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Wave Propagation in the Strip Plate with Longitudinal Stiffeners

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Key Words : strip stiffened plate, wave propagation, waveguide finite element method

ABSTRACT

It is important to understand the vibrating behavior of plate structures for many engineering applications. In this study, vibration characteristics of strip plates which have finite width and infinite length are investigated theoretically and numerically. The waveguide finite element approach is used in this study which is known as an effect tool for waveguide structures. WFE method requires only cross-sectional FE model and uses theoretical harmonic solutions for the wave propagation along the longitudinal direction. First of all for a simple strip plate, WFE results are compared with theoretical ones such as the dispersion diagrams, point mobilities, etc. to validate the numerical model. Then in the numerical analysis, the several different types of longitudinal stiffeners are included to the plate model to investigate the effects of the stiffeners in terms of the dispersion curves and mobilities.

1.Introduction

Many large structures like ships, trains, etc are built up with plates with complex stiffeners. These structures are often simplified as waveguides which have uniform cross-sections along their lengths. To be able to predict the vibrational responses of these waveguide structures, it is necessary to understand propagating behavior of the waves in stiffened plates as a basic element structure. In this paper, vibration characteristics of infinite length strip plates are investigated numerically by means of the waveguide finite element (WFE) method. The WFE method models only 2D cross-sections of the waveguide structures but takes into account the three dimensional nature of the infinite extent of the waveguide. In the numerical analysis, the several different types of longitudinal stiffeners are combined with strip plates to investigate the effects of stiffeners on the wave propagation in plates in terms of the dispersion diagrams and mobilities. Finally, a stiffened double plate is formed to examine its characteristics of wave propagation.

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2. Theoretical analysis

Before starting numerical analysis for strip plates, a theoretical analysis is performed for a simply supported strip plate without stiffeners in order to understand general behavior of the base strip plate. A strip plate considered in this study has width l_y in the y direction and infinitely long in the x direction, as illustrated in Fig. 1. Properties and dimensions of the strip plate are given in Table 1.

For a thin undamped plate, the z directional displacement $w(x,y,t)$ satisfies the following differential equation.

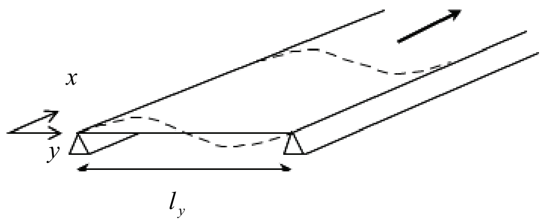

Fig. 1 A simply supported plate strip model.

Table 1. Dimensions and material parameters of plate strip

Parameters	Value	Units
E	7.1×10^{10}	N/m^2
υ	0.332	
h	6	mm
l_y	1	m
ρ	2.7×10^3	kg/m^3
η	0.1	

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^2 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\rho h\frac{\partial^4 w}{\partial t^4}=0 \quad (1)$$

where D is the bending stiffness, E is Young's modulus, h is the plate thickness, υ is Poission's ratio, and ρ is the mass density of the plate. Due to the simply supported boundaries at $y=0$ and $y=l_y$, the vertical displacement of the strip plate along the y direction has shapes of sine waves. It is assumed that the vertical displacement has time and spatial harmonic motion in the x direction. So $w(x,y,t)$ is defined as [1]

$$w(x,y,t)=\sum_{m=1}^{\infty}\sin(\frac{m\pi}{l_y})e^{-ik_{x,m}x}e^{i\omega t} \quad (2)$$

where m is the order of the strip plate's mode in the y direction. Substituting this into Eq. (1) yields

$$(k_{x,m}^2+k_{y,m}^2)^2=k_B^4 \quad (3)$$

$$k_{x1,m}=\pm\sqrt{k_B^2-k_{y,m}^2} \quad (4)$$

$$k_{x2,m}=\pm\sqrt{-k_B^2-k_{y,m}^2} \quad (5)$$

where k_B is the free bending wavenumber, k_x and k_y are the x and y directional wavenumbers of plate, respectively. Eq. (3) has four solutions of k_x which are divided into two different wave solutions for each m as given in Eq. (4) and Eq. (5). The frequency at which $k_B=k_y$ is referred to as m^{th} cut-on frequency ω_m. When a force is applied to the plate, the structural responses can be calculated from the sum of all the wave components sustained in the strip plate. For a force acting on $x=0$ and $y=y_0$, the point mobility is written by [2]

$$Y(\omega)=\sum_{m=1}^{\infty}\frac{\omega}{D' l_y k_{x1,m}(k_{x1,m}^2-k_{x2,m}^2)}\left[1-\frac{k_{x1,m}}{k_{x2,m}}\right]\sin^2\left(k_y y_0\right) \quad (6)$$

where $D'=E(1+i\eta)h^3/12(1-\upsilon^2)$. Dispersion diagrams and point mobility will be compared with the numerical ones later in this paper to validate the numerical results.

3. Waveguide finite element analysis

3.1 Equation of motion

The WFE equation is given by [3]

$$\left\{\mathbf{K_4}(-i\kappa)^4+\mathbf{K_2}(-i\kappa)^2+\mathbf{K_1}(-i\kappa)+\mathbf{K_0}-\omega^2\mathbf{M}\right\}\widetilde{\mathbf{\Phi}}=\mathbf{0} \quad (7)$$

where $\mathbf{K_4},\mathbf{K_2},\mathbf{K_1}$ and $\mathbf{K_0}$ are the matrixes that come from the stiffness of the structure, $\mathbf{M}$ is the mass matrix and $\widetilde{\mathbf{\Phi}}$ is the displacement vector representing shapes of cross-sectional deformation. Eq. (7) has two unknown parameters of frequency and wavenumber. If a wavenumber is given, Eq. (7) can be solved for the frequencies, which represent dispersion relations of the propagating waves. Conversely, if a frequency is given, Eq. (7) can be solved to get wavenumbers of all the waves including nearfield ones, which are required to predict forced responses.

Point mobility can be expressed at $x=0$ for $x>0$ [4]

$$Y(\omega)=i\omega\sum_{m=1}^{\infty}\mathbf{A_m}\widetilde{\mathbf{\Phi}}_\mathbf{m} \quad (8)$$

where $\mathbf{A_m}$ and $\widetilde{\mathbf{\Phi}}_\mathbf{m}$ are amplitude and deformation shape of the m^{th} wave.

3.2 Modeling of stiffened plate

The cross-sectional model of a plate with three stiffeners is shown in Fig.2. The base plate is simply supported at both ends and the properties of the base plate are the same as listed in Table 1. The thickness and height of the stiffeners are set to 0.6 cm and 5 cm, respectively. The distance between the adjacent stiffeners is set to 25 cm. In the cross-sectional FE modeling, plate elements are used as illustrated in Fig. 2. This WFE model consists of 55 elements with 56 nodes and has 218 degrees of freedoms. Fig. 2 illustrates a strip plate with three stiffeners but in the analysis the plates with a single and three stiffeners are investigated to evaluate the effects of the stiffeners.

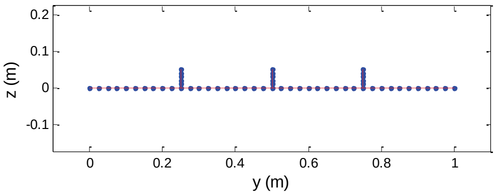

Fig. 2 Modeling a cross-section of stiffened plate.

3.3 Results for characteristics of plate with stiffeners

For the base plate without stiffeners, the dispersion diagram and point mobility predicted by the WFE method are shown in Fig. 3 and 4 and compared with the theoretical results. The dispersion diagram in Fig. 3 shows that there are many of propagating waves which are distinguished by the modes in the y direction as given in Eq. (2). As frequency increases, higher order modes are cut-on consecutively. The point mobility calculated at $x=0$ and y=0.425m shows that it has the maximum response at the cut-on frequency of the first mode. As compared in Fig. 3 and 4, the numerical results agree very well to the theoretical ones. Therefore it can be said that the WFE approach is useful to predict the structural response of the strip plate.

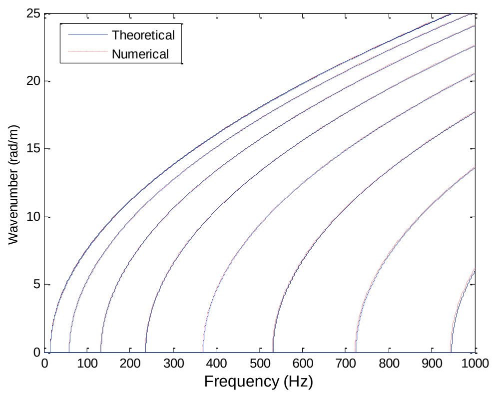

Fig. 3 The dispersion diagrams of strip plate.

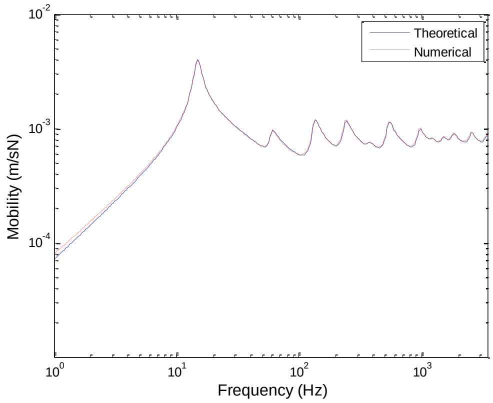

Fig. 4 The point mobilities of strip plate.

For a plate with a single stiffener, the dispersion diagrams can be obtained separately by giving the symmetric and anti-symmetric boundary conditions at the middle of the plate where the geometry becomes symmetry. To investigate an effect of a stiffener on the plate's dynamic response, a single stiffener of a height 12 cm is regarded. In the case of symmetric boundary condition in the middle of the plate, the dispersion relations are shown in Fig. 5, which correspond to the waves of possessing odd numbers of mode m in the y direction.

It can be seen from Fig. 5 that each dispersion curve can be divided into three regions of different slopes. In order to figure out the physical behavior of the each region, three equivalent systems are introduced and their dispersion curves are compared in Fig. 5 with those of the stiffened plate. The three systems regarded are an equivalent plate with the smeared mass of the stiffener, an equivalent beam [5] and a half width plate having a clamped support at one end and a simply support at the other end. It can be found from Fig. 5 that the dispersion curves of the stiffened plate are well approximated by these three equivalent structures for each frequency regions, respectively. Fig. 6 illustrates 3 dimensional deformation shapes of the second symmetric wave in Fig. 5 for two different frequencies. As shown in the upper plot in Fig. 6, displacement of the stiffened plate at 204 Hz, which belongs to the second region, has the similar deformation shape to that of an unstiffened plate but the bending stiffness must be predominantly governed by the stiffener. At a higher frequency of 350

Hz, on the contrary, displacement of the stiffener becomes nearly zero as shown in the lower plot in Fig. 6, which belongs to the third region of the dispersion curve. In this third region, dispersion curves of stiffened plate converge to those of the half width base plate simply supported at one end and clamped at the other end.

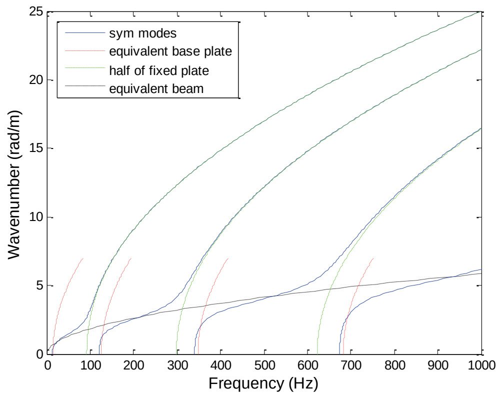

Fig. 5 Dispersion diagrams of symmetric mode of stiffened plate with a single stiffener

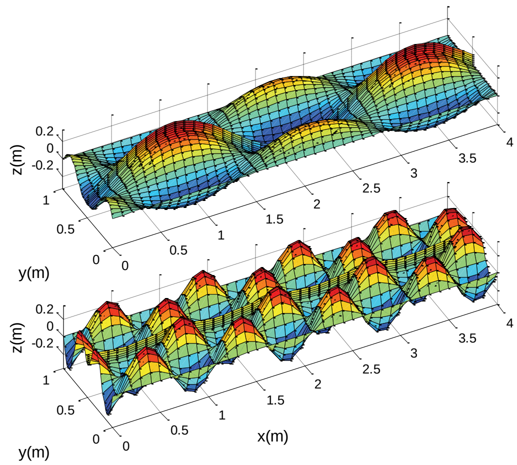

Fig. 6 Deformation shape of second symmetric modes at 204, 350Hz

In the case of anti-symmetric boundary condition in the middle of the plate, the dispersion relations are shown in Fig. 7. The dispersion curves in Fig. 7 are quite different from those in Fig. 5. The first curve in Fig. 7 is nearly the same as that of the unstiffened plate while the second one of the unstiffened plate is splitted into two dispersion curves in the stiffened plate case. This splitting is taken place by the vibration of the stiffener either in-phase or anti-phase with respect to the base plate. For this anti-symmetric wave, the stiffener works like a dynamic absorber.

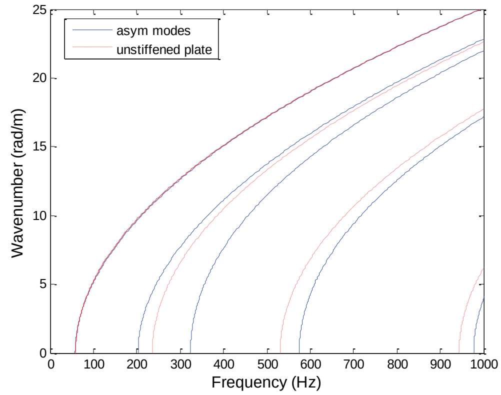

Fig. 7 Dispersion diagrams of asymmetric mode of stiffened plate with a single stiffener.

For the plate with three stiffeners shown in Fig. 2, the dispersion curves are illustrated in Fig. 8. As shown in Fig. 8, the dispersion curves are bounded into several grouped as wavenumber increases. Each group contains four waves in general, which correspond to the number of bays of the stiffened plate separated by three stiffeners. To approximate the dispersion curves of the stiffened plate, dispersion curves of the single bay structure are compared for simply supported-simply supported and clamped-clamped boundary conditions at both ends. Dispersion curves of the single bay of the stiffened plate are plotted in Fig. 8 together with those of the stiffened plate. It can be found in Fig. 8 that four waves in each group are bounded by those of the single bay. Note that there is an exceptional group (the third group) in Fig. 8 which is not bounded by the waves of the single bay structure. This group contains three curves, which represent bending waves propagating along the stiffeners. The deformation shape of the stiffened plate in the y direction in this group at 3922 Hz is shown in Fig. 9. Three stiffeners in Fig. 11 have quite large deformation in the y direction which does not appear in other groups. The stiffened region of the base plate also has the similar y -directional deformation as shown in Fig. 9

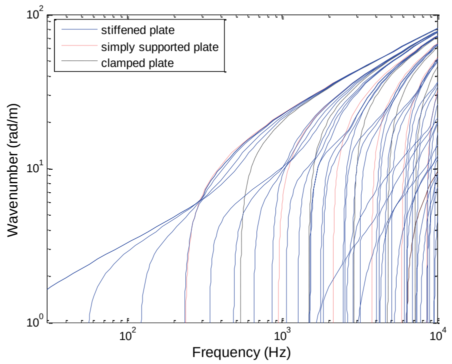

Fig. 8 Dispersion diagram of stiffened plate with three stiffeners.

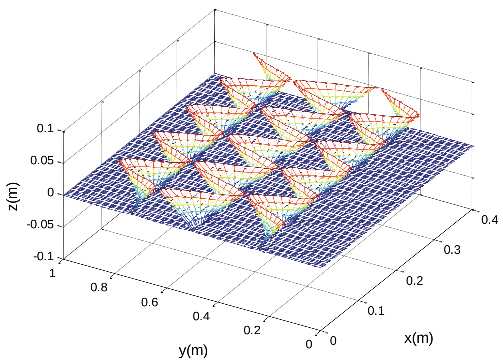

Fig. 9 Deformation shape in the y direction of the stiffened plate with three stiffeners.

Point mobilities were calculated for the unstiffened plate, the stiffened plate with single stiffener and three stiffeners at two different excitation points as shown in Fig. 10 and Fig. 11. The single stiffener has same properties and dimension to three stiffeners. When a point force is applied in the middle of the plate, the responses of these stiffened plates generally reduce compare to that of the unstiffened one, due to the presence of the stiffeners as shown in Fig. 10. If a point force is off the center of the plate, the point mobilities of these stiffened plates become lower than that of the unstiffened plate because of the stiffeners at low frequencies but at high frequencies these stiffened plates have a similar level of response with the unstiffened plate despite of the added stiffeners.

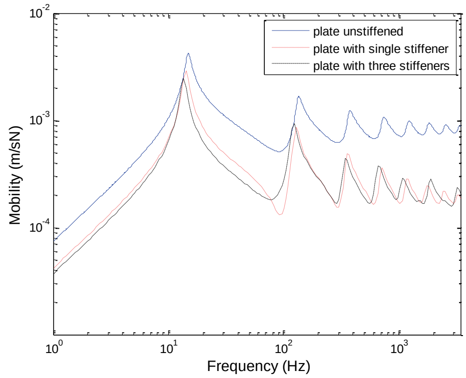

Fig. 10 Point mobilities of plates at y =0.5m.

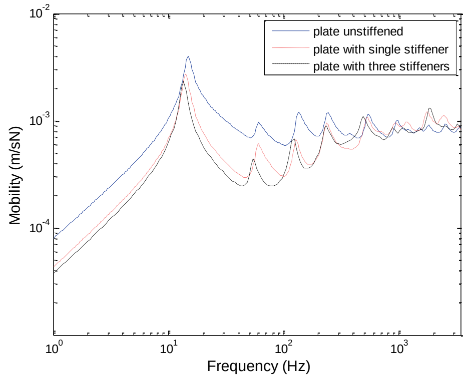

Fig. 11 Point mobilities of plates at y =0.425m.

3.4 Results for stiffened double plate

Wave propagation in a stiffened double plate is investigated by adding upper plate on top of the stiffeners as shown in Fig. 12. The upper plate has the same properties and dimension to the lower base plate. Dispersion curves of stiffened double plate are shown in Fig. 13. The curves in Fig. 13 may be classified into three types. The first two waves have flapping modes of the unconstrained bays in the upper plate. The next six waves correspond to those propagating along the six bays (four in the lower plate and two in the upper plate). These two types of waves are shown repeatedly in Fig. 13. Since the stiffeners are restricted by the lower and upper plates, the waves propagating along the stiffeners will occur at fairly high frequency so that not shown in Fig. 13. When the stiffened double plate is blocked by stiffeners at the both edges of the plate, dispersion curves of the stiffened double plate are shown in Fig. 14. Each group consists of eight waves in it as illustrated in Fig. 14. Flapping modes do not appear anymore because the upper plate is constrained by added stiffeneers. In each group, eight waves can be classified into two types. Four waves have in-phase, the rests have anti-phase between upper and lower plate. It can be found in Fig. 14 that eight waves in each group are bounded by those of the single bay as same as one of the stiffened plate with three stiffeners.

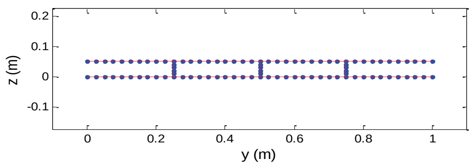

Fig. 12 Modeling a cross-section of stiffened double plate.

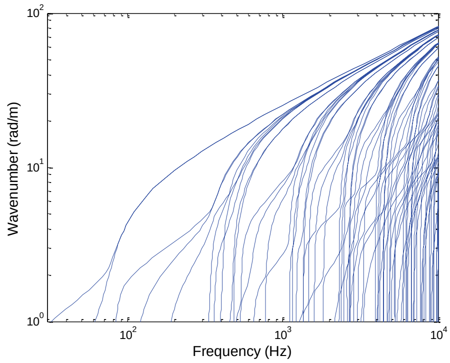

Fig. 13 Dispersion diagram of stiffened double plate.

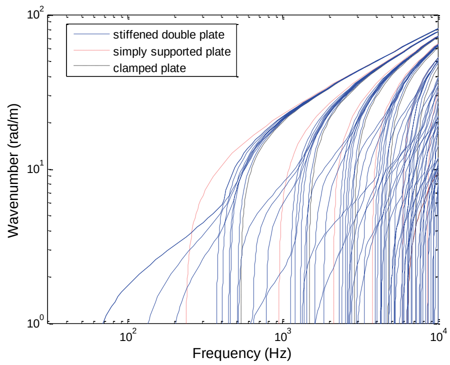

Fig. 14 Dispersion diagram of stiffened double plate blocked by stiffeners.

4. Conclusion

In this study, characteristics of wave propagation along plates with stiffeners were investigated numerically by using the WFE method. For a strip plate with a single stiffener, it was found that the stiffener works to increase the stiffness of the base plate at low frequencies while it acts like a fixed boundary at high frequencies. From the investigation for a plate with three stiffeners, it was observed that the wave behavior of the stiffened plate can be interpreted clearly with that of the bay segments. In terms of the mobility, it was seen that the stiffeners reduce the responses of the plates at low frequency but at high frequencies these stiffened plates may give similar level of response with one of the unstiffened plate depending on the excitation and receiving locations. Even for the stiffened double plates, the dispersion curves are grouped with the number of bay segments and well described by the wave behavior in the bay segment.

The forced responses will be calculated for the stiffened double plate as the next step. Based on the work presented in this paper, the sound radiation from stiffened plates is going to be calculated by coupling boundary element to the WFEs in further studies.

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