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Application SVD-Least Square Algorithm for solving astronomical ship position basing on circle of equal altitude equation

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Abstract : *This paper presents an improvement for calculating method of astronomical vessel position with circle of equal altitude equation based on using a virtual object in sun and two stars observation. In addition, to enhance the accuracy of ship position achieved from solving linear matrix system, and surmount the disadvantages on rank deficient matrices situation, the authors used singular value decomposition (SVD) in least square method instead of normal equation and QR decomposition, so, the solution of matrix system will be available in all situation. As proposal algorithm, astronomical ship position will give more accuracy than previous methods.*

Key words : *Celestial navigation, circle of equal altitude, vector-matrix method, SVD decomposition*

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SIMAC lab.

1. Introduction

- Nowadays, with development of high technology have created many modern equipment to compute ship position accurately, such as GPS, INS. But, nothing can ensure that there are not any accidents to these. So, we should maintain the methods to backup when these modern equipment can뭘use.
- In 2010, at the IMO conference in Manila, celestial navigation received sustained emphasis Manila amendment, there is an explicit encouragement for the usage of electronic nautical almanacs and celestial navigation calculation software in Section B-II/1 of Chapter 2, "*Guidance regarding the master and the deck department*"
- Previous research use Line of position (LOP) to compute ship position, but LOP is not exact locus of ship. Actually, the exact locus of ship lie on Circle of position (COP), the other use COP, but there are some problems as complex procedure, not consider to the movement of ship position

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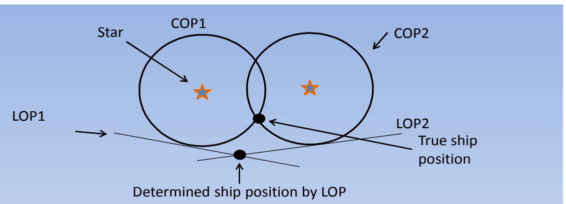

With above inference, we researched and developed celestial navigation method, it can use to backup. In this research, we use COP – vector matrix method and connect Singular Value Decomposition-least square method, virtual body to compute astronomical ship position.

As using COP, so determined ship position will give more accurate than previous method

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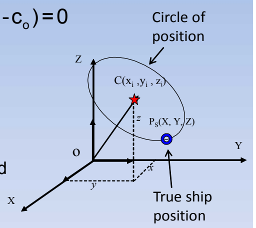
2. Establishing circle of equal altitude equation on Descartes coordinate system

The plane?equation of COP

$$x_i.(X-a_o)+y_i.(Y-b_o)+z_i.(Z-c_o)=0$$

$$\overrightarrow{OA} = k.\overrightarrow{OC_i}$$

$$A(a_o, b_o, c_o) = \overrightarrow{OA}(k.x_i, ky_i, kz_i)$$



k: scale coefficient of vectors was determined according to formula

$$k=\frac{|\overline{OA}|}{|\overline{OC}|}=\frac{|\overline{OA}|}{|\overline{OP_S}|}=\frac{|\overline{OP_S}|.\cos(90^0-h_S)}{|\overline{OP_S}|}=\sinh_S$$

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Expanding on equation of COP?plane and taking condition constrain

$$x_i^2+y_i^2+z_i^2=1$$

$$x_i.(X-\sinh.x_i)+y_i.(Y-\sinh.y_i)+z_i.(Z-\sinh.z_i)=0$$

We have equation of COP

$$x_i.X+y_i.Y+z_i.Z=\sinh$$

The above (Eq.2) is used to compute ship position directly.
If we are instead sphere coordinate of on Eq.2 that will receive

$$\sin H_O=\sin\varphi.\sin\delta+\cos\varphi.\cos\delta.\cos t_L$$

3. The SVD-Least square algorithm for solving astronomical ship position and virtual celestial body

Normal decomposition Least square algorithms

Matrix system for ship position

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ .. & .. & .. \\ x_n & y_n & z_n \end{bmatrix}.\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}+\begin{bmatrix} v_1 \\ v_2 \\ .. \\ v_n \end{bmatrix}=\begin{bmatrix} \sinh_1 \\ \sinh_2 \\ .. \\ \sinh_n \end{bmatrix}$$

$$A.x+v=b$$

$$\min\|v\|^2=(b-A.x)^T.(b-A.x)$$

$$A^T.A.\hat{x}=A^T.b$$

Solution $\hat{x}=(A^T.A)^{-1}.A^T.b$

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Singular Value Decomposition – least square algorithm for ship position

$$A=U.\sum V^T \text{where} \sum=\text{diag}(\varepsilon_1,\varepsilon_2,\varepsilon_3..\varepsilon_p)$$

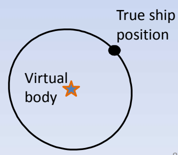
$$A=U.\sum V^T=\sum_{i=1}^{p}\sigma_i(u_i.v_i^T)$$ U(m x m), V(n x n) are orthogonal

Solution $x=V.\sum^{-1}U^T.b$

Using Singular Value Decomposition (SVD) have most accurate to solve matrix system for ship position

Virtual celestial object

Virtual celestial object is used in 2 stars and sun condition



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$$\sin(90^o-h_v)=\frac{v.t}{M}=\frac{v.t.(1-e^2.\sin^2\varphi_a)^{3/2}}{a.(1-e^2)}$$

$$\sin h_v=\sqrt{1-\left(\frac{v.t.(1-e^2.\sin^2\varphi_a)^{3/2}}{a.(1-e^2)}\right)^2}$$

The equation COP of virtual celestial object

$$x_v.X+y_v.Y+z_v.Z=\sinh_v$$

After that, from this system, we convert position which have been found to sphere coordinate system according to following formula

$$Lat=\text{atan}\frac{Z}{\sqrt{(X^2+Y^2)(1-e^2)}}$$

$$Long=\text{atan}\frac{Y}{X}$$

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4. Experiment results

- In this paper, some experiments were performed with numeric database of sun, stars on Do Son sea area – Hai Phong – Vietnam
- least square method can carry out in all situations of celestial bodies observation not fear celestial rank matrix deficient with most ship position? accuracy
- In case sun or two stars, It is insufficient equation for solving matrix system, It is necessary to use virtual body. In case, altitude observations are more than 2, we don칖need to use this body. Visual Basic 6.0 was also used to solve these problem

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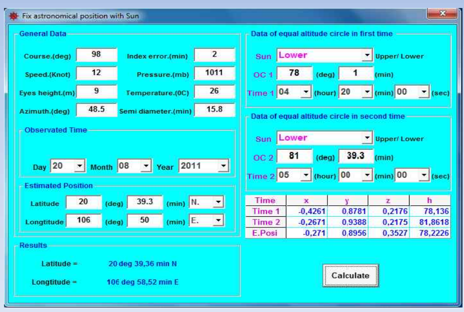
Case 1: fix by sun

UT	*Ho*	*x*	*y*	*z*
042000	780136	-0.4261	0.8781	0.2176
050000	8108618	-0.2671	0.9388	0.2175

ALat = $20^0$39? N, ALong = $106^0$50'E, TC = 980, *ZN* = + 07, (Aug 20,2011)



Interface for SVD vector matrix method with sun

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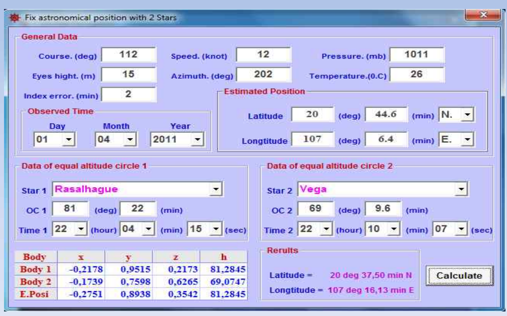
Case 2: fix by two stars

Body	UT	Ho	x	y	z
Rasalhague	*220415*	*81,2845*	*-0.2178*	*0.9515*	*0.2173*
Vega	221007	69,0747	- 0.1739	0.7598	0.6265

(Apr 1,2011), *ZN* = + 07, ALat = 20044? N, ALong = 107006? E, TC = 1120

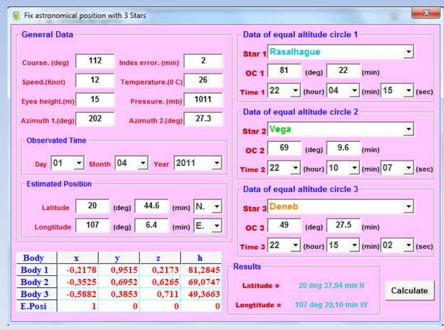


Interface for SVD vector matrix method with two stars

Case 3: fix by three stars

Body	UT	Ho	x	y	z
Rasalhague	*220415*	*81,2845*	*-0.2178*	*0.9515*	*0.2173*
Vega	221007	69,0747	- 0.1739	0.7598	0.6265
Deneb	221502	49.366	0.5882	0.3853	0.7110

(Apr 1,2011), *ZN* = + 07, ALat = 20044? N, ALong = 107006? E, TC = 1120



Interface for SVD vector matrix method with three stars

Intercept method (LOP)

Lat = $20^0$37? N, Long = $107^0$21? E

SVD – vector matrix method

Lat = $20^0$37? 4N, Long = $107^0$20? E

True position on chart

Lat = $20^0$38? N, Long = $107^0$19? E

Summary error in these above experiments to true position

Methods	Sun	2 stars	3 stars
Intercept method	4.45 NM	3.42 NM	1.649 NM
SVD-Least square algorithm	1.94 NM	2.46 NM	1.146 NM

5. Conclusion

The authors present an improvement to celestial navigation method to determine ship position basing on : circle of equal altitude equation, virtual body and SVD – least square method algorithms. The experiment results also show that ship position?accuracy to be more than traditional methods. This approach has the potential to be integrated with ECDIS

Although there are some improve, celestial navigation is not instead of GPS as low cost, high accuracy and reliability. However, It can be used to confirm and backup for GPS. So, In this paper, we carried out some development about celestial navigation method to be possible to take its become e - navigation system in the future.