한국소음진동공학회 2013년 춘계학술대회논문집, pp. 835~841

Avoidance of Internal Resonances in Hemispherical Resonator Assemblies from Fused Quartz Connected by Indium Solder

세르게이 사라플로프† · 이희남* · 박상진**
Sergii A. Sarapuloff, Huinam Rhee and Sang-Jin Park

Key Words : Inertial Navigation, hemispherical resonator gyroscope (HRG), designing of high-Q resonators, quartz glass, indium bonding, elastic shell, Rayleigh's isogeometric mode, internal resonance, FEM.

ABSTRACT

Modern solid-state gyroscopes (HRG) with hemispherical resonators from high-purity quartz glass and special surface superfinishing and ultrathin gold coating become the best instruments for precise-grade inertial reference units (IRU) targeting long-term space missions. Designing of these sensors could be a notable contribution into development of Korea as a space nation.

In participial, *40mm* diameter thin-shell resonator from high-purity fused quartz, fabricated as a single-piece with its supporting stem has been designed, machined, etched, tuned, tested, and delivered by *STM Co*. (*ATS of Ukraine*) several years ago; an extremely-high Q-factor (upto *10~20 millions*) has been shown. Understanding of the best way how to match such a unique sensor with inner glass assembly of the gyro means how to use the high potential in a maximal extent; and this has become the urgent task.

Inner quartz glass assembly has a very thin indium (*In*) layer soldered the resonator and its silica base (case), but effects of internal resonances between operational modal pair of the shell-cup and its side (parasitic) modes can notable degrade the potential of the sensor as a whole, instead of so low level of resonator's intrinsic losses. Unfortunately, there are special combinations of dimensions of the parts (so-called, "resonant sizes"), when intensive losses of energy occurs. The authors proposed to use the length of stem's fixture as an additional design parameter to avoid such cases.

So-called, a cyclic scheme of finite element method (FEM) and *ANSYS* software were employed to estimate different combinations of gyro assembly parameters. This variant has no mismatches of numerical origin due to FEM's discrete mesh. The optimum length and dangerous "resonant lengths" have been found.

The special attention has been paid to analyses of 3D effects in a cup-stem transient zone, including determination of a difference between the positions of geometrical Pole of the resonant hemisphere and of its "dynamical Pole", *i.e.*, its real zone of oscillation node. Boundary effects between the shell (cup) and 3D short "beams" (inner and outer stems) have been ranged. The results of the numerical experiments have been compared with the classic model of a quasi-hemispherical shell band with inextensional midsurface, and the solution using Rayleigh's functions of the 1st and 2nd kinds.

To guarantee the truth of the recommended sizes to a designer of the real device, the analytical and FEM results have been compared with experimental data for a party of real resonators. The consistency of the results obtained by different means has been shown with errors less than 5%. The results notably differ from the data published earlier by different researchers.

1. Introduction

A correct analytical model of an arbitrary axisymmetric shell resonator has been provided over a century ago in the classic treatment of J.W. Strutt[1]. It has been given as a family of isogeometric deformations of a

† 교신저자; 순천대학교 우주항공공학전공
E-mail : Sergii.sarapuloff@gmail.com
Tel : 061-750-3824, Fax : 061-750-3820
* 순천대학교 기계우주항공공학부
E-mail : hnrhee@sunchon.ac.kr
** 순천대학교 대학원 우주항공공학과

geometric surface, and this solution had been generalized by G.H. Bryan[2] for a rotating inextensional shell.

In more details, taking into account membrane strains inside shell' s midsurface, it can be done also using Kirchhoff-Love' s Shell Theory[3], including its corrected Novozhilov' s variant[4]. Eigen functions of Love-Novozhilov' s equations for a deep spherical shell had been written down using combinations of several Legendre' s and Heine' s functions having complex orders[5]. These very complicated solutions had been constructed for a cantilevered hemisphere as well as for spherical bands[6].

However, these sophisticated analytical solutions obtained and summarized in[7] shown that they have the same accuracy with the elementary functions used in earlier Rayleigh' s studies. So, for midsurface tensility effects, it is also possible to analyze starting from simple asymptotic solutions near a free edge of rotating shells[8], including their specific features for the equatorial rim in HRG[9].

Basing on classic achievements[10], newer smaller variants of Coriolis' vibratory gyroscopes with shell resonators are designing and fabricating[11]. The thinwallness ratio ($\varepsilon_h = h / R$ – thickness/radius) is notably increasing, and, so, analyses and methodology of correct assembling of resonators still looks urgent.

Unfortunately, the analytical and FEM studies published in different countries[11,12,13,14,15,16,17,18], and several other attempts to compare the numerical results and experiments shown a huge discrepancy (upto ±20~30%) that are rather high to be used by a designer in real practice. Such a problem is originated from errors of the numerical schematisation and approaches used, and it caused also by considerations of a non-complete variant of Rayleigh' s functions.

In the papers mentioned above, any influences of specific resonator' s fixture inside a test set-up or in a glass case (base) of inner HRG assembly connected by a metal bonding had not been analysed too. However, such influences are essential ones for a real practice, and they can be employed very effectively to avoid any internal resonances (or they cause a critical drop down of the *Q*-factor after assembling.)

2. Statement of Problem & Proposed Methods

Let us consider a precise inner HRG' s assembly made from high-purity quartz glass (fused quartz). A very thin bonding (In) has been used for such a connection of the quartz glass parts without and any dry contact of them.

Additional internal resonances (so-called, " hidden symmetry") in the assembled structure cause unexpected additional degeneration of the modes, and, therefore, they generate intensive flows of the operational energy from operational modal pair (usually, $n = 2$) into parasitic modes. Even if the resonator itself designed well and tuned perfectly, it can be degraded notably by such a wrong-designed assembly.

We propose to consider a length of stem' s fixture as the main varying design parameter, and a thickness of the connecting layer (indium bonding) as an additional design parameter. Several other minor parameters have been noted below too.

3. Analytical Solutions

Free-edge boundary conditions for an open (cantilevered) shell (usually, close to a hemisphere) let us use isogeometric deformations of the shell' s midsurface as an enough good approximation for the low-frequency eigen functions (so-called, Rayleigh' s modes).

Most of researchers used only Rayleigh' s functions of the 1st kind that is a correct step for a complete hemisphere only (*i.e.*, for a cup having no stem in its polar zone).

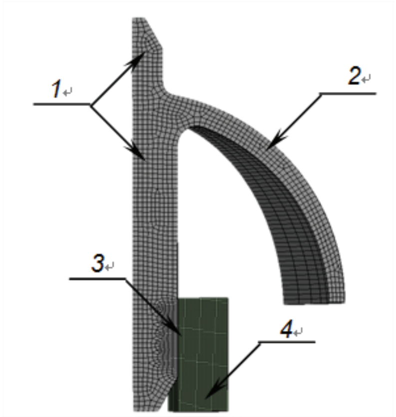

Figure 1 Scheme of the mesh for resonator's assembly (half of the axial cross-section): 1 – outer and inner stems fabricated together with 2 – a resonant shell cup, 3 – a connecting indium layer, 4 – an element of a glass case (not shown completely).

According initiative of Dr. D.D. Lynch, the complete Rayleigh analytic solutions for a hemispherical band had been studied in USA-Ukraine joint project[19]

$$U_{n\,\mathrm{R}}(\theta) = V_{n\,\mathrm{R}}(\theta) = -\sin\theta\,(\tan^n \frac{\theta}{2} + \delta_n \cot^n \frac{\theta}{2}),$$
$$W_{n\,\mathrm{R}}(\theta) = (n+\cos\theta)\tan^n \frac{\theta}{2} - \delta_n (n-\cos\theta)\cot^n \frac{\theta}{2} \;;$$

where $\vec{u}_{n\mathrm{R}}(\varphi,\theta) = \vec{U}_{n\mathrm{R}}(\theta) \begin{matrix}\cos\\ \sin\end{matrix} n\varphi$ is a displacement field for the n^{th} mode having the meridianal modal functions $\vec{U}_{n\mathrm{R}} = \{U_{n\,\mathrm{R}}, V_{n\,\mathrm{R}}, W_{n\,\mathrm{R}}\}$, and δ_n is a constant depending on stem's radius and stem-cup boundary condition.

An additional term due to tensility of the midsurface has been obtained too[20]. So, the classic Rayleigh formula for eigen frequencies

$$\omega_{n\,(\mathrm{R})} = \zeta_n \frac{h}{R^2}\sqrt{\frac{E}{(1+\nu)\rho}}$$

(with a non-dimensional constant ζ_n) gets a new modified form

$$\omega_n = \omega_{n\,(\mathrm{R})}\left(1+\frac{\zeta_{Cn}}{\varepsilon_h^{\,2}}\right),$$

where ζ_{Cn} is a non-dimensional constant related with midsurface tensility. Let us note that the both constants ζ_n and ζ_{Cn} depend on ratio of cup-stem diameters (*i.e.*, on the angle of polar zone) and have been tabulated.

These solutions can be further improved by a boundary condition of an elastic clamping at shell's polar zone (like the condition for "Kirchhoff's effective bending moment *vs.* meridianal angle"

$$M^*_{\theta} = a\,\frac{\partial w}{\partial \theta} \;;$$

where a is an effective stiffness of cup-stem connection). Such a stiffness constant has been estimated by FEM or by local 3D-asymptotic analysis (using Theory of Elasticity).

4. Numerical Experiments

Detailed numerical simulations have been done by finite element method (FEM), using well-known ANSYS software. A discrete mesh has been generated with the following main parameters: number of nodes is 73,007, number of FEs is 15,672, in particular, formation of 4 FE-layers in normal directions of the cup and of 2 FE-layers in the *In*-bonding layer. Special attentions have been paid to get FE-nodes at the midsurface and to have a corresponding mesh densification at the stress concentration domains.

The cyclic variant of FEM procedure has been used to avoid any false mistuning (mismatches) in the structure due to numerical errors. Three major variants of the fixtures has been considered: (*i*) a totally-free resonator, (*ii*) "a mushroom design" (one-side fixture by the inner stem only), and (*iii*) a two-stemmed fixture. All these variants used the same real Ø40mm-hemispherical resonator (with a small axial eccentricity of its inner and outer faces).

For comparison of FEM data and Rayleigh-type analytic solution, eigen functions have been normalized in the same way. Analytical results obtained for a complete-hemisphere and for shell-band solutions have been shown at the fig.2 on the background of the FEM numerical experiment.

The comparison shows that the maximal difference for the midsurface deflections is less are than 43% only. So, the Rayleigh-type approximation visually looks almost-perfect one for analyses of the displacement fields and principal frequencies.

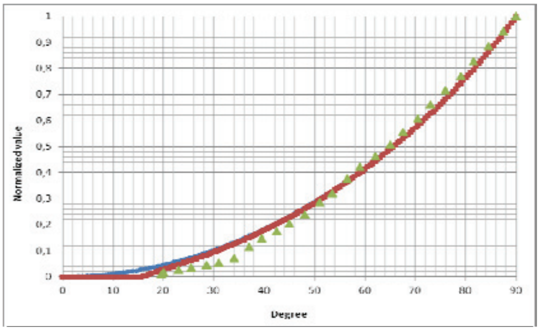

Figure 2 Rayleigh's analytics for a complete hemisphere (blue curve), for a polar zone (appr.16) (red curve), and data of the FEM numerical experiment (greens).

However, the small boundary effects in respect to midsurface strains are much more dangerous then it seems due to an essential drop down of resonator' s *Q*-factor by these effects. Let us note that intrinsic relaxational losses dominate in bulk silica, and surface losses dominate in usual free-suspended resonators. The local gradients of mean strains are responsible also for thermo-elastic damping in such structures[21].

Influences of midsurface tensility have been noted at a vicinities of (*a*) Polar zone and of (*b*) free-edge zone (so-called, shell edge effects). The shell under consideration has also non-linear distributions of strains along the normals to the midsurface (differences between 3D and classic Kirchhoff-Love' s shell model) that caused additional strains at some domains of the inner and outer faces of the cup.

Local tri-dimensional effects at the cup-stem transient zone (polar zone) have been studied also, such as: (*a*) a 3D-effect at the equatorial rim, (*b*) a shift of " a dynamic Pole" (a central axial node point) from its geometrical position to a location inside the inner stem, (*c*) characteristic lengths of penetration of 3D edge effects along the inner and outer stems, *etc*.

The mentioned 3D-effects are enough sensitive to local design details: (*i*) a pair of chamfers at the free edge of the cup, (*ii*) transient radii at a cup-stem monolithic connection; a combination of (*iii*) a chamfer at the hole of the case and of (*iv*) an indium transient " collar" and the entry section of stem-case connection.

5. Design Optimization

A rigid fixture of the stem can be defined as " *a theoretical limiting case*" of the assembly design. The corresponding optimal length, providing a maximal range between operational multiply eigen frequencies of the 2nd flexural mode and of the side (parasitic) modes, have been found for cases of a cantilevered " mushroom" design and taking into account a second fixture at the outer stem. The most dangerous resonant sizes are found too, and they look enough close ones to the optimum sizes. So, a high accuracy of the simulation is required to be really helpful for the engineering practice.

" *A practical limiting case*" like a perfect glass fixture with zero-thickness perfect connection (a complete acoustic contact) has been considered too and the corresponding optimal and dangerous lengths have been calculated also.

To be sure that " resonant sizes" have been estimated with the accuracy required for practice (*i.e.*, less than parts of a millimeter), and to know exactly that safety range for the frequencies (more than 1.5kHz) has been really-guaranteed for a designer, we investigated several variants of indium bonding, and a table with corresponding minor corrections of the optimum sizes have been obtained.

90°-geometry of the inside angle cannot be used for FEM simulations directly (we mean its pure-elastic variant) due to an infinite stress concentration. (It can be characterized by Irvin' s stress intensity coefficient.) So, optimization of *Q*-factor and plasticity inside the connecting layer are subjects of separate analyses.

Table 1

Variant	Opt Length, mm	Bad Lengths, mm
Rigid Case	10.5	5.9
Glass Case	12.7	7.9
0.1mm-Indium Layer	12.2	8.0
0.3mm-Indium Layer	12.8	8.1
0.5mm-Indium Layer	13.1	8.2

Let us note that Indium plastic flows are starting at a very low yield stress (about 2 orders less then steel), and it has a very high Poisson' s ratio (like a rubber) that causes additional strains inside the connecting layer under any loading. For a mistuned resonator assembly, pendular modes excited together with operational ones. So, such a flow can damped them and decreased the operational *Q* too.

To avoid a plastic flow of the indium in a vicinity of the angular line at the " entry" of the stem-case connection a transient rounding (" an *In*-collar") and a chamfer at the hole have been used. This design let us use softer requirement for the tuning targeting decreasing of the coupling between the pendular and operational modes.

6. Natural Experiments

Sensitivities of eigen frequencies to variations can be estimated(19), using physical constants defined by the standard(22) for quartz glass like КУ-1, and using data about Indium from(23).

Unique Ø40mm-resonators for HRG had been designed, fabricated, and tested(24). So, let us compare FEM data and experimental data averaged for a party of resonators(25).

An experimental test set-up and corresponding experimental technique have been developed by(26) for glass and sapphire samples, and later these matters have been developed for HRG too. For our matters, an original fixture has been designed and fabricated too(25).

Figure 3 STM's resonator for HRG made from high-purity quartz glass, as a single-piece of 40mm-hemispherical cup and stem through it

Even without any fitting of elastic moduli and mass density of the quartz glass, comparison of frequencies measured for the resonators shows a minor systematic difference between the eigen frequency (mean value for the party of resonators) and results of our FEM analyses. It is about 135Hz (<1.8%) only. It can be explained by a small residual stiffness induced by the experimental set-up or by minor variations of the mentioned physical constants for a specific lot of the glass.

The range of individual frequency differences of the resonators is more notable (appr. ±500Hz instead of small mismatches <0.5Hz), and it is a subject of more detail studies based on insights in technology used (residual stresses, surface tension, *etc.*), and taking into account manufacturing errors (asphericities, thickness variations, physical non-homogeneities, variations at the edges, *etc.*).

7. Conclusions

Statement of design of parasitic internal resonances problem for the resonator-case assembly has been formulated and corresponding design, analytical and numerical tools have been developed.

Consistency of the data obtained has been proven by comparison of the data obtained by analytical tools, by FEM software, and by direct tests of the HRG resonators. These techniques let us avoid any internal resonances as early as at a design phase of development, and provide a practical basement to estimate behaviors of the modern resonators and assemblies. So, the methods used are enough adequate and accurate to be applied effectively during the designing.

Optimization of the design in respect to its *Q*-factor, and ranging of the frequencies for a party of resonators and their assemblies has to be studied additionally.

Acknowledgements

The authors are grateful to students and staff at department of Aerospace Engineering at Sunchon National University of Korea for their assistance during numerous FEM simulations, and to engineers of the STM Co. / ATS of Ukraine providing original experimental technique and tools.

References

(1) Strutt J.W. (Baron Rayleigh). Theory of Sound. Vol.1. – 2nd edn. New York. Dover Publications. 1945. – pp.418-432.

(2) Bryan G.H. On the Beats in the Vibrations of a Revolving Cylinder or Bell //Proceedings of the Cambridge Philosophical Society. 1890, Nov.24. Vol.VII. – P.101-111.

(3) Love A.E.H. A Treatise on the Mathematical Theory of Elasticity. – 1st edn, Cambridge. University Press. 1892. – 4th edn, Dover Publications. New York. 1944.

(4) Novozhilov V.V. Estimation of Stresses in a Thin Spherical Shell under an Arbitrary Load //Dokladi USSR's Academy of Sciences. 1940. Vol.XXVII. No.6. (In Russian.)

(5) Hobson E.W. The Theory of Spherical and Ellipsoidal Harmonics. – Cambridge Press. Cambridge, England. 1931. – 500p.

(6) Luzhin O.V. On Free Vibrations of a Thin Spherical Shell //Structural Mechanics & Computations. 1961. No.3. – P.32-37. (In Russian.)

(7) Lizarev A.D., and Rostanina N.B. Vibration of Metal-Polymeric Homogeneous Spherical Shells. – Minsk. "Nauka i Technika" Publisher. 1984. –192p. (In Russian.)

(8) Smirnov A.L. Free Vibrations of the Rotating Shells of Revolution //Journal of Applied Mechanics. 1989. Vol.56. No.2. — P.423-429.

(9) Sarapuloff S.A., Kisilenko S.P., and Iosifov A.O. Rotation Influence on Imperfect Hemispherical Resonator Dynamics //"Mechanics of Gyroscopic Systems" Collection of Scientific Papers. Issue 7. 1988. Kyiv. – P.59-66. (In Russian.)

(10) Lynch D.D. Hemispherical Resonator Gyro //EOS Technical Journal. Vol.3. No.1. Winter 1995. – P.11-22.

(11) Meyer A., and Rozelle D. Milli-HRG Inertial Navigation System //Proceedings of IEEE/ION PLANS 2012. Myrtle Beach, South Carolina. April 2012. – P.24-29.

(12) Fan Shang-Chun, Liu Guang-Yu, and Wang Zhen-Jun. On Vibration of Hemispherical Shell by Using Finite Element Method //Applied Mathematics and Mechanics. English Edition. Vol.12. No.12. Dec. 1991. Published by SUT. Shanghai, China. – P.1169-1175.

(13) Matveev V.A., Naraykin O.S., and Ivanov I.P. Calculation of Hemispherical Resonator on Computer //Izvestiya Vuzov. Priborostroyeniye. 1987. No.7. – P.6-9. (In Russian.)

(14) Naraykin O.S. Method of Numerical Solution of Dynamical Shell Theory Problems //Applied Mechanics. 1988. Vol.24(34). No.2. – P.30-38. (In Russian.)

(15) Egarmin N.E., and Yurin V.E. Introduction to Theory of Vibratory Gyroscopes. – Moscow. Binom Publishing Co. 1993. (in Russian).

(16) Bodunov, B.P., Lopatin V.M., Lunin B.S., Lynch D.D., and Voros A. Inexpensive Hemispherical Resonator for Small-Size HRGs in Navigation Systems for Civil Application //Proceedings of the IV-th St.-Petersburg International Conference on Integrated Navigational Systems. 26-28 May 1995. St.-Petersburg, RF. – P.89-92.

(17) Lunin B.S., and Shatalov M.Y. Investigation into Low-Frequency Spectrum of Hemispherical Resonators of Vibrating Gyroscopes //Proc. the X-th Saint-Petersburg International Conference on Integrated Navigation Systems. St.-Petersburg, RF. 2003. – P.326-331.

(18) Antkowiak B. Simulation of Modal Vibration Pattern Variations due to Gyroscopic Effects in an Active Vibration Controlled Structure #2004-23 //Presentation at MSC Software Virtual Product Development Conference. Huntington Beach, CA, USA. 2004. – 15p.

(19) Sarapuloff S.A. Analyses of HRG Resonators. – Final Report of The STM Co. / ATS of Ukraine to Space & Launch Div. of Litton Guidance & Control Corp., CA, USA according STCU's Project #558 of 1998. – Science & Technology Council for Ukraine (STCU). Kyiv. 2000.

(20) Idem.

(21) Sarapuloff S.A. Development and Cost Reduction of High-Q Dielectric Resonators of Solid-State Gyroscopes //The VIII-th International Conference on Integrated Navigation Systems. – St. Petersburg. RF. 2001.

(22) USSR's Standard ГОСТ 15130-86 "Optic Quartz Glass". – Moscow. 1987. (In Russian.)

(23) Web-site of Indium Corp. of America.

(24) Sarapuloff S.A., et al. Design and Development of K-HRG. – Final Report The STM Co. / ATS of Ukraine to Doosan Infracore Corp., Korea. 2008.

(25) Sarapuloff S.A., et al. High-Q Hemispherical Resonators of Solid-State Coriolis' Gyroscopes for Space Mission and Other Applications. – P.35. In book: Catalogue of Novel Technologies. – Chapter. Sensors. – Published by Science & Technology Council for Ukraine (STCU). Kyiv.

(26) Braginskii V.B., Mitrofanov V.P., and Panov V.I. Systems with Small Dissipation. – Moscow. Nauka. 1981. – 144p.