# 압축센싱을 위한 필터선택 비교
# Comparison of Filter Selection for Compressed Sensing

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## Abstract

Compressed Sensing (CS) has been developed for several years. Among many CS algorithms for image, the Block-based Compressed Sensing with Smoothed Projected Landweber (BCS-SPL) demonstrates its excellent performance in low-complexity and near-optimal reconstruction. Several noise filtering algorithms of image reconstruction have been introduced such as the Wiener or the median filters, etc. In general, each filter has its own advantages and disadvantages depending on specific coding scheme. In this paper, we show that reconstruction performance can be varied according to the choice of filter. When a sub-rate value is changed, different filter causes different effect as well. Concerning the sub-rate, an inner filter can be chosen to improve the reconstructed image quality.

*Keyword – Compressed sensing, median filter, Wiener filter.*

## 1. Introduction

To address ever-rising demand for more communication channels, researchers have tried to have more compression since the more the compression achieves, the more users can share the same resource. In this line of efforts, the Compressed Sensing (CS) (also known as Compressive Sensing, Compressed Sampling) was introduced in 2004 by David Donoho, Emmanuel Candes, and Terence Tao [1], [2]. It relies on the two basic assumptions: signals are sparse and samples are linear functional. The compressed sensing allows the signal to be sampled at a sub-Nyquist rate while still attaining near-optimal reconstruction.

Several factors affect the overall quality of reconstructed images – sparsity of signal, sub-rate of encoder, incoherence between sparsifying transform and measurement basis, and smoothing algorithm (at the decoder’s side), etc.

Some researchers [3] have constructed an adaptive smoothing algorithm, but they mainly focused on the whole iteration which holds the error of signal's estimates below a specified threshold. Both theoretical and practical studies proved that the selection of a sub-rate would directly impact reconstructed result. Moreover, some authors introduced the effects of different inner filters in smoothing process [4].

In this paper, we investigate effectiveness of filter selection for reconstructing image at the receiver (decoder). Each filter is chosen regarding the sub-rate of transmitted signal to bring higher PSNR value. Our paper is organized as follows. Section II introduces some basic knowledge. Section III represents the different effects of median filter and Wiener filter. Section IV shows our experimental results. In the end of this paper, Section V concludes our work with some comments on future work.

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## 2. Background

*A. Compressed sensing*

In essence, the compressed sensing is a mathematical approach which reconstructs a signal vector of length N from just M measurements ($M \ll N$). The ratio $M/N$ is called the sub-rate, and y is a sampled vector represented by equation (1).

$$y = \Phi x \tag{1}$$

where $x \in R^N$ is a real-valued vector and $\Phi$ is an $M \times N$ measurement matrix. Although $M \ll N$, $x$ can be recovered exactly or approximately from $y$ if and only if $x$ is sparse. Note that $x$ is called K-sparse ($K \le M \ll N$) if there exist $\breve{x}$ satisfying $\breve{x} = \Psi x$, where $\breve{x}$ has at most K non-zero coefficients. $\Psi$ is known as a sparsifying transform which is a matrix $\left[\psi_1 | \psi_2 \ldots | \psi_N\right]$ with N columns $\left\{\psi_i | i = 1 \sim N\right\}$; each column is a basis vector of size $M \times 1$.

E.J Candes, J Romberg, and T Tao [2] proved a sufficient condition on the mutual incoherence between sparsifying transform $\Psi$ and measurement basis $\Phi$ to exactly or approximately reconstruct x from y. The matrix $\Phi$ is usually designed as a random matrix and represented in Gaussian distribution. Therefore, the main problem in CS is about a method to reconstruct exactly or approximately $\breve{x}$ from $y$. Reconstruction algorithms need to be enhanced somehow to reduce computational complexity and improve reconstructed image quality. Taking both the complexity and stability of image reconstruction process into account, BCS- SPL (Block-based Compressed Sensing with smoothed projected Landweber), an algorithm formed by successively projecting and thresholding [3], [5], provides reduced computational complexity and possibly offers incorporating additional optimization criteria.

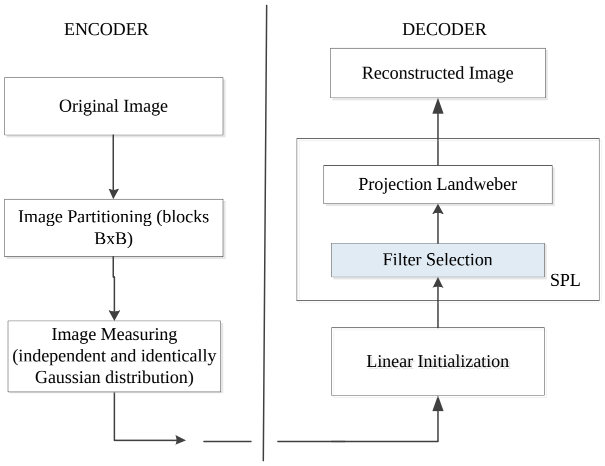


Figure 1. BCS-SPL

*B. Block-based Compressed Sensing with smoothed projected Landweber (BCS-SPL)*

Figure 1 shows a procedure which consists of two parts: encoder and decoder. The encoder divides an original image $(N\times N)$ into blocks $B\times B$ by Image Partitioning. Measurement matrix for each block $\Phi_B$ is defined as an orthonormalized *i.i.d* (independent and identically distributed) Gaussian matrix. The corresponding measurement $y_i$ with size $M_B\times 1$ is computed in Image Measuring as: $y_i=\Phi_B x_i$ . $M_B$ is the size of measurement for each block. So that, we just have to reconstruct $M_B\times B^2$ Gaussian matrix, much smaller than a full $M\times N$ one. Larger B means more storage space but brings better reconstruction performance. In this paper, we use $B=32$.

At the decoder side, the measurement vector is processed via some successive functional blocks:

- Linear Initialization: measurement initial vector.
- Filter selection: A filter is added into the algorithm to impose smoothness and remove noise in spatial domain.
- Projected Landweber (PL): iterative thresholding operator which determines how exact $x$ can be.

SPL reconstruction algorithm is described as below:

Initial vector: $x^0=\Phi^T y$

function $x^{(j+1)}=SPL(x^{(j)},y,\Phi_B,\Psi,\lambda)$

$x^j = filter(x^j)$

For each block i:

$\hat{x}_i^{(j)}=x_i^{(j)}+\Phi_B^T(y-\Phi_B x_i^{(j)})$

$\breve{x}^{(j)}=\Psi\hat{x}^{(j)}$

$\breve{x}^{(j)}=Threshold(\breve{x}^{(j)},\lambda)$

$\bar{x}^{(j)}=\Psi^{-1}\breve{x}^{(j)}$

$x_i^{(j+1)}=\bar{x}_i^{(j)}+\Phi_B^T(y-\Phi\bar{x}_i^{(j)})$

Here Threshold ( $\breve{x}^{(j)},\lambda$ ) is a hard threshold which is an estimator of $\breve{x}^{(j)}$ . The iteration will be finished when $\left|D^{(j+1)}-D^j\right|<10^{-4}$ where D is mean squared error (MSE):

$$D^j=1/\sqrt{N}\left\|\breve{x}^{(j)}-\breve{x}^{(j-1)}\right\|_2 \tag{2}$$

Finally, we get reconstructed image which is an approximation of the original image at the encoder side. Note that in this procedure, the transmitted signal is supposed to be ideally received at the decoder.

## 3. Comparison of Filter Selection

As mentioned in section 2.B, an initial approximation $x^0$ is calculated from y and $\Phi_B$ by the equation $x^0=\Phi_B^T y$. It is equivalent to:

$$x^0=\Phi^T\Phi x\Rightarrow x-x^0=(I-\Phi^T\Phi)x \tag{3}$$

In which $\Phi^T\Phi$ is called a pseudo-inverse matrix and $x-x^0$ is noise between original and initial values. The smaller the noise is, the better the initial value we get. That means when $\Phi^T\Phi$ is reduced, $(I-\Phi^T\Phi)$ increases and the noise will be larger.

When the sub-rate is low enough ( $M \ll N$ ), noise is supposed to be at high volume ( $\Phi^T\Phi$ is small). The difference between original and initial values is considerable, and noise will be like salt and pepper noise. In this case, a median filter is effective in removing the salt and pepper noise since peak pixel value is replaced by neighboring median value [6].

On the contrary, when the sub-rate becomes higher, the quality of initial image would be better; amplitude of initial value gets closer to original image. This will lead to a conclusion that filtering algorithm like median would not be effective. By the way, because the Wiener filter uses the average of all the local estimated variances to be a noise variance, with large variance, the filter performs little smoothing and more smoothing in case of small variance

[7]. With a high sub-rate, variance is small, so using the Wiener filter is expected to be more effective than the median filter. It can be concluded that at a small sub-rate, the median offers a higher Peak-Signal to Noise Ratio (PSNR); while in case of a high sub-rate, the Wiener filter has better PSNR in comparison with the median filter.

## 4. Experimental results

We evaluate the performance of both filters by using Matlab program. Experimental parameters are shown as in Table 1.

Table 1. Experimental Parameters

| Parameters | Value |
|---|---|
| Image size | 512x512 |
| CS Block size | 32x32 |
| Sparsifying transform | DWT (Discrete Wavelet Transform) |
| Sub-rate range | 0.1~ 0.5 |
| Filters | median, Wiener |
| Tested Images | Lena, Barbara. |

Figure 2 represents the difference between the two filters: the median and the Wiener filters for Barbara image. When the sub-rate is around 0.1, the median filter produces PSNR of about 0.5 dB higher than the Wiener filter (23dB in comparison with 22.5dB). On the contrary, when the sub-rate becomes near 0.5, the Wiener filter (28dB) offers a better PSNR than the median filter (27.2dB).

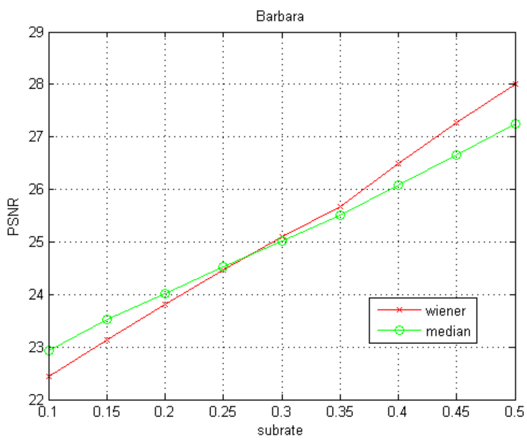


Figure 2. PSNR of Barbara image.

Similar to Barbara, results of coding Lena is given in the Figure 3. It also demonstrates effectiveness of a median filter at a low sub-rate (less than 0.35). Around the sub-rate 0.1, PSNR value by using the median filter is higher than that by the Wiener filter by about 1dB (28dB is comparison with 29dB). At a sub-rate near 0.5, the Wiener filter (36.2dB) produces PSNR higher than the median filter (36dB).

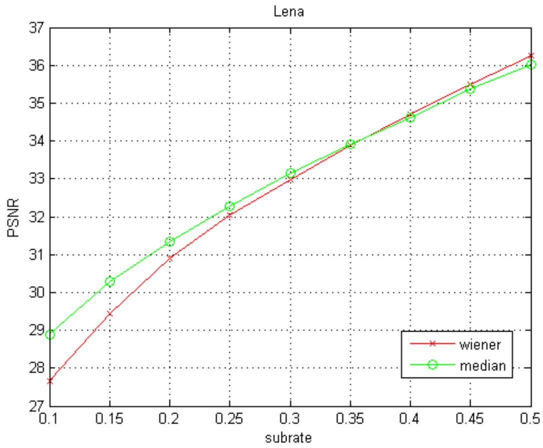


Figure 3. PSNR of Lena image

## 5. Conclusion

In this paper, we compared the effectiveness of the median filter and the Wiener filter in compressed sensing of image data. It is shown that the median filter is more effective on reconstructed image's quality at a low sub-rate than the Wiener filter. On the contrary, using the Wiener filter at a high sub-rate will give higher PSNR than using the median filter. In the near future, we plan to design an adaptive filtering scheme which can switch between the median filter and the Wiener filter based on a sub-rate. In addition to that, the sub-rate based filter determination will be examined in more detail in thresholding or the number of filters for choosing.

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