

# A Stochastic Linear Scheduling Method using Monte Carlo Simulation

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**Abstract:** *The linear scheduling method or line-of-balance (LOB) is a popular choice for projects that involve repetitive tasks during project execution. The method, however, produces deterministic schedule that does not convey a range of potential project outcomes under uncertainty. This results from the fact the basic scheduling parameters such as crew production rates are estimated to be deterministic based on single-point value inputs. The current linear scheduling technique, therefore, lacks the capability of reflecting the fluctuating nature of the project operation. In this paper the authors address the issue of how the variability of operation and production rates affects schedule outcomes and show a more realistic description of what might be a realistic picture of typical projects. The authors provide a solution by providing a more effective and comprehensive way of incorporating the crew performance variability using a Monte Carlo simulation technique. The simulation outcomes are discussed in terms of how this stochastic approach can overcome the shortcomings of the conventional linear scheduling technique and provide optimum schedule solutions.*

**Keywords:** *Linear Scheduling; Line-of-Balance Method; Monte Carlo Simulation; Project Uncertainties*

## I. INTRODUCTION

A common technique used to schedule linearly progressive projects in the construction industry is known as the linear schedule or Line-of-Balance (LOB) method. LOB was put into practice to schedule repetitive activities in such a way to maintain the continuity of work. Such construction projects that involve repetition of activities include those found in high-rise buildings, pipelines, roads and civil-infrastructure, tunnels, and other projects done in a physically linear fashion [1-4]. Importance is placed on work continuity in order to minimize work disruptions while maximizing the benefits of the learning curve for repetitive work [1, 5-7]. The LOB method, however, produces a deterministic schedule, meaning it does not show the wide range of potential project outcomes. This results from the fact that basic scheduling inputs, mainly crew production rates, are estimated using single-point average values. Current LOB techniques, therefore, lack the capability to reflect the critical fact that crew production changes constantly due to various factors of uncertainty. Linear scheduling methods currently used by project managers fail to recognize how the variability of crew production rates affect the project schedule.

When compared to critical path method (CPM) scheduling, there is an added dimension to LOB that is taken into consideration: location. Although CPM scheduling offers a high level of detail in terms of project information, projects that contain repetition are more suited for LOB. Other scheduling outputs, such as Gantt charts, have been found to be “inefficient if used in construction projects that consist of repetitive activities” [3]. When it comes to linear construction and the repetitive nature of the project, bar charts may lead to misleading and inaccurate information [8]. The inclusion of location within a project schedule allows a project manager to plan for work continuity,

which reduces the overall project cost and duration [5]. In addition to the major benefits presented, a minor benefit that is gained through the use of LOB is the graphical format with which information is presented. Information regarding the production rates of working crews can be easily interpreted by project managers and all other parties involved [1, 3, 7-8]. Several logic-based restraints are set within LOB schedules in order for activities to flow from one to the next. Linear schedules confine each activity to a time and space dependency. Activities are seen as ‘time-dependent’ when one is to be performed immediately after another, while others are ‘space-dependent’ when there is a sequential order each activity must follow [3, 7-9]. They must be performed with these restrictions applied; the project must follow these logical parameters in order to reach completion.

Current LOB scheduling utilizes historical data to identify the average productivity of a given working crew. In order to create a balanced schedule, each crew is assumed to maintain their natural rhythm throughout the project [1, 8, 10-11]. These averages are forecasted through field observation, technical specifications, or previously collected data [8]. Due to the fact that these averages are single-point values, with minimal information from which the value is derived, there is an assured level of uncertainty to the actual project duration as a result of underlying variation.

## II. UNCERTAINTY IDENTIFIED

Through an extensive literature review it has been found that most researchers make the assumption that linear schedules are generated using deterministic, single-point productivity averages, also known as the ‘natural rhythm’ of each task [1, 8, 10-12]. Variation lies within working crews’ daily performance. Incorporating uncertainty

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variables in worker productivity has not been a large topic of interest when it comes to LOB scheduling. Due to the fact that day-to-day productivity is unlikely to be consistent, the natural rhythm should not be used in scheduling calculations. This assumption can cause a domino effect of magnified inaccuracies throughout the entire duration of the project [8-9]. Pending the overall scope of the project, minor inaccuracies may lead to the inclusion of unnecessary cost and time. Additional information must be taken into consideration when calculating working averages to reach a range of various outcomes, giving more information the project manager can use in scheduling decisions.

Several models and methods in dealing with these uncertainties have been developed. The most basic way of dealing with uncertainty is known as reactive scheduling. Reactive scheduling is an effort in which the project manager revises the schedule throughout the duration of the project to incorporate any uncertainty as it may occur [13]. Models, which have been developed to incorporate uncertainty into linear scheduling calculations, include Duffy's linear scheduling model with variable production rates (LSMVPR) [14], and Maravas' fuzzy repetitive scheduling method (F-RSM) [15].

The LSMVPR model identifies the relationship between changes in productivity, time, and location, which is then used for prediction modeling of future tasks and scheduling. Similar to reactive scheduling, the model is used to adjust the schedule during the project as unforeseen conditions are introduced, allowing the project manager to schedule for anticipated project adjustments in order to stay on track and meet the predetermined project deadline. According to Ammar [1], this model allowed construction teams to "visualize the obstacles when and where they occur". This advancement in linear scheduling takes uncertainty into consideration by adding adjusted production rates to the schedule, in both time and location, to be used in predicting future project scheduling habits. Alternatively, Maravas and Pantouvakis developed a fuzzy repetitive scheduling method (F-RSM) based on fuzzy logic principles [15]. As explained by Ammar [1], "possible differences between repetitive units and variations in the performance of work crews are encoded into fuzzy unit production rates". Fuzzy logic places uncertainty variables in place of the production rates in order to visualize possible dominant activities. Having identified a schedule controlling activity, a project manager gains a better understanding of imprecision tolerances that can be applied to the non-critical activities. These models offer valuable information about productivity uncertainty but lack the incorporation of multiple iterations. In order to better predict project duration, many situations must be simulated to prevent static values.

### III. OPTIMIZATION OF PROJECT DURATION

Since the early 1970s, there have been several studies done to advance the methodology behind LOB from an optimization perspective [1, 3, 8]. Several methods of project optimization have been proposed in order to reach minimized project duration. Overtime, double-shifts, weekends, stronger work force, and adjustment of productivity have all been considered in order to shave days off of the project duration [3, 6-8]. Within the listed optimization options, there are many negative indirect outcomes. Several issues come into play when the project manager has crews working overtime and weekends. Not only is there an increase in unit labor cost, but also a study has come to the conclusion that working overtime and weekends lowers efficiency and productivity [6-8]. Multiple-shift workdays lead to a huge effort from management to ensure work transfers smoothly from shift to shift [8]. In order to get a stronger work-force, a crew must either be increase in size or increase in productivity. Both options lead to an increase in overall costs and can leave a workspace overcrowded, in turn lowering productivity [8]. Finally, the optimization strategy of adjusting productivity is unrealistic because a work-crew will not want to work at a very slow rate [7].

Several models have been proposed to incorporate interruption as a performance enhancer in one way or another. Ali and Elazouni [16] created a model to optimize financial performance within linearly progressive projects. Applicable to project duration, this model adds interruptions to activities where the predecessor is found to have a higher rate of productivity. Interrupting exceedingly productive tasks allows for the dominating activities to be condensed on the time table [7, 10, 16]. Ipsilandis [17] developed a model termed Multi Objective Linear Programming model (MOLP-LRP). This model can optimize a project schedule in relation to the overall duration, number of interruptions, or cost. The MOLP-LRP model can also generate a sensitivity analysis to show a cost tradeoff between each of the previously mentioned variables. Interruption duration and location is based on the project manager's strategy. Although these models introduced interruption to the project schedule, they were severely limited by the number of interruptions allowed as well as technology in place to perform these interruptions. When creating a linear schedule, the project manager must apply his or her strategy to the optimization techniques and manually adjust the schedules to show adjusted timetable changes.

### IV. MONTE-CARLO SIMULATION-BASED LOB CASE

In this paper, how the variability of crew production rates affects linear schedule outcomes is demonstrated from a schedule optimization perspective. This is necessary to present a realistic picture of what might be the case for the project outcomes. To present a more user-friendly and

comprehensive way of incorporating this production variability in LOB, a Monte-Carlo simulation technique is applied. The manual and computer simulation results are compared and discussed in terms of their pros and cons. This study will first create a linear schedule model that will integrate Monte-Carlo simulation with LOB as a way of accounting for variability in project duration. Additionally, an optimization schedule model of project duration is to be created in conjunction with the Monte-Carlo simulations by using the strategy of task interruption with buffer variables which could be decided on by project managers.

The Monte-Carlo simulation is a “computerized mathematical technique that allows professionals to account for risk in quantitative analysis and decision making” by considering multiple scenarios in a project through randomly selected values of input variables from distributions [18]. The current LOB scheduling technique lacks the capability to reflect variation and variability in crew productivity, which is what Monte-Carlo simulation is known for to provide. Probability distribution for each crew productivity input variable is chosen to account for such variation in crew performance. Results are generated through thousands of iterations of simulation from randomly selected values from these input distributions to create histograms of probable outcomes. The cumulative simulation outcomes through graphical displays can prove to be very useful in decision making of project managers.

The information presented here demonstrates why the current LOB method does not produce a realistic project picture for planning and how this deficiency can be overcome using a stochastic simulation methodology using the simulation technique. It helps answer the following questions: Can LOB be improved on to incorporate uncertainty found within the input variables of productivity using Monte-Carlo simulation? In addition to uncertainty, what can be done with a project schedule to aid in predicting project schedule outcomes.

For an analysis in this paper, the following highway construction scenario is used. There are ten different types of construction crews involved in the project. The construction crews and their respective average productivities (presented in in Table 1) are used for the linear scheduling. The average productivities are based on historical data collected through the previous experiences of each working crew.

Table 1. Crews Used in the Scheduling Case

ID	Working Trade	Average Productivity (Units/Day)
Crew 1	Land Preparation	10
Crew 2	Excavation	10
Crew 3	Soil Leveling/Compaction	27
Crew 4	Drainage/Sewer Installation	24
Crew 5	Subgrade layers	10
Crew 6	Subgrade Compaction	20
Crew 7	Reinforcement	19
Crew 8	Set Forms	20
Crew 9	Pour Concrete	25
Crew 10	Remove Forms	20

Also it is necessary to have additional inputs such as work buffer zones and project length to create a complete LOB schedule. Work buffer zones will be set in place to make sure no two crews are working on top of each other and there is enough staging room to ensure crews are not congested. Project length is included to define the quantity of work that is to be scheduled for and completed.

## V. COMPARISON OF SCHEDULES

Based on the case described, three different schedule solutions are created. The first solution is based on the conventional LOB practice where all the inputs are deterministic average values. The second solution is focused on implementing uncertainty via the Monte-Carlo simulation into LOB schedule. And the last schedule is targeted for an optimum solution in the Monte-Carlo simulation LOB schedule.

### A. Conventional Linear Scheduling

The conventional LOB schedule, as shown in Figure 1, does not consider the uncertainty in crew productivities. This is because all the lines that represent various crew productivities are straight and the line angles are fixed from the beginning of the crew work initiation until the end of their operation. In the case, each activity maintains a 100 unit buffer zone between working crews to keep work flow constant and efficient. According this LOB solution, the total project duration is predicted to be 222 days. This schedule is not an accurate representation of the project where crew productivities are dynamically changing throughout their operations. Although this is common practice for current linear scheduling methods, it means a overly simplified input assumptions and therefore creating unrealistic prediction on the total project duration.

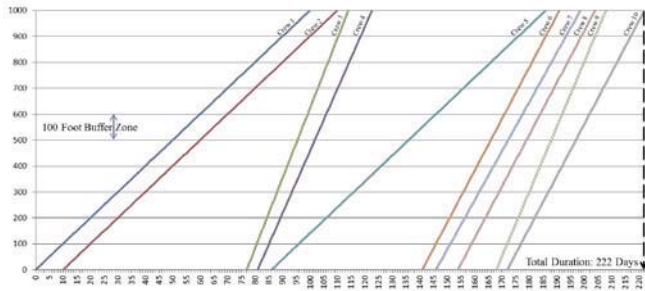


Figure 1. Conventional LOB Schedule for the Case

*B. Uncertainty-Incorporated Schedule*

To create a linear schedule based on the Monte-Carlo simulation, a software application was developed in the MS Excel environment by the authors, which runs on top of Monte-Carlo simulation simultaneously (for the screenshot, please see the next section in Figure 3). The program has complex formulas to tie Monte-Carlo simulated data into user-defined inputs to generate probabilistic analysis capability and graphical representations. The inputs available in this model include total number of iterations, minimum work buffer, number of working crews, and a total project length. The productivity of each working crew is defined with various probability distributions according to the characteristics of the productivity variations. Essentially, for each crew, the Monte-Carlo simulation uses randomly selected value from the crew's productivity distribution curve per simulation iteration. Therefore, every time a new iteration of simulation is run, the slope is likely going to have different value, meaning it gets steeper or flatter, which affects the ensuing crews' productivity lines. This is reflected graphically in the LOB schedule recalculating the expected project duration. The following user-defined inputs are used in the Monte-Carlo simulation: a 100 foot work buffer, 10 working crews, and a total project length of 1000 feet. The results of the cumulative simulation outcomes on the project duration are summarized in Figure 2.

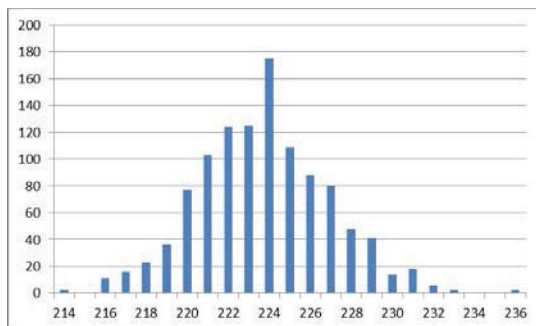


Figure 2. Project Duration Distribution based on Monte-Carlo Simulation-Based Schedule

This histogram shows that the mean project duration is around 224 days with a minimum of 214 days and a maximum of 236 days. Recall that when the static LOB with deterministic single-point average productivities described in the previous section, we are only given a single project duration time (222 days). However, in this analysis, the simulation displays a range of potential project durations and respective confidence levels. It is found that, for example, there would be less than 40% chance that the project could be done within the originally expected project duration (Note that 40% can be calculated by getting the percentage of the area occupying the left side of day 222 with respect to the total area in the histogram in Figure 2). This conversely means that there is more than 60% chance that the project would be completed beyond that original deterministic project completion with the worst case project duration being 236 days. By incorporating uncertainty it is shown that the deterministic project duration would be refined and estimated more realistically. In other words, depending on a desired level of confidence, the project manager could determine more realistic project duration. For example, if 90% of confidence is required in terms of project duration, then as shown in Figure 2, the expected project duration is likely extended to be around more than 230 days. Therefore, what is important is not a static single prediction on the project duration, but rather it is flexibility of the schedule information that allows managers to be able to filter information based on what is required.

*C. Optimization of Project Duration*

The last schedule presented in this section is to provide an optimum solution in terms of the project duration. The concept is very similar to the schedule in the previous section except that now each crew productivity line is allowed to be not straight as shown in Figure 3. As shown in the figure, the lines now are bent and erratic. This in fact is much more realistic representation because crews go through their own productivity fluctuations due to various factors at the job site. Work buffer zone constraints are, however, strictly applied to ensure crews do not interfere with each other.



Figure 3. Monte-Carlo Simulation-based LOB Program

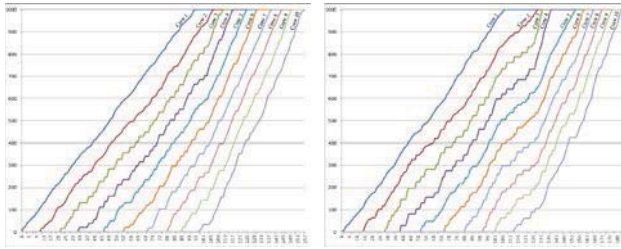


Figure 4. Optimized Linear Schedule Outcomes

Figure 4 shows simulated LOB schedule outcomes from two different simulation iterations. When thousands of iterations are performed and each outcome is accumulated, the project duration distribution curve can be created as shown in Figure 5.

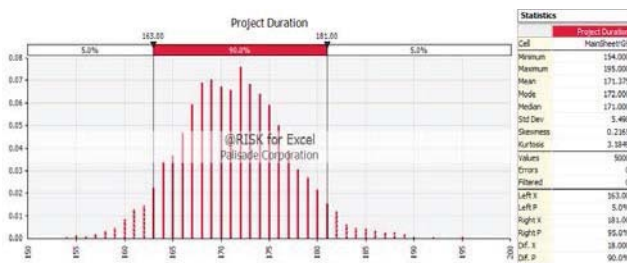


Figure 5 Project Duration Distribution When Allowing Productivity Interruption

The distribution curve in Figure 5 indicates that the project duration is shown to have a mean value of 171 days, a minimum duration of 154 days, and a maximum duration of 195 days. It also shows that the chance the project duration would be between 163 and 181 days would be about 90%, which is an improved solution compared to the solution presented in the previous section.

## VI. CONCLUSION

Incorporating productivity uncertainties into the conventional LOB practice can lead to more realistic scheduling prediction that can be easily understood by project managers. Monte-Carlo simulation along with the LOB scheduling methods lowers the risks involved with crew performance fluctuations because the crew productivity can be closely reflected through the distribution variables instead of deterministic average productivity numbers. In addition to uncertainty, by allowing the productivity line in LOB to be non-straight Monte Carlo simulation allows for realistic schedule compression as an optimum schedule solution. These methods are both practical and revolutionary to the current LOB scheduling methods and can change the way linearly progressive projects are scheduled in the near future. It is because the methodology provides more flexibility for project managers to inject their understanding in the form

of distribution-based input variables, not single average input values.

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