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카오스 신호를 이용한 보의 균열 탐지에 관한 연구
A study on the detection of cracks in beams using chaotic signals

유정수†
Jungsoo Ryue

1. Introduction

The work in this study is directed at exploring the feasibility of attractor-based methods using chaotic excitation to detect and identify cracks in a structure. As candidate crack indicators, two chaotic measures, i.e., the correlation dimension and Hausdorff distance are quantified for the response of a beam with a crack by means of numerical simulations. The solution of the Duffing equation is chosen as the chaotic input excitation signals which are applied to a cracked beam model in this study. The dynamical behaviour of a cracked cantilever beam is presented as an object structure. An low frequencies this beam can be modelled as a one degree-of-freedom bilinear system which has different stiffesses when stretching and compressing of the crack. The half-space correlation dimension and Hausdorff distance are predicted from the attractor geometries of the structure output in phase space with the different crack size.

2. Chaotic input signals

Chaotic solution of forced Duffing equation will be applied as an input excitation force to a cracked beam in this study.

$$\ddot{x} + c\,\dot{x} - k_1 x + k_2 x^3 = F\cos\omega t \qquad (1)$$

where c, k_1 and k_2 are system parameters of the Duffing oscillator, F is the amplitude of the forcing sinusoid and ω is the angular frequency of the forcing function. This equation can be converted to a set of equations with three state variables,

$$\dot{x}_1 = x_2\,,$$
$$\dot{x}_2 = F\,cos\omega t + k_1 x_1 - k_2 x_1^3 - c x_2\,, \qquad (2)$$
$$\dot{x}_3 = \omega$$

The correlation dimension is defined by the logarithmic ratio between correlation sum $C(r)$ and the radius r as

$$D_c = \lim_{r\to 0}\frac{\log_2 C(r)}{\log_2 r} \qquad (3)$$

The Hausdorff distance is a measure of similarity between two sets of points. It is expressed mathematically as

$$h(A,B) = \max\{d(A,B), d(B,A)\}$$

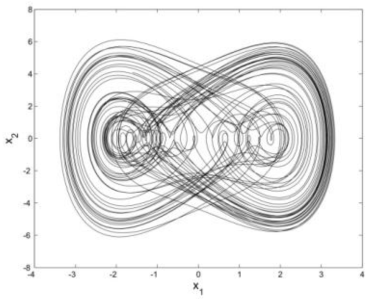

Fig. 1. Phase portrait of the chaotic solution of the Duffing equation projected onto the (x_1, x_2) plane.

3. Cracked beam modelling

A beam having a crack can be regarded as a continuous system during compression. On the contrary, during stretching the stiffness decreases as the crack opens. The equations of motion of a cracked cantilever beam are described as a single degree-of-freedom mass-spring-damper system at low frequency

$$\ddot{y} + 2\zeta_s\omega_s\,\dot{y} + \omega_s^2 y = F(t) \quad for\ y \geq 0 \qquad (4)$$
$$\ddot{y} + 2\zeta_c\omega_c\,\dot{y} + \omega_c^2 y = F(t) \quad for\ y < 0$$

In these equations the subscripts c and s denote

† 교신저자; 정회원, 울산대학교
E-mail : jsryue@ulsan.ac.kr
Tel : 052-259-2168, Fax : 052-259-2677

compression and stretching, respectively. For free oscillations, the ratio of the two oscillating frequencies of the cracked beam can be expressed as

$$\frac{\omega_s^2}{\omega_c^2} = \frac{k_s/m}{k_c/m} = \frac{k_c - \Delta k}{k_c} = 1 - \frac{\Delta l}{l} \qquad (5)$$

where $\Delta k = k_c - k_s$, l is the thickness of the beam and Δl is the crack depth. In this study, a cantilever beams which has a natural frequency of 15.7 Hz was used as an object structure.

4. Attractor-based Measures

Correlation dimensions and Hausdorff distances are estimated for the cracked beam model. Two correlation dimensions from a single phase portrait are estimated, corresponding to the compression and stretching regions of a crack. This local correlation dimension is named a *half-space correlation dimension* in this work. The normalized versions of reconstructed phase portraits are illustrated in Fig. 2 for two crack sizes of $\Delta l/l = 0$ and 0.25.

Two estimated half-space correlation dimensions are given in Fig. 3(a) for various relative crack sizes. These features, however, appear to be irrelevant to be an efficient crack

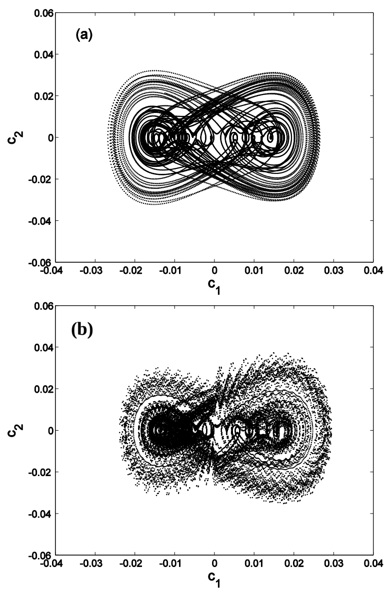

Fig. 2. Reconstructed attractors of the cracked beam model for the Duffing excitation in Fig. 1. (a) $\Delta l/l = 0$, (b) $\Delta l/l = 0.2$.

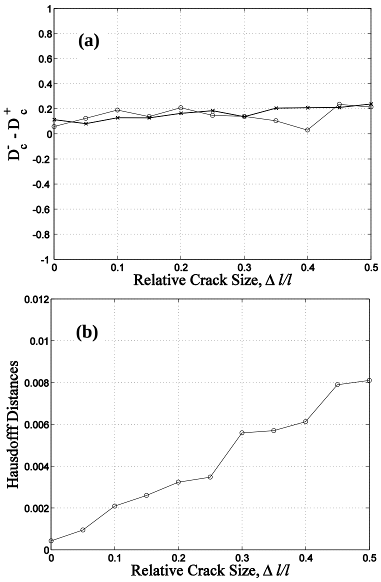

Fig. 3. (a) Correlation dimensions and (b) Hausdorff distances corresponding to the relative crack size for the cracked beam model.

indicator because the gaps between the two correlation dimensions in compression and stretching regions seem to be insensitive to relative crack sizes.

Hausdorff distances of output attractors are presented in Fig. 3(b) with respect to the relative crack size. It is clear that the Hausdorff distance gradually increases with the relative crack size. Therefore, based on the results in Fig. 3(b), the Hausdorff distance of a cracked beam provides a consistent rise with a crack size.

4. Conclusion

Through this investigation, it was shown that the half-space correlation dimension for stretching and compression region was relatively insensitive to changes in crack size in the beam. Therefore, the half-space correlation dimension is an inappropriate measure for identifying the relative size of a crack in beams. Alternatively, the Hausdorff distance is continuously increasing with the relative size of the crack. Hence, the Hausdorff distance appears to be an adequate measure for quantifying the presence and size of cracks in structures.