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Oscillation of Microbeam Structure with Irregular Mass Distribution

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Key Words : Euler-Bernoulli beam, thermoelastic damping, attached mass, Q-factor

ABSTRACT

In this study, an analytical model of micro-beam structure including thermoelastic damping with irregularly distributed masses is investigated. The significance of thermoelastic damping for micro-scale mechanical resonators is evaluated to design with high quality factor(Q-factor). The beam model of this work is based on Euler-Bernoulli beam theory. In order to determine the natural frequency of the model, energy method is applied. Also, the thermoelatic damping effects are considered by using heat conduction equations, and the Q-factor can be determined. To derive the equation of motion, non-dimensionalization is employed for systematic analysis. Results of the model are verified, and present mode shapes and Q-factors for the micro-beam with thermoelastic damping including random point masses.

본 연구에서는, 해석적 모델로 불규칙하게 분포된 질량을 가진 열탄성 댐핑을 포함하는 마이크로빔 구조물을 연구하였다. 마이크로 스케일의 기계적 공명체(mechanical resonator)에 대한 열탄성 댐핑의 중요성은 높은 Q-factor 를 설계하는데 고려된다. 본 연구에서의 빔 모델은 Euler-Bernoulli 빔 이론을 기초로 한다. 빔의 고유 진동수를 결정하기 위하여, 에너지 기법이 적용되었다. 또한, 열탄성 댐핑 효과는 열전도 방정식을 사용함으로써 고려되었고, Q-factor 가 결정될 수 있었다. 운동방정식의 유도에는 체계적인 무차원화를 수행하였다. 임의의 집중된 질량을 포함하는 열탄성 댐핑을 가진 마이크로빔에 대해 모델의 결과값을 입증하였고 mode shape 과 Q-factor 를 제시하였다.

1. Introduction

Sensitive devices such as resonators have been developed for rate sensors, and requires high Quality factors(Q-factors). The factor is defined as the ratio of the total kinetic and potential energy of the system to dissipation that occurs due to various damping mechanisms [1]. Now, it has been well known that one of the most important energy loss factors is thermoelastic damping in micro- and nano-structures. Moreover, thermoelastic damping effect is one of the most important factors to estimate the quality of the resonators.

A number of studies on thermoelastic damping for microstructures have been performed by Lifshitz and Roukes [2] performed general research on thermoelastic damping for the micro- and nano-mechanical systems(MEMS and NEMS). Yi [3] investigated the geometric effect on thermoelastic damping in MEMS resonators. Wong et al. [4] presented the damping of the in-plane vibration of thin silicon ring.

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Practical point of view, imperfection of the resonator may be inevitable. Thus, the structures include the imperfection as point masses. Wu and Lin [5] performed free vibration analysis of a uniform cantilever beam with point masses. Dwivedy and Kar [6] analyzed a natural frequency and combined effects from internal resonance of slender beam with an attached mass. Wu and Chen [7] studied bending vibrations of wedge beams with any number of point masses.

In this paper, Euler-Bernoulli beam including attached masses is investigated. To obtain the natural frequency of the beam, energy method is adopted. This work is concerned with the determination of the fundamental bending frequencies and effect of Q-factors for a beam with point masses. Results of the model are verified and present Q-factors for the micro-beam with thermoelastic damping including random point masses. As a beam model, the clamped-clamped beam is used.

2. Formulations

Thermoelastic damping of a beam with random point masses is considered to analyze the Q-factor of the model. The model is a thin beam with rectangular cross-section. Fig. 1. shows a model with attached irregular point masses.

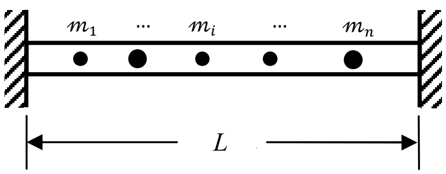

Fig. 1. A model of beam with n attached point masses m_i.

2.1 Thermoelastic damping equation of motion

For beam theory, plane stresses σ and strains ε relation by Hooke's law are

$$\varepsilon_{xx} = \frac{1}{E}\sigma_{xx} + \alpha\theta \qquad (1.a)$$

$$\varepsilon_{yy} = \varepsilon_{zz} = -\frac{\nu}{E}\sigma_{xx} + \alpha\theta \qquad (1.b)$$

$$\varepsilon_{xy} = \varepsilon_{yz} = \varepsilon_{zx} = 0 \qquad (1.c)$$

In here, $\varepsilon_{\alpha\beta}$ and θ are the functions of x and y. Specifically, θ and ν denote the variation in temperature from the ambient temperature and Poisson's ratio, respectively. And α is the linear thermal-expansion coefficient.

For Euler-Bernoulli beam, the thermoelastic equation of motion is given by Ref. [2] and attached masses are adopted as [7]

$$\rho A\frac{\partial^2 Y(x,t)}{\partial t^2} + \sum_{i-1}^{n} m_i \frac{\partial^2 Y(x,t)}{\partial t^2}\delta(x-x_i) + \frac{\partial^2}{\partial x^2}\left(EI\frac{Y(x,t)}{\partial x^2} + E\alpha I_T(x,t)\right) = 0 \qquad (2)$$

where

$$I = \int_A y^2 dydz \quad and \quad I_T(x) = \int_A y\theta(x,y,t)dydz \qquad (3)$$

In here, x is the axial coordinate, Y is the transverse deflection, E is Young's modulus, ρ is the mass density of material, A is the cross-sectional of the beam and t is time. I and I_T are integrals over the cross section of the beam giving the mechanical and the thermal contributions to its moment of inertia. And, m_i denotes i th concentrated mass, and $\delta(x-x_i)$ is the Dirac delta function at x_i .

$$\left(1+2\Delta_E\frac{1+\sigma}{1-2\sigma}\right)\frac{\partial\theta(x,y,t)}{\partial t} = \chi\frac{\partial\theta(x,y,t)}{\partial y^2} + y\frac{\Delta_E}{\alpha}\frac{\partial}{\partial t}\left(\frac{\partial Y(x,t)}{\partial x^2}\right) \qquad (4)$$

where χ is the thermal diffusivity.

2.2 Harmonic vibrations

To consider the effect of thermoelastic coupling on the vibrations of a beam, equations (2) and (4) are to solve. Kinetic energy T and strain energy V can be written as

$$T = \int_0^T \left(\begin{aligned} &\frac{1}{2}\rho A \left(\frac{\partial Y(x,t)}{\partial t} \right)^2 \\ &+ \frac{1}{2}\sum_{i=1}^{n} m_i \left(\frac{\partial Y_0(x,t)}{\partial t} \right)^2 \delta(x - x_i) \end{aligned} \right) dt \tag{5}$$

$$V = \int_0^T \left(\frac{1}{2} EI \left(\frac{\partial^2 Y(x,t)}{\partial x^2} \right)^2 + \frac{1}{2} E\alpha \left(I_T(x,t) \right)^2 \right) dt \tag{6}$$

$Y(x,t)$ and $\theta(x,y,t)$ are assumed as Ref. [2]

$$Y(x,t) = Y_0(x)e^{i\omega t}, \quad \theta(x,y,t) = \theta_0(x,y)e^{i\omega t} \tag{7}$$

Substituting Eq. (7) into the heat equation (4) and neglecting the term of order Δ_E on its left-hand side.

Condition no flow of heat across the boundaries of the beam such as $\frac{\partial \theta_0}{\partial y} = 0$ at $y = \pm b/2$. The temperature profile across the beam is then given by

$$\theta_0(x,y) = \frac{\Delta_E}{\alpha} \frac{\partial^2 Y_0(x)}{\partial x^2} S(y) \tag{8}$$

where

$$S(y) = y - \frac{\sin(ky)}{k\cos(bk/2)}, \quad k = \sqrt{i\frac{\omega}{\chi}} = (1+i)\sqrt{\frac{\omega}{2\chi}}$$

Substituting Eq. (3) into the Eq. (6), the equation can be obtained as

$$V = \frac{1}{2} EI \left(\frac{\partial^2 Y_0(x)}{\partial x^2} \right)^2 + \frac{1}{2} E\Delta_E \left(\frac{\partial^2 Y_0(x)}{\partial x^2} \right)^2 \int_A yS(y)dA \tag{9}$$

Adopting a clamped-clamped beam, boundary conditions is $Y_0(x) = \frac{\partial Y_0(x)}{\partial x} = 0$. Therefore, the natural frequency relation can be obtained as

$$\begin{aligned} &\left(1 + \sum_{i=1}^{n} \frac{m_i}{\rho A} \delta(x - x_i) \right) \omega^2 Y_0(x) \\ &= \frac{EI}{\rho A} \left(1 + \frac{\Delta_E c}{I} \int_{-b/2}^{b/2} yS(y)dy \right) \frac{\partial^4 Y_0(x)}{\partial x^4} \end{aligned} \tag{10}$$

In order to perform systematic analysis, it is convenient to transform the parameters to non-dimensionalized form as below

$$\begin{aligned} &X = \frac{x}{L}, \quad X_i = \frac{x_i}{L}, \quad Y = \frac{y}{H}, \quad M_i = \frac{m_i}{M}, \quad B^* = \frac{b}{B}, \\ &C^* = \frac{c}{C}, \quad I^* = \frac{I}{L^4}, \quad \tau = \frac{t}{L^2}\sqrt{\frac{EI}{\rho A}}, \quad \mu = \frac{m}{\rho AL} \end{aligned} \tag{11}$$

Substituting the parameters of Eq. (11) into Eq. (10), the equation is rewritten as

$$\begin{aligned} &\left(1 + \sum_{i=1}^{n} \mu M_i \delta(X - X_i) \right) \omega^2 Y_0(x) \\ &= \tau^2 \left(1 + \frac{\Delta_E C^*}{I^*} \int_{-b/2}^{b/2} YS(Y)dy \right) \frac{\partial^4 Y_0(x)}{\partial x^4} \end{aligned} \tag{12}$$

Eq. (12) can be rewritten as

$$\begin{aligned} &\left(1 + \sum_{i=1}^{n} \mu M_i \delta(X - X_i) \right) \omega^2 Y_0(x) \\ &= \tau^2 \left\{ 1 + \Delta_E \left[1 + f(\omega) \right] \right\} \frac{\partial^4 Y_0(x)}{\partial x^4} \end{aligned} \tag{13}$$

where the complex function $f(\omega)$ is given by

$$f(\omega) = f(k(\omega)) = \frac{24}{b^3 k^3} \left[\frac{bk}{2} - \tan\left(\frac{bk}{2} \right) \right]$$

In Eq. (13), because Young's modulus E is a frequency-dependent, it is replaced by

$$E_\omega = E\left\{ 1 + \Delta_E \left[1 + f(\omega) \right] \right\} \tag{14}$$

The dispersion relation between ω and q_n for the thermoelastic beam is given by

$$\omega = \sqrt{\frac{E_\omega I}{\rho A\left(1+\sum_{i=1}^{n}\mu M_i\delta(X-X_i)\right)}q_n^2} \tag{15}$$

$$= \omega_0\sqrt{\frac{1+\Delta_E\left[1+f(\omega)\right]}{1+\sum_{i=1}^{n}\mu M_i\delta(X-X_i)}}$$

Using Taylor series, the dispersion relation becomes

$$\omega = \omega_0\left[\frac{1+\frac{\Delta_E}{2}\left[1+f(\omega_0)\right]}{1+\sum_{i=1}^{n}\mu M_i\delta(X-X_i)}\right] \tag{16}$$

From Eq. (16), the real and imaginary parts can be extracted as

$$\mathrm{Re}(\omega) = \omega_0\left[\frac{1+\frac{\Delta_E}{2}\left(1-\frac{6}{\xi^3}\frac{\sinh\xi-\sin\xi}{\cosh\xi+\cos\xi}\right)}{1+\sum_{i=1}^{n}\mu M_i\delta(X-X_i)}\right] \tag{17.a}$$

$$\mathrm{Im}(\omega) = \omega_0\frac{\frac{\Delta_E}{2}\left(\frac{6}{\xi^3}\frac{\sinh\xi+\sin\xi}{\cosh\xi+\cos\xi}-\frac{6}{\xi^2}\right)}{1+\sum_{i=1}^{n}\mu M_i\delta(X-X_i)} \tag{17.b}$$

where

$$\xi = b\sqrt{\frac{\omega_0}{2\chi}}$$

The amount of thermoelastic damping is expressed as inverse of the Q-factor.

$$Q^{-1} = \frac{E\alpha^2 T_0}{C}\left(\frac{6}{\xi^2}-\frac{6}{\xi^3}\frac{\sinh\xi+\sin\xi}{\cosh\xi+\cos\xi}\right) \tag{18}$$

where

$$Q^{-1} = 2\left|\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right|$$

3. Results and Discussion

In this chapter, the Q-factor of the model is verified and numerical study is performed. Through the verification of formulations, acceptable results can be confirmed.

3.1 Verification

Without the thermoelastic term, the Eq. (10) becomes the frequency of a beam with attached masses as in Ref. [8]. This equation is equivalent to the Ref. [8] using Rayleigh's energy method.

Considering a beam with thermoelastic property only, then Eq. (13) has exactly equal form with previous work.

In this work, in order to perform the systematic analysis, non-dimensionalization for equations is used. As a material of model, silicon is adopted and thermoelastic damping effect is investigated. Fig. 2. shows thermoelastic damping in silicon thin rectangular beam.

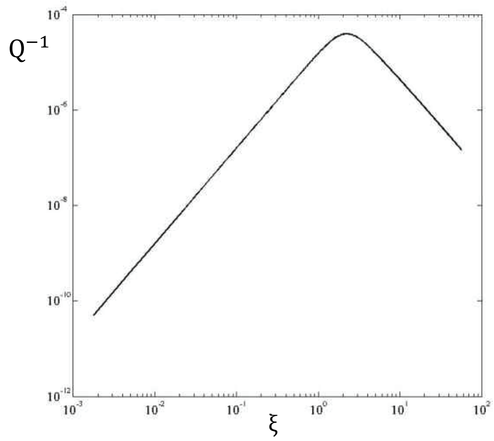

Fig. 2. Thermoelastic damping in silicon thin rectangular beam.

In here, the configuration of thermoelastic damping curve is same with previous study [2].

3.2 Effect of attached masses

The universal behavior of the normalized frequency shift $[\mathrm{Re}(\omega)-\omega_0]/\omega_0\Delta_E$ and of the normalized attenuation $\mathrm{Im}(\omega)/\omega_0\Delta_E$ as functions of the dimensionless variable ξ are shown in Fig. 3.

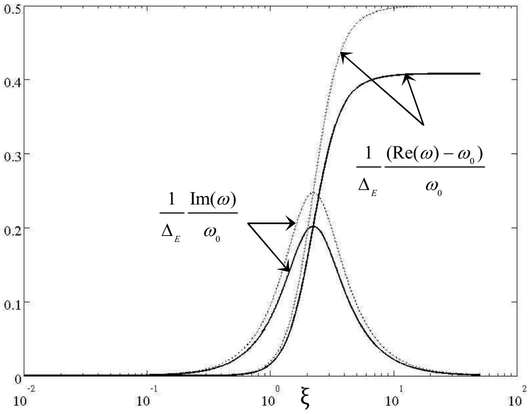

Fig. 3. The frequency shift and attenuation of small flexural vibrations in beams due to thermoelastic coupling.

In here, dotted lines are frequencies of thermoelastic damping in thin rectangular beams [2], and solid lines denote frequencies with random point masses. For the proportion of attached masses, 5% of total mass is applied. In case of concentrated masses, the magnitude of vibration is decresed. As a result, though the amplitude of the frequencies for beams can be different as shown Fig. 3., but thermoelastic damping effect is same.

4. Conclusions

In this work, natural frequencies and Q-factor of micro-beam considering thermoelastic damping and attached point masses are investigated. Based on the thermoelastic coupling, the equation of motion is derived. And, as comparing with previous studies, thermoelastic effect in beam is presented. Also, in order to show the generalized tendency of thermoelastic damping, non-dimensionalization is performed. Comparing with the previous study of thermoelastic damping, the amplitude of vibration for the beam with attached masses is decreased. And, though the magnitude of the frequencies for beams(with attached masses and without attached masses) can be different, but thermoelastic damping effect is same. As a result, for thermoelastic damping of beam with attached point masses, the imaginary and real values of Q-factor are variable, but its ratio is constant.

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