

# 직관내 기포의 흐름에 대한 2차원 수치 모의

## 2D Numerical Simulations of Bubble Flow in Straight Pipes

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### Abstract

Water aeration is an effective water treatment process, which involves the injection of air or air-water mixture into water treatment reservoir commonly through pipes. The main purpose of water aeration is to maintain healthy levels of dissolved oxygen (DO), which is the most important water quality factor. The pipes' operating conditions are important for controlling the efficiency and effectiveness of aeration process. Many studies have been conducted on two-phase flows in pipes, however, there are a few studies to deal with small scale in millimeter. The main objective of this study is to perform 2-dimensional two-phase simulations inside various straight pipes using the computational fluid dynamics (CFD) OpenFOAM (Open source Field Operation And Manipulation) tools to examine the influence of flow patterns on bubble size, which is closely related to DO concentration in a water body. The both flow regimes, laminar and turbulence, have been considered in this study. For turbulence, Reynolds-averaged Navier-Stokes (RANS) has been applied. The coalescence and breakage of bubbles caused by random collisions and turbulent eddies, respectively, are considered in this research. Sauter mean bubble diameter and water velocity are compared against experimental data. The simulation results are in good agreement with the experimental measurements.

*Key words:* 2D CFD Modeling, Two-Phase Pipe Flow, Small Scale Bubble Formation,

## 1. Introduction

Water aeration is an effective water treatment process, which involves the injection of air or air-water mixture into water most commonly through pipes. Through the process, dissolved oxygen (DO) concentration within a body of water can be increased or decreased. DO can be found in small scale bubbles that are mixed in water and it is an important parameter in assessing water quality. The main objective of this study is to perform 2-dimensional two-phase simulations inside various straight pipes using the computational fluid dynamics (CFD) OpenFOAM (Open source Field Operation And Manipulation) tools to examine the influence of flow patterns on bubble size, which is closely related to DO concentration in a water body. The simulations are performed using High Performance Computing (HPC) system to achieve fast and precise results from data-intensive calculations.

## 2. Mathematical Modeling

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## 2.1 RANS Governing Equations

The Reynolds-Averaged Navier - Stokes equations (or RANS equations) are time-averaged equations of motion for fluid flow. Each variable is decomposed into its time-averaged and fluctuating quantities and RANS equations are primarily used to describe behaviors of turbulent flows. Applying Reynolds time-averaging to the incompressible form of the Navier-Stokes equations leads to the RANS equations describing the time variation of mean flow quantities. the RANS describing the time-evolution of the mean flow quantities  $U_i$  and  $P$  can be written as:

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (2.1.1)$$

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} + \frac{1}{\rho} \frac{\partial P}{\partial x_i} = \frac{1}{\rho} \frac{\partial}{\partial x_j} (\bar{\tau}_{ij} + \lambda_{ij}) \quad (2.1.2)$$

where  $\bar{\tau}_{ij}$  is the fluid stress tensor evaluated in terms of the mean flow quantities and  $\lambda_{ij}$  is the Reynolds or turbulent stress tensor.

## 2.2 Turbulence Model: k- $\epsilon$

The OpenFOAM solver used in this research combines the population balance method with break-up and coalescence models in order to determine the bubble size of the dispersed phase. It uses the Eulerian-Eulerian two-fluid model with the k- $\epsilon$  turbulence model.

For turbulent kinetic energy,  $k$ :

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho k u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x_j} \right] + 2\mu_t E_{ij} E_{ij} - \rho \epsilon \quad (2.2.1)$$

For dissipation,  $\epsilon$ :

$$\frac{\partial(\rho \epsilon)}{\partial t} + \frac{\partial(\rho \epsilon u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \frac{\mu_t}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_j} \right] + C_{1\epsilon} \frac{\epsilon}{k} 2\mu_t E_{ij} E_{ij} - C_{2\epsilon} \rho \frac{\epsilon^2}{k} \quad (2.2.2)$$

where  $u_i$  represents the velocity component in corresponding direction,  $E_{ij}$ , the component of rate of deformation,  $\mu_t$ , the eddy viscosity ( $\mu_t = \rho C_\mu \frac{k^2}{\epsilon}$ ), and some adjustable constants:  $C_\mu = 0.09$ ,  $\sigma_k = 1.00$ ,  $\sigma_\epsilon = 1.30$ ,  $C_{1\epsilon} = 1.44$ , and  $C_{2\epsilon} = 1.92$ .

## 2.3 Bubble Coalescence and Breakup

The coalescence rate of bubbles of radii  $r_{bi}$  and  $r_{bj}$  ( $\Gamma_{ij}$ ) is given by the total collision frequency multiplied by the efficiency (Prince, 1990).

$$\Gamma_{ij} = (\theta_{ij}^T + \theta_{ij}^B + \theta_{ij}^{LS}) \times \exp(-t_{ij}/\tau_{ij}) \quad (2.2.1)$$

From the equation above, an expression for the overall coalescence rate can be obtained and it is the following (Prince, 1990).

$$\Gamma_T = \frac{1}{2} \sum_i \sum_j ((\theta_{ij}^T + \theta_{ij}^B + \theta_{ij}^{LS}) \times \exp(-t_{ij}/\tau_{ij})) \quad (2.2.2)$$

where  $\theta_{ij}^T$  represents the turbulent collision rate,  $\theta_{ij}^B$ , the bouyant collision rate,  $\theta_{ij}^{LS}$ , the collision rate due to laminar shear,  $t_{ij}$ , the time required for coalescence of bubbles of radius  $r_{bi}$  and  $r_{bj}$ , and  $\tau_{ij}$ , the contact time for the two bubbles.

The break-up rate for a bubble radius  $r_{bi}$  is given by (Prince, 1990):

$$\beta_i = \sum_e (\theta_{ie} \exp(-u_{ci}^2/u_{te}^2)) \quad (2.3.1)$$

The total break-up rate for all bubbles is (Prince, 1990):

$$\beta_T = \sum_i \sum_e \theta_{ie} \exp(-u_{ci}^2/u_{te}^2) \quad (2.3.2)$$

where  $\theta_{ie}$  represents the collision rate suggested by Kennard,  $u_{ci}$ , the critical eddy velocity, and  $u_{te}$ , the turbulent velocity of an eddy of radius  $r_e$ .

## 2.5 Sauter Mean Diameter

The Sauter Mean Diameter (d32) values are obtained using a bubble size tracking model called IATE (Interfacial Area Transport Equation) bubble diameter model. It solves for the interfacial curvature per unit volume of the phase rather than interfacial area per unit volume to avoid stability issues relating to the consistency requirements between the phase fraction and interfacial area per unit volume (OpenFOAM, 2016). The transport equations for the particle number, void fraction, and interfacial area concentration can be obtained respectively as [2.3.1 - 2.3.3] (Ishii, 2004):

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \nu_{pm}) = \sum_j R_j + R_{ph} \quad (2.3.1)$$

$$\alpha(x, t) = \int_{V_{\min}}^{V_{\max}} f(V, x, t) V dV \quad (2.3.2)$$

$$\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \nu_i) = \frac{2}{3} \left( \frac{a_i}{\alpha} \right) \left( \frac{\partial \alpha}{\partial t} + \nabla \cdot \alpha \nu_g - \eta_{ph} \right) + \frac{1}{3\psi} \left( \frac{\alpha}{a_i} \right)^2 \sum_j R_j + \pi D_{bc}^2 R_{ph} \quad (2.3.3)$$

where  $n$  represents the fluid particle number per unit mixture volume,  $\nu_{pm}$ , the average local particle velocity weighted by the particle number,  $R_j$ , the number source/sink rates due to particle interaction,  $R_{ph}$ , the number source rate due to the phase change,  $\alpha$ , gas void fraction,  $V$ , the fluid particle volume,  $a_i$ , the interfacial area concentration,  $\nu_i$ , fluid particle velocity,  $\nu_g$ , the gas velocity,  $\eta_{ph}$ , the rate of volume generated by nucleation source per unit mixture volume,  $\psi$ , the term accounts for the shapes of the fluid particles of interest, and  $D_{bc}$ , the critical bubble size.

## 3. Results

Table 3.1. Parameters

Figure Number	Superficial Gas Velocity (m/s)	Superficial Water Velocity (m/s)	Gas Volume Fraction
3.1 & 3.2	0.25	5.1	0.043
3.3 & 3.4	0.5	5.1	0.08
3.5 & 3.6	0.8	5.1	0.139
3.7 & 3.8	1.34	5.1	0.204

For each case, the time-averaged Sauter mean bubble diameter and water velocity values computed at the location  $L/D=253$  of 9 m straight pipes are compared against the experimental data of Kocamustafaogullari and Huang (1994). The relative mean errors of the Sauter Mean Diameter and axial water velocity are shown below.

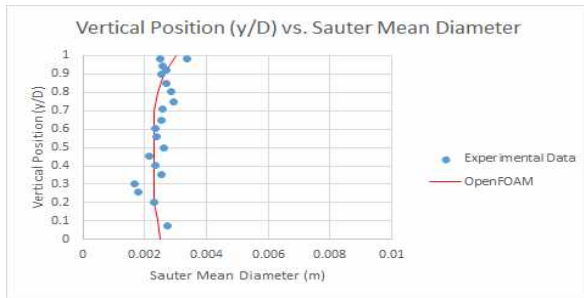


Fig 3.1. Sauter Mean Diameter Plot  
(Relative Mean Error: 14%)

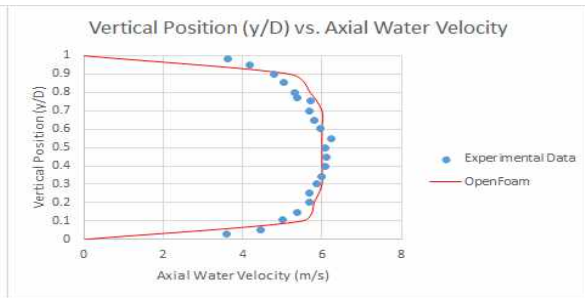


Fig 3.2. Axial Water Velocity Plot  
(Relative Mean Error: 4%)

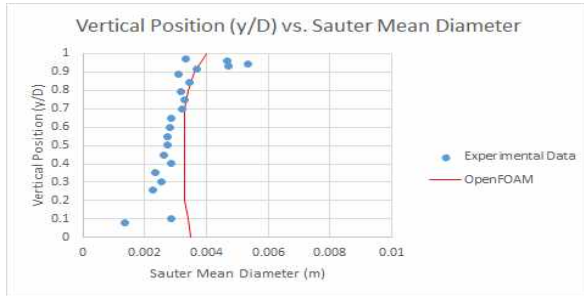


Fig 3.3. Sauter Mean Diameter Plot  
(Relative Mean Error: 26%)

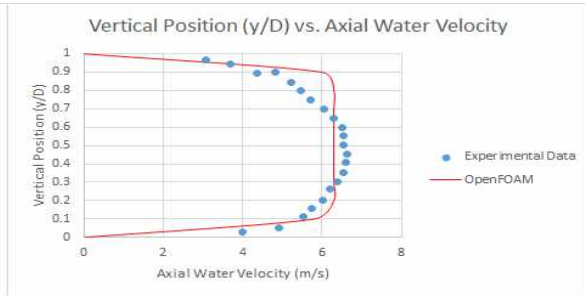


Fig 3.4. Axial Water Velocity Plot  
(Relative Mean Error: 9%)

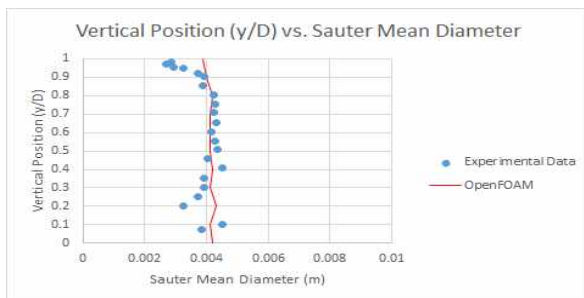


Fig 3.5. Sauter Mean Diameter Plot  
(Relative Mean Error: 14%)

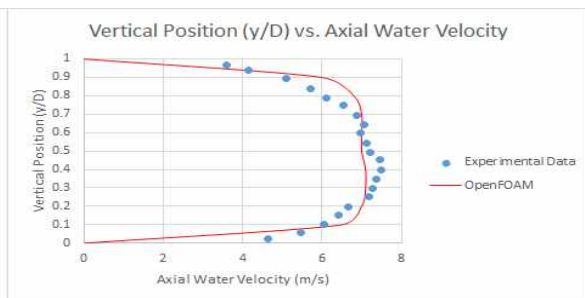
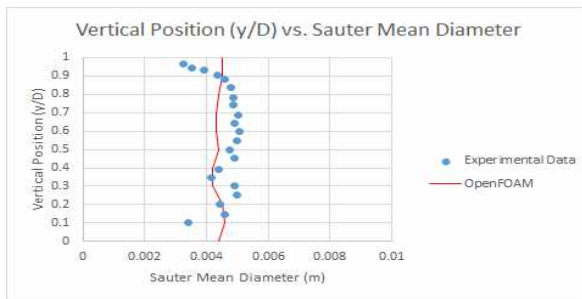
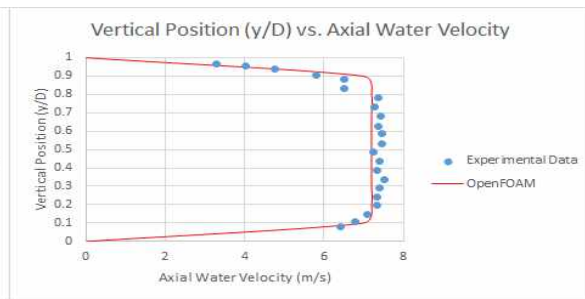


Fig 3.6. Axial Water Velocity Plot  
(Relative Mean Error: 7%)



**Fig 3.7. Sauter Mean Diameter Plot**  
(Relative Mean Error: 11%)



**Fig 3.8. Axial Water Velocity Plot**  
(Relative Mean Error: 8%)

## 4. Conclusion

The experimental data and the computational results show that the lower superficial gas velocity and gas volume fraction cause relatively smaller bubbles to form inside the straight pipes while the axial water velocity is kept constant. It is also shown that under these conditions, bubbly flow is formed inside the pipes and IATE diameter model is capable of capturing these bubbles and their diameters, but improvement is required for better prediction results.

## Reference

1. Ekambara, K., Sanders, R. S., Nandakumar, K., & Masliyah, J. H. (2012). CFD Modeling of Gas-Liquid Bubbly Flow in Horizontal Pipes: Influence of Bubble Coalescence and Breakup. *International Journal of Chemical Engineering*, 2012, 1-20. doi:10.1155/2012/620463
2. Ishii, M., & Kim, S. (2005). Development of Interfacial Area Transport Equation. *Nuclear Engineering and Technology.*, 37(6), 526-536. doi:10.13182/nse01-69
3. Kocamustafaogullari, G., & Huang, W. (1994). Internal structure and interfacial velocity development for bubbly two-phase flow. *Nuclear Engineering and Design*, 151(1), 79-101. doi:10.1016/0029-5493(94)90035-3
4. Luo, H., & Svendsen, H. F. (1996). Theoretical model for drop and bubble breakup in turbulent dispersions. *AIChE Journal AIChE J.*, 42(5), 1225-1233. doi:10.1002/aic.690420505.
5. OpenFOAM/OpenFOAM-2.3.x.(n.d.).Retrieved April 05, 2016, <https://github.com/OpenFOAM/OpenFOAM-2.3.x/blob/master/applications/solvers/multiphase/twoPhaseEulerFoam/twoPhaseSystem/diameterModels/IATE/IATE.H>
6. Prince, M. J., & Blanch, H. W. (1990). Bubble coalescence and break-up in air-sparged bubble columns. *AIChE Journal AIChE J.*, 36(10), 1485-1499. doi:10.1002/aic.690361004