# Polarization Precession Effects for Shear Elastic Waves in Rotated Solids

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ABSTRACT

Developments of Solid-State Gyroscopy during last decades are impressive and were based on thin-walled shell resonators like HRG or CRG made from fused quartz or leuko-sapphire. However, a number of design choices for inertial-grade gyroscopes, which can be used for high-g applications and for mass- or middle- scale production, is still very limited. So, considerations of fundamental physical effects in solids that can be used for development of a miniature, completely solid-state, and lower-cost sensor look urgent.

There is a variety of different types of bulk acoustic (elastic) waves (BAW) in anisotropic solids. Shear waves with different variants of their polarization have to be studied especially carefully, because shear sounds in glasses and crystals are sensitive to a turn of the solid as a whole, and, so, they can be used for development of gyroscopic sensors.

For an isotropic medium (for a glass or a fine polycrystalline body), classic Lamé's theorem (so-called, a general solution of Elasticity Theory or Green-Lamé's representation) has been modified for enough general case: an elastic medium rotated about an arbitrary set of axes.

Travelling, standing, and mixed shear waves propagating in an infinite isotopic medium (or between a pair of parallel reflecting surfaces) have been considered too. An analogy with classic Foucault's pendulum has been underlined for the effect of a turn of a polarizational plane (i.e., an integration effect for an input angular rate) due to a medium's turn about the axis of the wave propagation. These cases demonstrate a whole-angle regime of gyroscopic operation.

Single-crystals are anisotropic media, and, therefore, to reflect influence of the crystal's rotation, classic Christoffel-Green's tensors have been modified.

Cases of acoustic axes corresponding to equal velocities for a pair of the pure-transverse (shear) waves have of an evident applied interest. For such a special direction in a crystal, different polarizations of waves are possible, and the gyroscopic effect of "polarizational precession" can be observed like for a glass.

Naturally, formation of a wave pattern in a massive elastic body is much more complex due to reflections from its boundaries. Some of these complexities can be eliminated. However, a non-homogeneity has a fundamental nature for any amorphous medium due to its thermodynamically-unstable micro-structure, having fluctuations of the rapidly-frozen liquid. For single-crystalline structures, blockness (walls of dislocations) plays a similar role.

Physical nature and kinematic particularities of several typical "drifts" in polarizational BAW gyros (P-BAW) have been considered briefly too. They include irregular precessions ("polarizational beats") due to: non-homogeneity of mass density and elastic moduli, dissymmetry of intrinsic losses, and an angular mismatch between propagation and acoustic axes.

## 1. Introduction

Studies of fundamental effects of precession in polarizational states of shear elastic waves in solids have been forwarded from publically-closed reports to wide discussions by a scientific community as early as mid 80s at[2], at "Gulyaev's conference"[2], and in article[3], *etc.* Similar effects has been studied also for anisotropic media, and have been discussed later at "Peshekhonov's conference"[4] and in article[5]. Selected aspects of gyroscopic sensing have been noted also in reports[6].

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Actually, these effects[7] have a lot of analogies with effects discovered by G.H.Bryan for oscillations of elastic shells[8] and by E. Fermi for electromagnetic waves[9].

Their math model forms a basis for studies of bulk acoustic waves (BAW) and for considerations of similar effects for surface acoustic waves (SAW) [10,11,12,13]. So, a number of patents, directly using these matters is rapidly increasing in different countries[14,15),(16,17], *etc.*

## 2. Basic Polarizational Effect

Pressure elastic (" acoustic" ) waves (" sounds" , " ultrasounds" , *etc.*) in fluids and solids are longitudinal waves. But shear elastic waves (" shear sounds" ) in glasses are transverse waves, and, therefore, they have different polarizations: linear (LP), circular (CW – clockwise or CCW – counter-clockwise), or combined (elliptic). There are many enough complex types of elastic waves in crystals due to their anisotropy of these media and a variety of syngonies (different classes of symmetry). On the other hand, polarizational effects in a crystal can be very similar to a glass, if an observer considers propagations along special directions (so-called, acoustic axes).

From a first glance, the basic effect looks simple: When a solid body is rotated as a whole about an acoustic axis, the transverse wave propagating in this direction is under an influence of Coriolis' acceleration, and such a linear-polarized wave is precessing similar to well-known Foucault' s pendulum.

## 3. Modified Christoffel–Green's Tensor

Acoustics of non-rotated crystals has been considered in numerous articles and several monographs (See, *e.g.*, [18], *etc.*). Studies of anisotropic elastic media intensively use formalism of tensors.

The classic Christoffel-Green' s tensors for an arbitrary anisotropic solid have been modified [4-6] for a case of its rotation with an angular rate $\vec{\Omega} = (\Omega_i)$ about an arbitrary axis $\vec{m} = (m_i)$

$$\Gamma_{il}^{\ *} = \Gamma_{il} + \frac{\rho}{k^2}(\Omega_i \Omega_l - \Omega^2 \delta_{il}) = \Gamma_{il} + \Omega_*^{\ 2}(m_i m_l - \delta_{il}),$$

where $\hat{\Gamma} = (\Gamma_{il}) = (c_{ijkl}\, n_j n_k)$ is the classic Christoffel-Green tensor of an unrotated medium having elastic constants $\hat{c} = (c_{ijkl})$ and mass density $\rho$. This tensor is defined for a wave propagating in a direction $\vec{n} = (n_i)$.

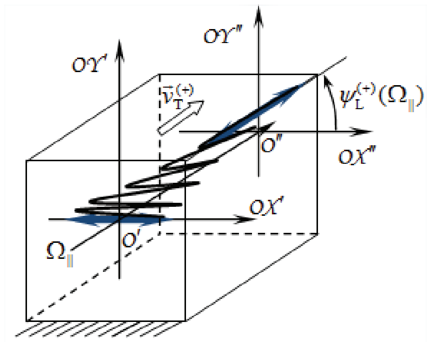


**Figure 1** Precession of polarizational plane of a shear ultrasound propagating inside a rotated crystal or a glass sample.2. Analytical Solutions

If a solid has an axis of symmetry with equal phase velocities of quasi-transverse elastic waves (*i.e.*, it has an acoustic axis)

$$v'_{QT} = v''_{QT} \quad ,$$

this case is a subject of our special applied interest, because it can be used for development of miniature solid-state gyroscopic sensors.

## 4. Modified Lamé–Green' s Representation

Let us pay more attention to a case of an isotropic medium (*i.e.*, an amorphous solid or polycrystalline one with fine and randomly-oriented grains). For such a case, the basic Navier' s equations of Elasticity Theory can be reduced to a system of wave equations (so-called, Lamé-Green' s representation theorem) [19,20]. This classic theorem has been modified [3] for a case of rotation of elastic body. For this case, using Helmholtz' expansions $\vec{u}=\vec{u}_{\rm L}+\vec{u}_{\rm T}=\nabla\phi+\nabla\times\vec{\psi}$, we get

$$c_{\rm L}^2\Delta\phi=\ddot{\phi}+\vec{G}\cdot\vec{\psi}+U\,,$$
$$c_{\rm T}^2\Delta\vec{\psi}=\ddot{\vec{\psi}}+\vec{G}\times\vec{\psi}-\vec{G}\phi+\vec{A}\,;$$

where $c_{\rm L}=\sqrt{\dfrac{\lambda+2\mu}{\rho}}$, $c_{\rm T}=\sqrt{\dfrac{\mu}{\rho}}$ are velocities of pressure and shear sounds, $\vec{f}=\nabla U+\nabla\times\vec{A}$ is a representation of a load distributed inside the material (if any) onto potential and solenoidal components, and $\lambda,\mu$ are Lamé' s elastic constants; the action of gyroscopic operator is defined according the relation $\vec{G}\times\vec{u}\equiv 2\vec{\Omega}\times\dot{\vec{u}}+\dot{\vec{\Omega}}\times\vec{u}$.

Such a representation of the basic equations can be further transformed to write down so-called " a general solution" of Elasticity Theory, which is very useful for many analytical studies. For motions like $\varphi=p_0(t)F_\chi$, $\vec{\psi}=\vec{p}(t)F_\chi$, if to represent the loads in the same way $\mathrm{U}=f_0(t)F_\chi$, $\vec{A}=\vec{f}(t)F_\chi$, we have a single quaternion ordinary differential equation

$$\ddot{\mathbf{q}}+\mathbf{G}\circ\mathbf{q}=A\mathbf{q}+\mathbf{g}\quad,$$

where $F_\chi$ is any eigen function defined by $\Delta F_\chi=\chi F_\chi$ and an eigen value $\chi$. Quaternions $\mathbf{q}=(p_0\;\vec{p})$, $\mathbf{g}=(f_0\;\vec{f})$, and $\mathbf{G}=(0\;\vec{G})$ correspond to generalized (modal) coordinates, forces, and to the gyroscopic operator. Here $A=\chi\,\mathrm{diag}(c_{\rm L}^2\;c_{\rm T}^2\;c_{\rm T}^2\;c_{\rm T}^2)$ is a matrix.

Materials like high-purity quartz glasses (fused quartz) are enough isotropic, homogeneous, and they have extremely-low CTE and levels of acoustic losses less. So, silica or titania can be potentially good candidates for a polarizational sensor.

## 5. Modified Love' s Representation for Planar Problem of Elasticity Theory

For a Planar problems of Elasticity Theory, using the results given above, we have

$$c_{\rm L}^2\,\Delta\phi=\ddot{\phi}+G_z\psi_z+U\quad,$$
$$c_{\rm T}^2\,\Delta\psi_z=\ddot{\psi}_z-G_z\phi+A_z\quad.$$

But classic Love' s representation[21] in respect to mean strain $\varepsilon=\frac{1}{2}(u_{,x}+v_{,y})$, and shear $\gamma=\frac{1}{2}(v_{,x}-u_{,y})$ is also very convenient. It has been generalised [3] for an arbitrary rotated media too

$$\Delta\varepsilon=D\varepsilon-G_*\gamma\,,$$
$$(1-\nu)\Delta\gamma=D\gamma+G_*\varepsilon\,;$$

where $D=\partial_\tau^2-\Omega_*^2$, $G_*=2\Omega_*\partial_\tau+\partial_\tau\Omega_*$, $\Omega_*=\left(\dfrac{E}{(1-\nu^2)\rho}\right)^{-\frac{1}{2}}\Omega$, $\tau=\dfrac{E}{(1-\nu^2)\rho}t$; and $E,\nu$ are Young' s modulus and Poisson' s ratio.

For strains, using the spatial functions $F_\chi$ introduced above, we have $\varepsilon=p_\chi(t)F_\chi$, $\gamma=q_\chi(t)F_\chi$; and the following system of ordinary differential equations has been given

$$Dp_\chi-G_*q+\chi p_\chi=0\,,$$
$$Dq_\chi+G_*p_\chi+(1-\nu)\chi q_\chi=0\,.$$

This equations contain $p_\chi=p_\chi(t)$, and $q_\chi=q_\chi(t)$ as functions of time; so, they can be easily modified for cases of different external actions (forced and/or parametric excitations, controlled waves, *etc.*).

Enough similar ways of transformations can be used both for a planar strain state and for a planar stress state.

## 6. Precession of Polarization for a Traveling Wave

Any travelling transverse LP-wave can be represented as a superposition of a mirror pair consisting from CW and CCW wave components.

For an arbitrary wave inside a solid under a rotation about the axis of its propagation, a mismatch between the frequencies of its CW and CCW components appears (so-called, gyroscopically-induced acoustic " activity" of a medium). In particularly, if a sum of such waves forms an LP-wave, the plane of its polarization will be slowly-turning (precessing) during propagation of the wave, and this process shows an angular rate proportional one to an input angular rate.

Thermal errors are not really essential for such sensing thanks to their almost-identical influence on the both CW and CCW components.

For $\nu = 0$, it is easy to find a general solution of the system given above for an arbitrary input rate relation $\Omega = \Omega(t)$ (similar to Foucault' s pendulum).

For $\Omega(t) = \mathrm{const}$, the characteristic equation has the form

$$(\chi - \Omega_*^2 - \omega_*^2)[(1-\nu)\chi - \Omega_*^2 - \omega_*^2] - 4\Omega_*^2\,\omega_*^2 = 0,$$

and the solution has the form of a harmonic process

$$p_\chi(t) = P_\chi\,\mathrm{e}^{\mathrm{i}\omega_*\tau} + \mathrm{c.c.},\quad q_\chi(t) = Q_\chi\,\mathrm{e}^{\mathrm{i}\omega_*\tau} + \mathrm{c.c.}$$

In particular, a combination of such solutions can be applied to a problem of planar oscillations of a rotated disc or for circular modes of a rotated infinite rod. To meet specific boundary conditions, a multiplette of such waves has to be formed. It can give enough complex total wave patterns.

## 7. Precession of Polarization for a Standing Wave in an Elastic Layer

If a solid medium is limited by a pair of parallel planes (faces) which are orthogonal to an acoustic axis of this elastic body, one can form a stationary wave oscillating between the mentioned planes inside such a layer (" an acoustic laser" ). Depending on a type of excitation and on coefficients of normal reflection at these planes, such a wave can be in different states (generally-speaking, it will be mixed and elliptically-polarized). In particularly, such " acoustic mirrors" can form a pure-standing transverse LP-wave that can be represented as a mirror pair of CW and CCW waves. The eigen frequency of such a wave depends on a distance between the faces and on a modal number(s).

Any standing wave can be represented by a pair of counter-travelling components. Under transverse angular rates, side effects like a change of their phase velocities can be observed. So, generally speaking, the total wave will have a mixed (stranding-travelling) character.

Under a rotation of the solid about the axis of the body, with an arbitrary wave inside it, a mismatch between the eigen frequencies of CW and CCW components of this wave appears. In particularly, a sum of such waves can form an LP-type pattern, whose effective polarization plane will be slowly-turning during the propagation with an angular rate proportional to an input angular rate.

## 8. On Rotation of a Polarizated Acoustic Beam

For an acoustic beam of a finite diameter, equations in respect to slow-varying variables have been written down too.

The transverse structure of the wave acoustic beam can be like a planar wave. However, more complex modal structures can be used to get low sensitivities to different side factors. Hence, transverse modal numbers can be treated as design parameters.

Thus, we can observe a polarizational precession of a shear wave beam travelling in a glass rotating about an axis of the propagation.

## 9. On Schemes of Gyroscopic Sensors

Any acoustic beam has a diffraction, but a transverse (radial) sizes of a sensor can be enough wide (or covered by damping layers) to neglect potential reflections from them, and to cause only a minor influence on *Q*-factor of the main acoustic beam. So, such a sample can be treated as a medium having " an infinite nature" in transverse directions. So, these sides of the solid body can be fixed enough rigidly without any essential influence onto damping of the operational wave, *etc*.

Waves can be propagating between a pair of parallel flat surfaces used for the excitation and readout. These planes can be also slightly-curved like lenses to decrease diffraction and to provide a fine concentration of acoustic energy along the axis of this solid " cavity" (and, so, to limit transverse dimensions of the sensor). Such designs are especially-useful for the variants employing piezoelectric solids, and they have numerous analogies among piezoelectric frequency generators.

The proposed element forms an " open" linear resonator that looks much simpler in respect to circular (or closed in another way) interferometers required for gyroscopes using SAW and BAW[22]. Such a " linear" resonator can be excited in regimes of travelling, standing or mixed waves depending on pattern of electromechanical system deposited at its faces.

To form a polarization-sensitive driving and pick-offs for such a resonator, a system of thin-film strip electrodes (*e.g.*, $Au$ covered onto $Ti$ sublayer on silica) operating under a magnetic field from stable and rigidly-fixed permanent magnets (*e.g.*, $SaCo$ group) has been tested[23]. Several other original planar variants have been proposed too, but not discussed in details in published articles.

There are two basic regimes of operation: (*a*) a rate regime, when a compensative feedback (" a force-to-rebalance loop" ) returns the polarization into its initial plane, and, so, the sensor operates as a rate gyroscope; (*b*) a whole-angle regime, when the polarization can change its orientation in a free manner, and the sensor operates as a gyroscope integrating an input angular rate (" a whole-angle regime" ). Control loops, having structures similar to HRG, stabilising parameters of the polarizational resonator can be added too.

## 10. Main Drifts of a Polarizational Gyroscope

Drifts of polarizational gyro are similar to drifts for HRG well-known from early 90s[24,25]. There are: (*a*) a mismatch of the velocities of the travelling waves having orthogonal polarizations (or a mismatch of eigen frequencies for design variants with standing waves) due to inertial-elastic non-homogeneities of the operational medium; (*b*) a mismatch between the axis of propagation and an acoustic axis (if a crystal has been used instead of glass); (*c*) a mismatch of dissipations for the orthogonal polarizations; (*d*) mismatches between counter-propagating travelling waves (*i.e.*, a small " destruction" of a pure-standing wave); (*e*) errors due to parasitic pressure waves; (*f*) mismatches of electrodes, electronic channels, *etc*. Non-linearity of the resonant medium can cause a regular polarizational precession too, if the excited/reflected waves are not pure-linear-polarized.

Increasing of $Q$-factor of the resonator gives a lot of design profits and helps to overcome many difficulties, and it do less dependent on fixture (in respect to similar HRG[26] or CRG[27] designs).

Fundamental limits of such a sensor connected with mechanical thermal noise. For a glass used as the operational medium, there is also Rayleigh' s backscattering from non-homogeneity of the material density. Such fluctuations have fundamental nature due to restriction on a potential rate of freezing of the melted material used to get a glass. Fused quartz has also relaxational losses due to bi-stability of the micro-structure observed in this type of glass. For a crystal like leuko-sapphire, $Q$-factor along its $c$-axis can be higher in order of value (in respect to fused quartz), but density of dislocations and blockness of the crystalline lattice have to be taken into account

## 11. Conclusions

Fundamental effect of polarizational precession has been studied for different media and transverse acoustic waves.

This effect can be used for designing of miniature and solid-state gyroscopic sensors to be used in high-$g$ applications and as low-cost variants useful for mass production.

Main potential errors for such sensors have been noted.

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