한국소음진동공학회 2013년 추계학술대회논문집, pp. 848~849

Nonlinear Vibration Analysis of a Deploying and Spinning Beam

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1. Introduction

The nonlinear vibration of a spinning beam with deployment is analyzed when it is axially moving out from a fixed rigid hub. The vibration analysis of a spinning beam is important since it can be applied to the spinning structures such as robot manipulators, space vehicles, drilling machines and so on.

There are not many studies on a spinning beam with axial motion, especially only a few studies on a moving and spinning beam with variable beam lengths are searched from the author's literature survey. When dealing with the vibration analysis of a spinning beam with axial motion, previous studies neglected the axial displacement since it is small compared to lateral displacements. However, the axial displacement should be considered seriously since it is important when the spinning beam has axial motion. On the other hand, deployment has not received enough attentions. Present study includes the axial displacement and deployment to study the vibration behavior more exactly and comprehensively than previous studies.

The investigation procedure is carried out in following steps. First, a spinning beam with deployment is model ed and governing differential equations of motion are d erived by Extended Hamilton's Principle. The axial and lateral dispalcements are considered but the torsional di splacement is neglected. Then, the weak forms are disc retized by the Galerkin method. Finally, time response will be obtained by Newmark method. The differences between linear and nonlinear models are investigated. F uthermore, the special time-varying beat phenomenon is also studied and the reason of occurrence is also invest igated.

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2. Dynamic Modeling

The dynamic model for a spinning beam with depl oyment is shown in Fig. 1. The beam is axisymmetric and uniform. The length outside the hub l depends on time. Axial displacement u and two lateral displacements v, w are considered. The torsional displacement may be neglected since the beam has doubly symmetric cross-section and isotropic material. The beam is extruded with axial moving velocity $V(t)$ and constant angular velocity Ω by an external force $F(t)$ which is applied at the left end. It is assumed that the friction force between the hub and beam is neglected.

The position vector of a general point on the centerline outside the fixed hub can be expressed in terms of the axial and two lateral displacements while the point inside the hub does not have lateral displacements, so the position vector o f centerline outside the hub can be given as

$$\mathbf{r} = (x+u)\mathbf{i} + v\mathbf{j} + w\mathbf{k} \tag{1}$$

The velocity vector outside the hub can obtained based on the above position vector.

$$\mathbf{v} = \left(V + \frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x}\right)\mathbf{i} + \left(\frac{\partial v}{\partial t} + V\frac{\partial v}{\partial x} - \Omega w\right)\mathbf{j} + \left(\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x} + \Omega v\right)\mathbf{k} \tag{2}$$

The beam is assumed to be slender enough so the effe cts of shear deformation and rotary are ignored. Euler-Bernoulli beam theory and von Karman strain theory are adopted to get the nonlinear strain and linearized stress. The linearized stress outside the hub is given as

$$\sigma_x^L = E\left(\frac{\partial u}{\partial x} - y\frac{\partial^2 v}{\partial x^2} + z\frac{\partial^2 w}{\partial x^2}\right) \tag{3}$$

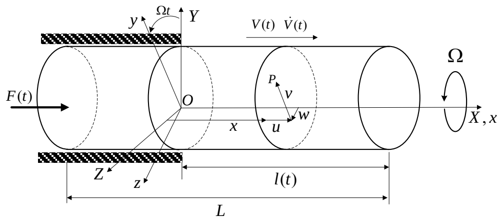

Fig.1.Modeling of a spinning beam with deployment

In order to derive the governing equation, velocity vect or is used to get the kinetic energy; linearized strain and non linear stress are applied to derive the potential energy.

$$T = \frac{1}{2}\int_0^l \int_A \rho \mathbf{v} \cdot \mathbf{v} dAdx \tag{4}$$

$$U = \int_0^l \int_A \sigma_x \varepsilon_x^L dAdx \tag{5}$$

The governing equations of motion are derived by Exte neded Hamilton's principle

$$\int_{t_1}^{t_2} \left(\delta T - \delta U + \delta W - \delta M \right) dt = 0 \tag{6}$$

3. Discretization

In order to obtain dynamic time response of the no nlinear coupled equations, governing equations of motio n are transformed into variational equations, i.e., weak forms and the Galerkin method is applied to derive the discretized equations of above weak forms. Based on b oundary conditions, it is easy to get the admissible fun ctions for the axial equation and comparison functions for lateral equations, by which, the components of disc retized equations can be computed. The discretized equ ations are given as

$$\sum_{j=1}^{J} \left\{ m_{ij}\ddot{T}_j^u + 2(g_{ij}^a + Vg_{ij}^b)\dot{T}_j^u + \left[k_{ij}^a + 2Vk_{ij}^b + \left(1 - \frac{V^2}{c^2}\right)k_{ij}^c + \dot{V}k_{ij}^d \right] T_j^u \right\} = -\dot{V}f_i^a + F\left(1 - \frac{V^2}{c^2}\right) f_i^b \tag{7}$$

$$\sum_{n=1}^{N}\sum_{q=1}^{Q} \left\{ \begin{aligned} & m_{mn}\ddot{T}_n^v + 2(g_{mn}^a + Vg_{mn}^b)\dot{T}_n^v \\ & + \left[k_{mn}^a + 2Vk_{mn}^b + V^2k_{mn}^c + \dot{V}k_{mn}^d + k_{mn}^e - \Omega^2 k_{mn}^f + \sum_{j=1}^{J} \alpha_{jmn} T_j^u \right] T_n^v \\ & + \left(-2\Omega g_{mq}\right)\dot{T}_q^w + \left(-2\Omega k_{mq}^a - 2V\Omega k_{mq}^b\right) T_q^w \end{aligned} \right\} = 0 \tag{8}$$

$$\sum_{q=1}^{Q}\sum_{n=1}^{N} \left\{ \begin{aligned} & m_{pq}\ddot{T}_q^w + 2(g_{pq}^a + Vg_{pq}^b)\dot{T}_q^w \\ & + \left[k_{pq}^a + 2Vk_{pq}^b + V^2k_{pq}^c + \dot{V}k_{pq}^d + k_{pq}^e - \Omega^2 k_{pq}^f + \sum_{j=1}^{J} \alpha_{jpq} T_j^u \right] T_q^w \\ & + \left(2\Omega g_{pn}\right)\dot{T}_n^v + \left(2\Omega k_{pn}^a + 2V\Omega k_{pn}^b\right) T_n^v \end{aligned} \right\} = 0 \tag{9}$$

4. Dynamic Time Response

Based on above discretized equations, dynamic time res ponse can be computed by Newmark time integration meth od. The nonlinear and linear dynamic time response for the beam tip trajectories are obtained.

It is observed that the differences between nonlinea r and linear models. We can also observe that the time -varying beat phenomenon occurs. The beat period and amplitude increase by time during deployment. In order to investigate the beat phenomenon, FFT analysis is us ed to find the two close frequencies. By frequency spe ctra, it is found that the beat phenomenon occurs since the first and second natural frequency get close to each other during deployement.

5. Conclusion

Considering the nonlinear effects caused by axial displa cement to two lateral displacements and coupling effects be tween two lateral directions, the equations of motion and dy namic time response have been obtained. The nonlinear effe cts caused by axial displacement to two lateral displacement s cannot be neglected under some geometry parameters and motion conditions. The differences between nonlinear an d linear models are observed. It is also found that the beat phenomenon occurs since the first and second nat ural frequency get close to each other.

Acknowledgment

This work was supported by a National Research Found ation of Korea (NRF) grant funded by the Korea governme nt (MEST) (No.2011-0017408).