3-widing coupled inductor

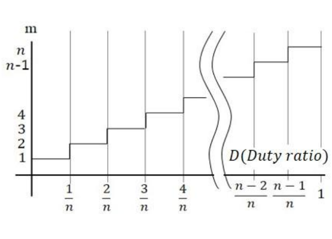

Fig 6 Relationship between m and duty ratio (D) of n-phase coupled inductor

$$k_{opt} = f(D, n, m) = -\frac{Y^2 - \sqrt{Y^2 - XZ}}{X} \quad (11)$$

$$\text{X} = D^2n^2 - D^2n - 2Dmn^2 + 2Dmn + Dn^3 - Dn^2 + m^2n - m^2 - mn + m \quad (12)$$

$$\text{Y} = D^2n^2 - D^2n - Dn^2 + Dn \quad (13)$$

$$\text{Z} = D^2n^2 - D^2n + 2Dn - 2Dmn + m^2 - m \quad (14)$$

4. System Verification

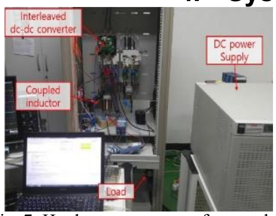

Fig 7 Hardware test set-up for coupled inductor testing

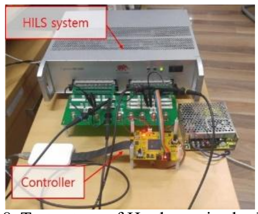

Fig 8 Test set-up of Hardware-in-the-loop simulator for coupled inductor testing

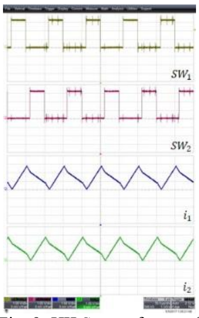

Fig 9 HILS waveforms of coupled inductor currents under duty = 0 4 and coupling factor = -0 3 (i_1, i_2 [2A/div, 50us/div])

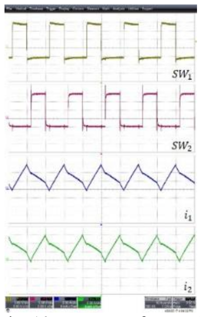

Fig 10 Exp waveforms of coupled inductor currents under duty = 0 4 and coupling factor = -0 3 (i_1, i_2 [2A/div, 50us/div])

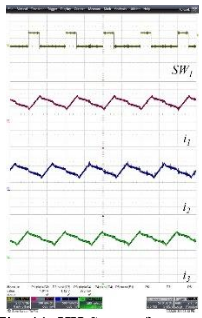

Fig 11 HILS waveforms of coupled inductor currents under duty = 0 3 and coupling factor = -0 3 (i_1,i_2,i_3[10A/div, 50us/div])

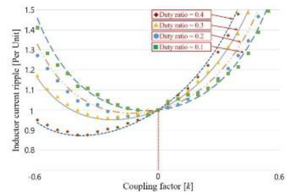

Fig 12 Graph of inductor current ripple vs coupling factor in 2-winding under duty < 0 5 [line: mathematical, dot: experiment]

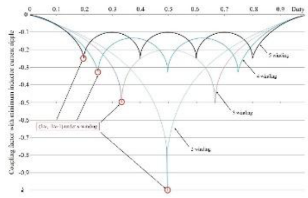

Fig 13 Graph of optimized coupling factor for inductor current ripple vs duty under four different inductor types (2, 3, 4, 5-winding cases)

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5. Conclusion

This paper investigates the design of coupled inductor for minimum inductor current ripple in rapid traction battery charger systems. Based on the general circuit model of coupled inductor together with the operating principles of dc-dc converter, the relationship between the ripple size of inductor current and the coupling factor is derived. Simulation and experimental result verify the theoretical derivation of the optimal coupling factor. The design guideline for selecting optimal coupling factor can be very useful in interleaved dc-dc converters. As a result, coupled inductors having optimal coupling factor can minimize the ripple current of inductor.

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