Non-parametric Linear MMSE Filter in Wireless Ad-Hoc Networks

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**ABSTRACT**

In this paper, we propose a method pursuing robustness in ad hoc network system when the CSI of interferers is unavailable. The non-parametric linear minimum mean square error filter is exploited to achieve large fraction of the MMSE filter transmission capacity employing the perfect covariance matrix information. From the numerical results, we show that the proposed scheme brings substantial transmission capacity gain over conventional MMSE filter using sample covariance matrix.

1. INTRODUCTION

Recently, Jindal et al. proposed that the sample covariance matrix can be estimated by listening to the observations including interference and noise under the imperfect CSI assumption [1]. Although this scheme provides a relatively accurate covariance matrix, the data rate loss when the covariance matrix is sampled with the small observations is substantial. In this paper, we propose a method equivalent to that of the conventional MMSE method employing non-parametric linear minimum mean square error (MMSE) receive filter. We show that the maximum SINR of the proposed method is equivalent to that of the conventional MMSE filter when the covariance matrix is perfectly estimated. Using this non-parametric MMSE filter, the proposed method achieves large fraction of the MMSE filter transmission capacity without transmission data rate loss.

2. SYSTEM MODEL AND NON-PARAMETRIC LINEAR MMSE FILTER

In ad hoc network, the transmitters are located according to a 2-D homogeneous Poisson point process (PPP) of density \( \lambda \) (interferers/m), and each receiver is randomly located at 0 meters away from the corresponding transmitter. Due to the stationarity of the Poisson process, we focus on a typical transmitter-receiver, denoted by \( T_a \) and \( R_a \). In the perspective of \( R_a \), the set of interferers also forms a homogeneous PPP, denoted by \( A = \{(X_i, h_i), \lambda, i \in N\} \) where \( X_i \) and \( h_i \) are the location and channel vector of the \( i \)-th transmitting node with respect to the typical receiver [2].

Under the frequency-flat channel model, the \( N \)-dimensional received signal can be described by

\[
y = d^{\alpha/2} h_d s_d + \sum_{i \in A} |X_i|^{-\alpha/2} h_i s_i + w \tag{1}
\]

where \( \alpha \) is a path-loss exponent (\( \alpha > 2 \)). If a unit norm receive filter \( v_d \) is employed, the estimated desired symbol becomes \( s_d = v_d^H y \) and hence resulting signal-to-interference-and-noise (SINR) ratio becomes

\[
\text{SINR} = \frac{\rho d^{-\alpha} |v_d^H h_d|^2}{\|v_d^H (\sigma^2 I + \rho \sum_{i \in A} |X_i|^{-\alpha} h_i h_i^H) v_d \|} \tag{2}
\]

The outage probability at SINR threshold \( \beta \) is \( P_{out}(\lambda) = P[\text{SINR} \leq \beta] \), the maximum interferer density is \( \lambda = \max \{ \lambda : P_{out}(\lambda) \leq \epsilon \} \) where \( \epsilon \) is outage constraint, and the transmission capacity of the ad hoc network is

\[
C(\epsilon) = \lambda (1 - \epsilon) \log_2(1 + \beta) \text{bps/Hz/m}^2 \tag{1}
\]

It is well known that the MMSE filter optimally pursues balance between signal boost and interference suppression for maximizing the SINR [1]. The normalized MMSE receive filter is given by \( v_d = \Sigma^{-1} h_d / \| \Sigma^{-1} h_d \| \), where

\[
\Sigma = \frac{1}{\text{SINR}} \mathbb{1} + d^\alpha \sum_{i \in A(\lambda)} |X_i|^{-\alpha} h_i h_i^H \tag{2}
\]

which is the spatial co-variance of the interference plus noise and \( \text{SINR} = \frac{\rho d^{-\alpha}}{\|v_d^H (\sigma^2 I + \rho \sum_{i \in A} |X_i|^{-\alpha} h_i h_i^H) v_d \|} \). Plugging \( v_d \) into (2), the maximum received SINR of the MMSE filter becomes

\[
\text{SINR}_{\text{MMSE}} = h_d^H \Sigma^{-1} h_d \tag{3}
\]

The MMSE with imperfect CSI [1] estimates the sample covariance matrix by listening to interferer transmissions in the absence of desired signal. If the desired transmitter remains inactive for \( K \) symbol durations when the receiver estimates \( K \) observations. Second, if the channel (under block fading) is changing per \( T \) symbol period,
Using the eigendecomposition \([5]\), (10) can be expressed as

\[
\mathbf{v}_d = (1 + \Sigma_h)^{-1} h_d \Sigma^{-1}
\]

Since \( \mathbf{R}_y = \mathbf{R}_y^\dagger \) when \( \frac{1}{\text{SNR}} \rightarrow 0 \), (9) can be reduced to

\[
v_d = \mathbf{R}_y^\dagger h_d (\mathbf{h}_d^H \mathbf{h}_d)^{-1}
\]

where \( \mathbf{R}_y = \frac{1}{M} \mathbf{Y} \mathbf{Y}^H \) is the sample correlation matrix obtained from the received signal set \( \mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_M] \).

In order to obtain an accurate sample covariance matrix in (4), the number of observations \( K \) should be very large. To do so, the transmitter should remain in inactive mode for \( K \) symbol duration. Whereas, no such requirement is necessary for the proposed approach in (12). In contrast to the fact that \( K \) is the overhead in the packet transmission, \( M \) in (12) can be freely chosen within the range of the packet length. Due to the fact that the length of the packet is typically on the order of hundreds of symbols, the expected SINR and transmission capacity using \( \mathbf{R}_y \) is larger than employing \( \Sigma \).

3. SIMULATION AND DISCUSSION

The simulation setup is based on the 2-D PPP transmitters. In Fig. 2, we plot the transmission capacity as a function of \( N \). Note that \( K = 10 \) (10\% of packet length) and \( M \) is the packet length. In particular, although the proposed filter leaves a performance gap from MMSE, the transmission capacity of the proposed method is larger than the MMSE with \( K \) samples and PZF, and the gain gets larger as \( N \) increases. Due to the scaling factor of \( 1 - \frac{K}{K+1} \), the expected SINR of the MMSE with \( K \) samples is smaller than that of the MMSE with full CSI. The transmission capacity of the MMSE with \( K \) samples is decreased when \( K \) is a fixed number and \( N \) goes to large number.

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