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실험적 모드의 자유도 확장을 위한 유한요소 모드의 선택 방법 및 유한요소모델의 오차위치검출에의 적용

A Procedure for Selecting FE Modes to Expand Experimental Modes and its Application to Error Location

김경호 · 유연식 · 김종인 · 박윤식

Gyeongho Kim, Yeon-Sik Yoo, Jong In Kim and Youn-sik Park

1. Introduction

The predicted dynamic behavior of a finite element (FE) model often differs from experimental data of a target structure. Thus, an FE model needs to be verified or modified by experimental data. However, the number of degrees of freedom (DOFs) exceeds by far that of measured data. The inevitable DOF incompatibility makes it difficult to compare the two data sets. This problem can be resolved by expanding the experimental data to the full set DOF of the corresponding FE model.

Among various expansion techniques, modal coordinate expansion (MCE), where an experimental mod is defined as a linear combination of FE modes, is considered in this paper. Although it is a straightforward and physically appealing approach, the expanded result is critically dependent on the selected set of FE modes. Nevertheless, the appropriateness of the expanded mode is not verified unless additional experiments are performed to get experimental data at some representative unmeasured points and these data are compared with the expanded result.

This paper presents a systematic method to select an adequate set of FE modes for MCE. For each measured DOF, this method assumes it unmeasured and expands the experimental data to the full set of the measured DOFs. Then, it compares the experimental data to such expanded results to evaluate the appropriateness of a selected set of FE modes. Several simulated cases studies indicate that an appropriate set of FE modes can be selected for MCE by using the suggested method.

2. Modal Coordinate Expansion

MCE assumes that each experimental mode is constructed from a linear combination of FE modes. Thus, an expanded experimental mode is expressed as:

$$\begin{Bmatrix} \psi_r^{1*} \\ \psi_r^{2*} \end{Bmatrix} = \begin{bmatrix} \varphi_j^1 \varphi_k^1 \cdots \varphi_m^1 \\ \varphi_j^2 \varphi_k^2 \quad \varphi_m^2 \end{bmatrix} T = \begin{bmatrix} \Phi^1 \\ \Phi^2 \end{bmatrix} T$$

where ψ_r is the r^{th} experimental mode, φ_k and Φ are k^{th} FE mode and modal matrix. The superscript 1, 2 denotes the measured and unmeasured DOFs respectively. Finally, the superscript $*$ indicates that the experimental mode is smoothed or estimated. The unknown set of coefficients T is calculated as:

$$T = [\Phi^1]^+ \psi_r^1.$$

Here, the number of the FE modes (n) is less than the number of the measured DOFs (p). Thus, the estimated experimental mode ψ_r^{1*} fits the experimental mode ψ_r^1 in a least-squares sense.

As mentioned above, the success of this modal projection approach is critically dependent on the selected set of FE modes, which must include a reasonable counterpart to each experimental mode.

3. FE Mode Selection for MCE

For the i^{th} measured DOF of the experimental mode, we assume it unmeasured and expand the mode to the full set of the measured DOFs:

$$\begin{Bmatrix} {}_i\psi_r^{1*} \\ {}_i\psi_{ri}^{1*} \end{Bmatrix} = \begin{bmatrix} {}_i\varphi_j^1 & {}_i\varphi_k^1 & \cdots & {}_i\varphi_m^1 \\ \varphi_{ji}^1 & \varphi_{ki}^1 & & \varphi_{mi}^1 \end{bmatrix} {}_iT = \begin{bmatrix} {}_i\Phi^1 \\ \varphi_{ji}^1\ \varphi_{ki}^1 \cdots \varphi_{mi}^1 \end{bmatrix} {}_iT$$

where the subscript in front of a symbol means the corresponding DOF is removed from the full set of the measured DOFs, and ψ_{ri} is the i^{th} component of the mode ψ_r. Note that ${}_iT$ is calculated by:

$${}_iT = \left[\ {}_i\Phi^1\right]^+\ {}_i\psi_r^1.$$

The effectiveness of the selected FE modes can by evaluated by comparing ${}_i\psi_{ri}^{1*}$ and ψ_{ri}^1 for all $i = 1, \cdots, p$. For this purpose, an index J is developed as:

$$J = \sum_{i=1}^{p}\sum_{l=1}^{p}\sum_{k=l+1}^{p}\frac{1}{d_{kl}}\left|\left({}_i\psi_{rk}^{1*} - {}_i\psi_{rl}^{1*}\right) - \left(\psi_{rk}^1 - \psi_{rl}^1\right)\right|$$

where d_{kl} is the physical distance between node k and l. Thus, the FE mode selection for MCE is defined as:

Find a subset from FE modes $[\varphi_j\ \varphi_k\ \cdots \varphi_m]$ which minimize the index J

4. Case Study

A simple plate with a crack is provided to demonstrate the effectiveness of the FE mode selection method (Fig. 1). To simulate the experimental data, a fine FE model with 3126 DOFs is constructed. It is assumed that out-of-plane vibrations are measured at 36 points as marked in Fig. 1. The experimental mode ψ_r^1 is expanded to the FE model DOFs and, then, the expanded mode ψ_r^* is compared with the simulated experimental mode ψ_r, which is the true value, based on the normalized modulus difference (NMD):

$$\frac{\left\|\psi_r - \psi_r^*\right\|}{\left\|\psi_r\right\|}$$

In Fig. 2, the horizontal line denotes the NMD value of the expanded mode by the suggested technique. The NMD value of the case number k is calculated by the expanded mode using the first k FE modes. Finally, the error location of the FE model is plotted in Fig. 3, using the expanded modes from the suggested technique.

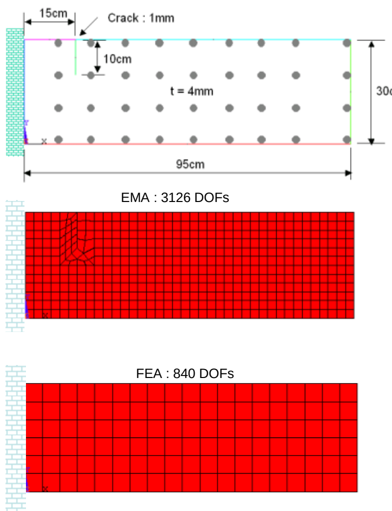

Fig. 1 Test plate with a crack

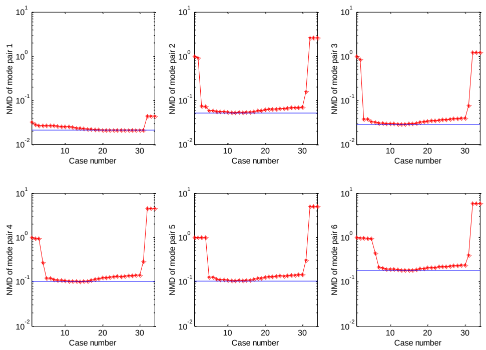

Fig. 2 Normalized modulus difference

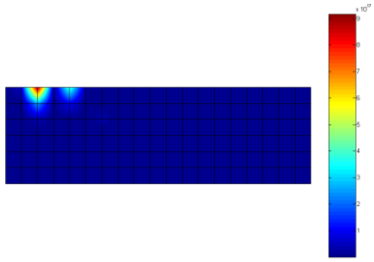

Fig. 3 Error location