# A Study on Errors in the Free-Gyro Positioning \& Directional System(II) 

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#### Abstract

This paper is to develop \& analyze the position \& direction error equations in the free-gyro positioning \& directional system by using two free gyros and is to find out the amount of the errors. First, the position \& direction error equations are introduced and developed, based on the position \& direction equations. Second, the value of errors is discussed based on sensors errors.


KEY WORDS : free gyro, positioning \& directional system, error equation, sensor error,

## 1. Introduction

A free gyro positioning \& directional system (FPS) is to determine the position of a vehicle by using two free gyros(Park \& Jeong, 2004). The free-gyro positionong \& directional system consists of a set of units with a vertical gyro, two free gyros and the computer that calculates navigational information.

The position \& directional error has not been decided yet. The system is thought to have position errors caused by the variation in Earth rate, measurement of time, and sensor errors, etc. In this paper the position \& directional error equations are introduced and developed, based on the definition of the type of errors which may be involved in the FPS. Then the amount of the errors of position \& direction by two free gyros were analyzed by the transformation of the coordinate frame.

## 2. Determination of Vehicle's Position \& <br> Direction by Two Free Gyros

2.1 Determination of Vehicle's position

$$
\begin{aligned}
& C_{i}^{n}=C_{e}^{n} C_{i}^{e} \\
& =\left[\begin{array}{lll}
-\sin \phi \cos \lambda & -\sin \phi \sin \lambda & \cos \phi \\
-\sin \lambda & \cos \lambda & 0 \\
-\cos \phi \cos \lambda & -\cos \phi \sin \lambda & -\sin \phi
\end{array}\right]\left[\begin{array}{lll}
\cos \omega_{e} t & \sin \omega_{e} t & 0 \\
-\sin \omega_{e} t & \cos \omega_{e} t & 0 \\
0 & 0 & 1
\end{array}\right] \\
& =\left[\begin{array}{lll}
-\sin \phi \cos \left(l+\varpi_{e} t\right) & -\sin \phi \sin \left(\lambda+\varpi_{e} t\right) & \cos \phi \\
-\sin \left(\lambda+\varpi_{e} t\right) & \cos \left(\lambda+\varpi_{e} t\right) & 0 \\
-\cos \phi \cos \left(\lambda+\varpi_{e} t\right) & -\cos \phi \sin \left(\lambda+\varpi_{e} t\right) & -\sin \phi
\end{array}\right] \\
& g_{v}^{n}=C_{i}^{n} g_{v}^{i} \\
& =\left[\begin{array}{lll}
-\sin \phi \cos \left(\lambda+\varpi_{e} t\right) & -\sin \phi \sin \left(\lambda+\varpi_{e} t\right) & \cos \phi \\
-\sin \left(\lambda+\varpi_{e} t\right) & \cos \left(\lambda+\varpi_{e} t\right) & 0 \\
-\cos \phi \cos \left(\lambda+\varpi_{e} t\right) & -\cos \phi \sin \left(\lambda+\varpi_{e} t\right) & -\sin \phi
\end{array}\right]\left[\begin{array}{l}
u_{x} \\
u_{y} \\
u_{z}
\end{array}\right] \\
& =\left[\begin{array}{l}
-u_{x} \sin \phi \cos \left(\lambda+\varpi_{e} t\right)-u_{y} \sin \phi \sin \left(\lambda+\varpi_{e} t\right)+u_{z} \cos \phi \\
-u_{x} \sin \left(\lambda+\varpi_{e} t\right)+u_{y} \cos \left(\lambda+\varpi_{e} t\right) \\
-u_{x} \cos \phi \cos \left(\lambda+\varpi_{e} t\right)-u_{y} \cos \phi \sin \left(\lambda+\bar{\omega}_{e} t\right)-u_{z} \sin \phi
\end{array}\right]=\left[\begin{array}{l}
N_{u} \\
E_{u} \\
D_{u}
\end{array}\right]
\end{aligned}
$$

$\cos \theta=\frac{U_{u}}{\left|g_{v}\right|}=-u_{x} \cos \phi \cos \left(\lambda+\bar{\sigma}_{e} t\right)-u_{y} \cos \phi \sin \left(\lambda+\bar{\sigma}_{e} t\right)-u_{z} \sin \phi$
(3)
$\tan \alpha=\frac{E_{u}}{N_{u}}=\frac{-u_{x} \sin \left(\lambda+\varpi_{e} t\right)+u_{y} \cos \left(\lambda+\varpi_{e} t\right)}{-u_{x} \sin \phi \cos \left(\lambda+\varpi_{e} t\right)-u_{y} \sin \phi \sin \left(\lambda+\varpi_{e} t\right)+u_{z} \cos \phi}$
2.2 Determination of Vehicle's Heading

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$$
\omega_{x}=\varpi_{e} \cos \phi \cos \psi
$$
$$
\omega_{y}=-\varpi_{e} \cos \phi \sin \psi
$$
$$
\psi=\arctan \frac{\omega_{y}}{\omega_{x}}
$$
$$
\theta=\int_{t_{1}}^{t_{2}} \omega_{H} d t
$$

## 3. Error Equations

The error equations of position and direction are obtained by using the following transformation matrices.

$$
\begin{aligned}
& C_{g}^{p}=\left[\begin{array}{ccc}
\cos \xi \cos \eta & -\sin \xi & \cos \xi \sin \eta \\
\sin \xi \cos \eta & \cos \xi & \sin \xi \sin \eta \\
-\sin \eta & 0 & \cos \eta
\end{array}\right] \\
& C_{p}^{n}=\left[\begin{array}{ccc}
\cos P & \sin R \sin P & \cos R \sin P \\
0 & \cos R & -\sin R \\
-\sin P & \sin R \cos P & \cos R \cos P
\end{array}\right]
\end{aligned}
$$

## Position error equation

## 

$\Delta z \in D x \quad \Leftrightarrow \quad \Delta=\underline{\underline{l}}-1^{\Delta}$

## 4. Conclusions

In this paper based on the position \& direction equation by two free gyros error equations of position and direction were analyzed
by the transformation of the coordinate frames. And the errors involved in the free-gyro positioning \& direction system were defined. Finally the error amount was theoretically investigated.

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