Non-parametric Linear MMSE Filter in Wireless Ad-Hoc Networks

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ABSTRACT

In this paper, we propose a method pursuing robustness in ad hoc network system when the CSI of interferers is unavailable. The non-parametric linear minimum mean square error filter is exploited to achieve large fraction of the MMSE filter transmission capacity employing the perfect covariance matrix information. From the numerical results, we show that the proposed scheme brings substantial transmission capacity gain over conventional MMSE filter using sample covariance matrix.

1. INTRODUCTION

Recently, Jindal et al. proposed that the sample covariance matrix can be estimated by listening to the observations including interference and noise under the imperfect CSI assumption [1]. Although this scheme provides a relatively accurate covariance matrix, the data rate loss when the covariance matrix is sampled with the small observations is substantial. In this paper, we put forth an approach improving the network-wide throughput by employing non-parametric linear minimum mean square error (MMSE) receive filter. We show that the maximum SINR of the proposed method is equivalent to that of the conventional MMSE filter when the covariance matrix is perfectly estimated. Using this non-parametric MMSE filter, the proposed method achieves l

arge fraction of MMSE filter transmission capacity without transmission data rate loss.

2. SYSTEM MODEL AND NON-PARAMETRIC LINEAR MMSE FILTER

In ad hoc network, the transmitters are located according to a 2-D homogeneous Poisson point process (PPP) of density λ (interferers/m^2), and each receiver is randomly located at d meters away from the corresponding transmitter. Due to the stationarity of the Poisson process, we focus on a typical transmitter-receiver, denoted by Tx_d and Rx_d. In the perspective of Rx_d, the set of interferers also forms a homogeneous PPP, denoted by $A=\{(X_i,h_i),\lambda,i\in\mathbb{N}\}$ where X_i and h_i are the location and channel vector of the i-th transmitting node with respect to the typical receiver [2]. Under the frequency-flat channel model, the N-dimensional received signal y can be described by

$$\mathrm{y}=d^{-\alpha/2}\mathrm{h}_d s_d+\sum_{i\in A(\lambda)}|X_i|^{-\alpha/2}\mathrm{h}_i s_i+\mathrm{w} \tag{1}$$

where α is a path-loss exponent (α > 2). If a unit norm receive filter vd is employed, the estimated desired symbol becomes $\hat{s}_d=\mathrm{v}_d^H\mathrm{y}$ and hence resulting signal-to-interference-and-noise (SINR) ratio becomes

$$\mathrm{SINR}=\frac{\rho d^{-\alpha}\mathrm{v}_d^H\mathrm{h}_d\mathrm{h}_d^H\mathrm{v}_d}{\mathrm{v}_d^H(\sigma^2\mathrm{I}+\rho\sum_{i\in A(\lambda)}|X_i|^{-\alpha}\mathrm{h}_i\mathrm{h}_i^H)\mathrm{v}_d} \tag{2}$$

The outage probability at SINR threshold β is $P_{out}(\lambda)$ = P[SINR ≤ β], the maximum interferer density is $\lambda_\epsilon=\max\{\lambda:\mathrm{P}_{\mathrm{out}}(\lambda)\le\epsilon\}$ where ϵ is outage constraint, and the transmission capacity of the ad hoc network is $C(\epsilon)=\lambda_\epsilon(1-\epsilon)\log_2(1+\beta)$ bps/Hz/m^2 [1].

It is well known that the MMSE filter optimally pursues balance between signal boost and interference suppression for maximizing the SINR [1]. The normalized MMSE receive filter is given by $\mathrm{v}_d=\Sigma^{-1}\mathrm{h}_d/\|\Sigma^{-1}\mathrm{h}_d\|$ where $\Sigma=\frac{1}{\mathrm{SNR}}\mathrm{I}+d^{\alpha}\sum_{i\in A(\lambda)}|X_i|^{-\alpha}\mathrm{h}_i\mathrm{h}_i^H$ is the spatial co-variance of the interference plus noise and $\mathrm{SNR}=\frac{\rho d^{-\alpha}}{\sigma^2}$. Plugging v_d into (2), the maximum received SINR of the MMSE filter becomes $\mathrm{SINR}_{\mathrm{MMSE}}=\mathrm{h}_d^H\Sigma^{-1}\mathrm{h}_d$.

The MMSE with imperfect CSI [1] estimates the sampled covariance matrix by listening to interferer transmissions in the absence of desired signal. If the desired transmitter remains inactive for K symbols, the receiver can employ the K observations to form the sample covariance as

$$\hat{\Sigma}=\frac{1}{K}\sum_{i=1}^{K}\mathrm{r}_i\mathrm{r}_i^H \tag{3}$$

where r_i represents the i-th observation of the noise plus interference. By replacing $\hat{\Sigma}$ with Σ, the resulting SINR becomes $\mathrm{SINR}=\frac{(\mathrm{h}_d^H\hat{\Sigma}^{-1}\mathrm{h}_d)^2}{\mathrm{h}_d^H\hat{\Sigma}^{-H}\Sigma\hat{\Sigma}^{-1}\mathrm{h}_d}$. Under the assumption that all interferers transmit independent Gaussian symbols, the expected SINR with respect to the $\hat{\Sigma}$ distribution is [3]

$$E_{\hat{\Sigma}}\left[\frac{(\mathrm{h}_d^H\hat{\Sigma}^{-1}\mathrm{h}_d)^2}{\mathrm{h}_d^H\hat{\Sigma}^{-H}\Sigma\hat{\Sigma}^{-1}\mathrm{h}_d}\right]=\left(1-\frac{N-1}{K+1}\right)\mathrm{h}_d^H\Sigma^{-1}\mathrm{h}_d \tag{4}$$

Note that the expected SINR of the MMSE with imperfect CSI is smaller than that of MMSE based on the perfect CSI due to the scaling factor $1-\frac{N-1}{K+1}$. Also, when the number of observations is very large, $\hat{\Sigma}$ becomes close to Σ.

In practice, the receiver is hard to employ the parametric MMSE filter (??) since it is not easy for the receiver estimates CSI of all the interferers in a decentralized network. Although the MMSE with imperfect CSI method yields a relatively exact covariance matrix and achieves the large fraction of the perfect CSI transmission capacity [1], it has the following drawbacks. First, the desired transmitter should be in inactive mode for K symbol durations when the receiver estimates K observations. Second, if the channel (under block fading) is changing per T symbol period,

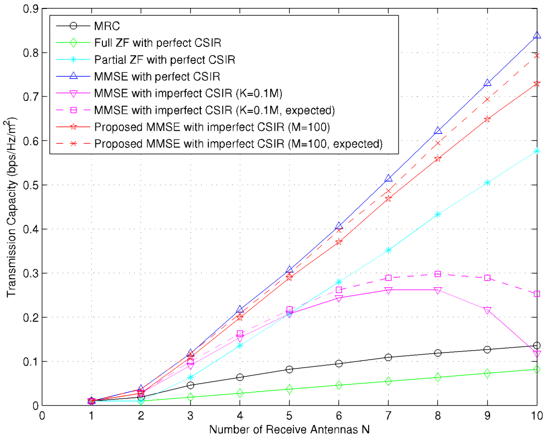

Fig. 1 Transmission Capacity versus N for ϵ = 0.1, β = 1, α = 3, d = 1, and K = 10.

then effective data rate is reduced by the factor of $\frac{T-K}{T}$.

As a way to overcome these shortcomings, we propose the receive filter based on the non-parametric linear MMSE estimation. In the proposed method, we will show that the covariance matrix can be replaced with autocorrelation of received signal $\mathbf{y}$ including desired channel information $\mathbf{h}_d$.
From [4], the output of the linear MMSE is

$$\hat{s}_d = \mathrm{v}_d^H \mathrm{y} = \mathrm{R}_{s_d y} \mathrm{R}_{yy}^{-1} \mathrm{y} = \mathrm{h}_d^H (\Sigma + \mathrm{h}_d \mathrm{h}_d^H)^{-1} \mathrm{y} \quad (5)$$

Regardless of the inclusion of desired channel information, we prove that the linear MMSE filter in (5) can achieve the maximum SINR of conventional MMSE filter in the following theorem.

Theorem 2.1: The linear MMSE filter using non-parametric autocorrelation R_{yy} is

$$\mathrm{v}_d = (\Sigma + \mathrm{h}_d \mathrm{h}_d^H)^{-1} \mathrm{h}_d \quad (6)$$

and the corresponding SINR is $\mathrm{SINR} = \mathrm{h}_d^H \Sigma^{-1} \mathrm{h}_d$

proof 2.2: From (2), the SINR of the MMSE filter (6) can be rewritten as

$$\mathrm{SINR} = \frac{(\mathrm{h}_d^H (\Sigma + \mathrm{h}_d \mathrm{h}_d^H)^{-1} \mathrm{h}_d)^2}{\mathrm{h}_d^H (\Sigma + \mathrm{h}_d \mathrm{h}_d^H)^{-1} \Sigma (\Sigma + \mathrm{h}_d \mathrm{h}_d^H)^{-1} \mathrm{h}_d} \quad (7)$$

By the Sherman-Morrison formula [4], an inverse of covariance matrix $(\Sigma + \mathrm{h}_d \mathrm{h}_d^H)$ is

$$(\Sigma + \mathrm{h}_d \mathrm{h}_d^H)^{-1} = \Sigma^{-1} - \frac{\Sigma^{-1} \mathrm{h}_d \mathrm{h}_d^H \Sigma^{-1}}{1 + \mathrm{h}_d^H \Sigma^{-1} \mathrm{h}_d} \quad (8)$$

Let $\Sigma_h = \mathrm{h}_d^H \Sigma^{-1} \mathrm{h}_d$, then (7) becomes

$$\mathrm{SINR} = \frac{\Sigma_h^2}{\Sigma_h} = \Sigma_h = \mathrm{h}_d^H \Sigma^{-1} \mathrm{h}_d \quad (9)$$

Theorem 2.1 tells us that we can achieve the maximum transmission capacity of the linear MMSE filter (6) even with the variation of covariance matrix. However, since the exist of R_{yy}^{-1} is not always guaranteed, we will find the alternative form of the non-parametric linear MMSE filter.

From (8), we can find that

$$\mathrm{v}_d^H = (1 + \Sigma_h)^{-1} \mathrm{h}_d^H \Sigma^{-1} \quad (10)$$

Using the eigendecomposition [5], (10) can be expressed as

$$\mathrm{v}_d^H = (\mathrm{h}_d^H \mathrm{R}_A^{\dagger} \mathrm{h}_d)^{-1} \mathrm{h}_d^H \mathrm{R}_A^{\dagger} \quad (11)$$

Since $\mathrm{R}_{yy}^{\dagger} = \mathrm{R}_A^{\dagger}$ when $\frac{1}{\mathrm{SNR}} \to 0$, (6) can be reduced to

$$\mathrm{v}_d = \hat{\mathrm{R}}_{yy}^{\dagger} \mathrm{h}_d (\mathrm{h}_d^H R_{yy}^{\dagger} \mathrm{h}_d)^{-1} \quad (12)$$

where $\hat{\mathrm{R}}_{yy} = \frac{1}{M} \mathrm{Y}\mathrm{Y}^H$ is the sample correlation matrix obtained from the received signal set $\mathrm{Y} = [\mathrm{y}_1, \mathrm{y}_2, \cdots \mathrm{y}_M]$.
In order to obtain an accurate sample covariance matrix in (4), the number of observations K should be very large. To do so, the transmitter should remain in inactive mode for K symbol duration. Whereas, no such requirement is necessary for the proposed approach in (12). In contrast to the fact that K is the overhead in the packet transmission, M in (12) can be freely chosen within the range of the packet length. Due to the fact that the length of the packet is typically on the order of hundreds of symbols, the expected SINR and transmission capacity using $\hat{\mathrm{R}}_{yy}$ is larger than employing $\hat{\Sigma}$.

3. SIMULATION AND DISCUSSION

The simulation setup is based on the 2-D PPP transmitters. In Fig. 2, we plot the transmission capacity as a function of N. Note that K = 10 (10% of packet length) and M is the packet length. In particular, although the proposed filter leaves a performance gap from MMSE, the transmission capacity of the proposed method is larger than the MMSE with K samples and PZF, and the gain gets larger as N increases. Due to the scaling factor of $1 - \frac{N-1}{K+1}$, the expected SINR of the MMSE with K samples is smaller than that of the MMSE with full CSI, the transmission capacity of the MMSE with K samples is decreased when K is a fixed number and N goes to large number.

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