

사물 인터넷의 최적화를 위한 행렬 완성 알고리즘

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Matrix Completion Algorithm for Internet of Things Localization

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요 약

In this paper, we propose a matrix completion algorithm for Internet of Things (IoT) localization. The proposed algorithm recovers the Gram matrix of sensors by performing optimization over the Riemannian manifold of fixed-rank positive semidefinite matrices. We compute and show the closed forms of all the differentially geometric components required for applying nonlinear conjugate gradients combined with Armijo line search method. The numerical experiments show that the performance of the proposed algorithm in solving IoT localization is outstanding compared with the state-of-the-art matrix completion algorithms both in noise and noiseless scenarios.

1. Introduction

Recently, Internet of Things (IoT) has received much attention for its plethora of applications, such as healthcare, surveillance, automatic metering, environmental monitoring, to name just a few. In sensing the environmental information wireless sensor network consisting of hundreds to thousands sensor nodes is often used. In order to interpret the environmental information and then make a proper reaction, the data center (e.g., access point, basestation) should have the location of sensor nodes [1, 2]. Traditionally, location information has been obtained at the sensor node via GPS-based triangulation. For the IoT network using small, cheap, and low-power sensor nodes, however, GPS-based approach is not so appealing.

In recent years, an approach to identify the location information at the data center has been received much attention. In this approach, basically, each sensor node measures the distance information of adjacent nodes and then send it to the data center. Once obtaining the distance information, the data center use multidimensional scaling (MDS) methods [3]. In measuring the distance, various physical layer

techniques have been used including RSSI, TDoA, and AoA. Since these techniques are based on terrestrial communication, they have many benefits over the GPS technology (e.g., cost and complexity).

When implementing this approach, one well-known concern is that the available distance information at the data center is limited due to various reasons including the power outage of the sensor node or limitation of radio communication range, obstructing the accurate identification of location information. The practical limitation of sensors motivates the design of efficient algorithms for reconstructing all the pairwise distances from its small subset so that MDS can then be used to find the location of sensors.

In this paper, we propose the matrix completion algorithm to find the location of sensors. The proposed algorithm applies the generalization of nonlinear conjugate gradient (CG) method over the Riemannian manifold of fixed-rank symmetric positive semidefinite matrices. In term of well-defined Riemannian manifold, following the framework of retractionbased optimization [4, 5], we show that the proposed algorithm can recover the missing pairwise distances and also compute the location of sensor nodes with high accuracy.

We briefly summarize notations used in this paper. $\text{diag}(\mathbf{A})$ is the vector formed by the main diagonal of \mathbf{A} , and $\text{Sym}(\mathbf{A})$ is the matrix formed by $\text{Sym}(\mathbf{A}) = 1/2(\mathbf{A} + \mathbf{A}^T)$ for any square matrix \mathbf{A} . The matrix $\text{eye}(\mathbf{a})$ is the adjoint operator of $\text{diag}()$.

$E = \{(i, j) : \|x_i - x_j\|_2 \leq r\}$ is the set of the observed indices, and the sampling operator P_E given by $[P_E(\mathbf{A})]_{ij} = A_{ij}$ if $(i, j) \in E$, and zero otherwise. The observed matrix is $D_{\text{obs}} = P_E(D)$.

2. System Model and Problem Description

Let $x_i \in \mathbb{R}^k$ ($i = 1, \dots, n$) be the coordinate vectors of n sensor nodes randomly distributed in k -dimensional Euclidean space (typically $k = 2$ or 3), and the coordinate matrix $X = [x_1 \ x_2 \ \dots \ x_n]^T$. Then the Euclidean distance matrix $D \in \mathbb{R}^{n \times n}$ is defined as

$$D = \begin{bmatrix} 0 & d_{12}^2 & \dots & d_{1n}^2 \\ d_{21}^2 & 0 & \dots & d_{2n}^2 \\ \dots & \dots & \dots & \dots \\ d_{n1}^2 & d_{n2}^2 & \dots & 0 \end{bmatrix}$$

where $d_{ij} = \|x_i - x_j\|_2^2$ is the distance between two nodes x_i and x_j . It can be shown that $\text{rank}(D) \leq k + 2$, and that $D = \kappa(Y)$ where $Y = XX^T$ and $\kappa(Y) = \mathbf{1}\text{diag}(Y)^T + \text{diag}(Y)\mathbf{1}^T - 2Y$.

Thus, one can reconstruct D by solving

$$(P_1) \begin{cases} \min_{Y \in S_+^{n \times n}} \frac{1}{2} \|P_E(\kappa(Y)) - D_{\text{obs}}\|_F^2 \\ \text{s.t.} \quad \text{rank}(Y) \leq k \end{cases}$$

The main idea of (P_1) is to reconstruct D by searching a low rank matrix $Y \in S_+^{n \times n}$ such that $P_E(\kappa(Y))$ is consistent with D_{obs} . From the eigenvalue decomposition $Y = Q\Lambda Q^T$, we obtain the coordinate matrix $X = Q\Lambda^{1/2}$. Note that due to the preservation of distance, there are many coordinate matrix X satisfying $D = \kappa(Y) = \kappa(XX^T)$, and that our target is to find one of them. In the sequel, we let $f(\tilde{Y}) = \frac{1}{2} \|P_E(\kappa(\tilde{Y})) - D_{\text{obs}}\|_F^2$. By defining $M_k^+ = \{Y \in S_+^{n \times n} : \text{rank}(Y) = k\}$, the problem (P_1) is recast as $(P_2) \min_{\tilde{Y} \in M_k^+} f(\tilde{Y})$.

3. The Proposed Algorithm

The key idea of the proposed algorithm is to sequentially find the solution \tilde{Y} of (P_2) using the generalization of nonlinear conjugate gradient method with Armijo line search based on the framework of retraction-based optimization [4, 5]. Once obtaining the Gram matrix Y , we then compute X immediately using the eigenvalue decomposition of Y . The proposed algorithm is summarized in Table 1. Due to the page limitation, we skip the proofs of lemmas and propositions (see [6] for details).

3.1. The Riemannian Manifold M_k^+

Let $St_k^+ = \{Q \in \mathbb{R}^{n \times k} : Q^T Q = I_k\}$

be defined as the Stiefel manifold and let

$$L = \{\text{eye}([\sigma_1 \ \dots \ \sigma_k]^T) : \sigma_1 \geq \dots \geq \sigma_k > 0\}.$$

Then we can represent

$$M_k^+ = \{Q\Lambda Q^T : Q \in St_k^+, \Lambda \in L\}$$

The smooth manifold structure of M_k^+ as well as the closed form of its tangent space is shown in the following proposition. Note that this proposition is a reformulation of Proposition 2.1 in [4] in terms of the newly defined manifold.

Proposition 3.1. The set M_k^+ is a smooth submanifold of dimension $1/2(2n - k + 1)k$ embedded in $\mathbb{R}^{n \times n}$. Its tangent space $T_Y M_k^+$ at $Y = Q\Lambda Q^T$ is given by

$$\begin{aligned} T_Y M_k^+ &= \left\{ [Q \ Q_\perp] \begin{bmatrix} B & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} Q^T \\ Q_\perp^T \end{bmatrix} \right\} \\ &= \{QBQ^T + QQ^T B + Q_\perp C^T : B \in \mathbb{R}^{k \times k}, B = B, Q_\perp \\ &= Q_\perp C, C \in \mathbb{R}^{(n-k) \times k}\} \end{aligned}$$

And the projection $P_{T_Y M_k^+}(A)$ of a matrix A is computed as

$$P_{T_Y M_k^+}(A) = P_Q \text{Sym}(A) + \text{Sym}(A)P_Q - P_Q \text{Sym}(A)P_Q$$

where we define $P_Q = QQ^T$.

In order to perform optimization over M_k^+ , we equipped M_k^+ with the Riemannian metric

$$g_Y = \langle \xi_1, \xi_2 \rangle, \quad (2)$$

where $\xi_1, \xi_2 \in T_Y M_k^+$. Further, we introduce the metric projection R_Y as a local smooth mapping from the tangent space $T_Y M_k^+$ to M_k^+ ,

$$R_Y: \xi \rightarrow R_Y(\xi) \triangleq P_{M_k^+}(Y + \xi) \\ = \arg \min_{Z \in M_k^+} \|Y + \xi - Z\|_F$$

It can be shown that R_Y satisfies the definition of the retraction in [7], based on Lemma 2.2 in [8].

3.2. Nonlinear CG Method

Nonlinear CG method solves the problem (P₂) iteratively. The update equation is $Y_{i+1} = R_{Y_i}(\alpha_i \eta_i)$ where α_i is the step size, and $\eta_i \in T_{Y_i} M_k^+$ is the search direction. The search direction η_i is the linear combination of the previous search direction η_{i-1} and the Riemannian gradient defined as the tangent vector $grad(Y) \in T_{Y_i} M_k^+$ satisfying

$$g_Y \langle \xi, grad f(Y) \rangle = \langle \xi, \nabla_Y f(Y) \rangle,$$

for any tangent vector $\xi \in T_Y M_k^+$, and $\nabla_Y f(Y)$ is the Euclidean gradient determined in the following lemma.

Lemma 3.2. The Euclidean gradient of $f(Y)$ is $\nabla_Y f(Y) = 2eye(Sym(Z)1) - 2Z$ where $Z = 2P_E(Sym(1diag(Y)^T) - Y) - D_{obs}$.

We transport η_{i-1} to $T_{Y_i} M_k^+$ using the mapping $T_{Y_{i-1} \rightarrow Y_i}$ defined in [4].

Table 1. The proposed algorithm

Input: D_{obs} , P_E , τ : tolerance, $\beta \in (0, 1)$ T: number of iterations.

Initialize: $Y_1 \in M_k^+$, $i = 1$, tangent vector η_0 .

While $i \leq T$ do

1: Compute the Riemannian gradient ξ_i

2: Compute a conjugate direction η_i

3: Use Armijo line-search to find the step size α_i

4: $Y_{i+1} = R_{Y_i}(\alpha_i \eta_i)$

5: $D_{i+1} = 2[\text{Sym}(1diag(Y_{i+1})^T) - Y_{i+1}]$

If $\|P_E(D_{i+1}) - D_{obs}\|_F \leq \tau$ then break

6: $i = i + 1$

End

7: Obtain the eigen-decomposition $\tilde{Y} = Q\Lambda Q^T$

8: $\tilde{X} = Q\Lambda^{\frac{1}{2}}$

Output: \tilde{X}

In determining the step size α_i we use a fast line search scheme so called Armijo's rule, i.e. $\alpha_i = 0.5^m t_i$ where t_i is the initial step size and $m \geq 0$ is the smallest integer satisfying

$$f(Y_i) - f(R_{Y_i}(0.5^m t_i \eta_i)) \geq -0.5^m t_i \tau \langle \xi_i, \eta_i \rangle,$$

where τ is a given scalar in $(0, 1)$.

4. Simulation and Discussion

For comprehensive view, we observe the performance of matrix completion algorithms including the proposed algorithm, ADMiRA [9], LMaFit [10], LRGeomCG [4], and TNN-ADMM [11] through empirical simulations with the maximal number of sensor nodes to $n = 100$, the number of trials to 30, and test the performance, measured in terms of the root mean square error (RMSE) defined as

$$RMSE = \frac{1}{n} \|L\tilde{X}\tilde{X}^T L - LX_0 X_0^T L\|_F, \quad (6)$$

where $L = I - \frac{1}{n} \mathbf{1}\mathbf{1}^T$ and X_0 is the true coordinate matrix of the sensor nodes.

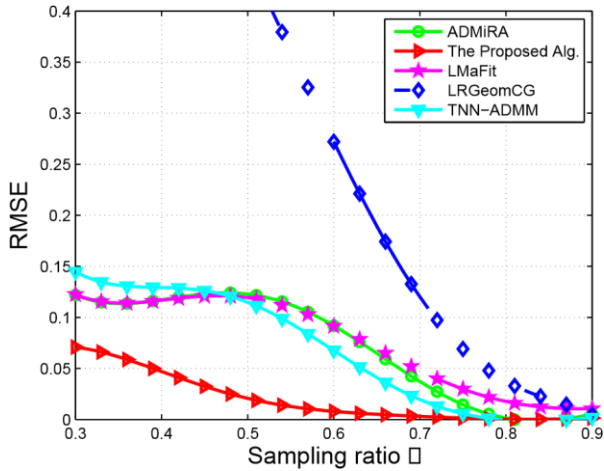


Fig. 1. Performance of various matrix completion algorithms in noiseless scenario

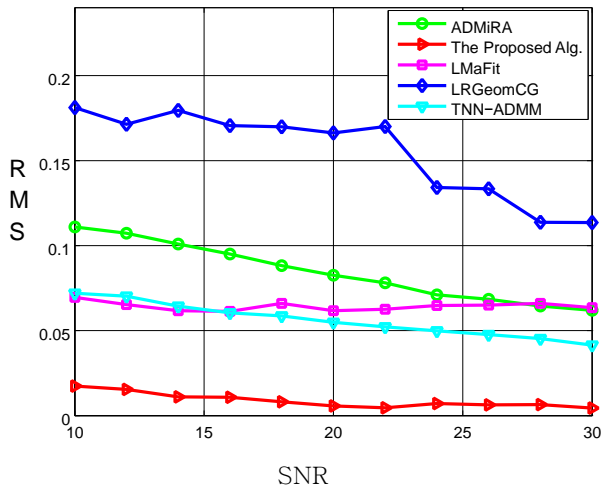


Fig. 2. Performance of various matrix completion algorithms for $\rho = 0.7$ in noise scenario

Fig. 1 provides the RMSE with respect to the expectation of the sampling ratio ρ [6]. In general, we observe that the RMSE improves with ρ . The performance of the proposed algorithm dominates for small ρ . In Fig. 2, we plot RMSE results in noise scenario for $\rho = 0.7$. Overall, we see that the performance of matrix completion algorithms is a bit worse than the noiseless scenario, and the RMSE of the proposed algorithm is the lowest one.

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5. References

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