

Combining Four Elements of Precipitation Loss in a Watershed 유역내 네가지 강수손실 성분들의 합성

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Abstract

In engineering hydrology, an estimation of precipitation loss is one of the most important issues for successful modeling to forecast flooding or evaluate water resources for both surface and subsurface flows in a watershed. An accurate estimation of precipitation loss is required for successful implementation of rainfall-runoff models. Precipitation loss or hydrological abstraction may be defined as the portion of the precipitation that does not contribute to the direct runoff. It may consist of several loss elements or abstractions of precipitation such as infiltration, depression storage, evaporation or evapotranspiration, and interception.

A composite loss rate model that combines four loss rates over time is derived as a lumped form of a continuous time function for a storm event. The composite loss rate model developed is an exponential model similar to Horton's infiltration model, but its parameters have different meanings. In this model, the initial loss rate is related to antecedent precipitation amounts prior to a storm event, and the decay factor of the loss rate is a composite decay of four losses.

Key words: Precipitation Losses, Precipitation Abstractions, Composite Loss Rate, Effective Rainfall, Direct Runoff

1. Introduction

Precipitation loss can be estimated by lumping/combining the loss elements evaluated using physical or empirical models, but this is not easily accomplished. In many cases, runoff models based on storm event infiltration models, as listed in Table 1, have been used for estimation of all the losses, i.e. the infiltration and the other loss elements—depression storage, evaporation, and interception—the latter of which are treated as a minor. A more exact determination of infiltration and other losses improves the water balance analysis in a watershed, even in the case of a storm (Gupta 1989). Although these minor losses may be small in cases based on storm events, an estimation of all the losses, including minor losses, can account for the effective rainfall in a storm event in a watershed more logically.

Table 1. Infiltration models used for obtaining precipitation loss based on a storm event

Runoff model	Author	Infiltration model used
HM	Huggins and Monke (1968)	Holtan's equation
MIT	Maddaus and Eagleson (1969)	Any suitable model
SWMM	Metcalf and Eddy (1971)	Horton's equation
USGS	Dawdy et al.(1972)	Philip's equation
SCS TR-20	SCS (1973)	SCS curve number method
HYMO	Williams and Hann (1973)	SCS curve number method
GAWSER	Ghate and Whiteley (1977)	Horton's equation
WBNM	Boyd et al. (1979)	Φ Index
IHM	Morris (1980)	Richard's equation
FHSM	Foround and Broughton (1981)	Modified Horton's equation
HEC-1	HEC (1981)	Variable loss rate method
RORB	Laurenson and Mein (1983)	Contant and variable loss rate method

In this study, a composite loss rate model that logically combines four loss rates is presented as an exponential function of time for runoff modeling based on a storm event in a watershed.

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2. Development of a composite loss rate model

In this section, precipitation losses (i.e., hydrological abstractions) are described as temporal rate models as in HEC-1 (HEC 1981). These losses, including infiltration, depression storage, evaporation or evapotranspiration, and interception, are briefly explained and reviewed in relation to previous models. Subsequently, a composite loss rate model combining the four losses is derived.

Infiltration loss

Infiltration is the process of the entry of water into the soil through the soil surface, and the water then may percolate into the base flow. Infiltration has been often treated as a major loss element of precipitation that does not contribute to direct runoff. Estimation of infiltration is required to determine the effective rainfall in a watershed. Among many infiltration estimation models, the Horton model (Horton 1940) is one of the best known infiltration models in hydrology. It is regarded as suitable for deriving a loss rate in this study because of its exponentially decaying behavior with time. In this model, the infiltration rate f , which means the infiltration capacity of the soil, may be described as

$$f = f_c + (f_0 - f_c) \exp(-k_1 t) \quad (1)$$

where f_c is the final constant rate of infiltration, f_0 is the initial infiltration rate, k_1 is a constant for the decay factor, \exp is an expression of the exponential function, and t is time.

Depression storage

Depression storage is the amount of rain water retained in depressions during a rainfall storm event. The rate of depression storage intensity v (Linsley et al. 1975; Singh 1989; Bras 1990) is

$$v = (i - f) \exp\left(\frac{-P_e}{S_d}\right) \quad (2)$$

expressed as

where i is the rainfall intensity, P_e is the rainfall excess, and S_d is the maximum storage capacity. As a simplification of the development of a lumped model, let i_a be assumed to be a constant value averaging $(i - f)$ over the storm duration, t_r .

The depression storage, Eq. (2), can be transformed into Eq. (3) by the study (Yoo, 2012).

$$v = A_2 \exp(-k_2 t) \quad (3)$$

where $A_2 = k_2 S_d$ and $k_2 = \frac{i_a}{S_d}$, $i_a = \frac{\int_0^{t_r} (i - f) dt}{t_r}$ and these two values are treated as constants. It is shown that the rate of this loss decreases exponentially with time during a storm as indicated in Eq. (3).

Evaporation and transpiration

Evaporation is the process by which water is transferred from land and bodies of water to the atmosphere. Evapotranspiration, expressed as E_r , is the sum of the volume of evaporation from the soil and the water used by vegetation (transpiration) described as

$$E_r = E + C_v \quad (4)$$

where E is the evaporation estimated by many empirical equations and is simplified as Eq. (5):

$$E = C_3 f(u) (e_s - e) \quad (5)$$

where e_s is the vapor pressure of saturated air at the temperature of the water surface, e is the actual vapor pressure of air at some height above the surface, $f(u)$ is a function of the wind speed, u , and

C_3 is a constant value associated with solar radiation. Eq. (5) can be transformed into Eq. (6) by the study (Yoo, 2012).

$$E = C_3 f(u) (e_s - e_0) \exp(-k_2 t) \quad (6)$$

Interception

Interception is defined as the portion of precipitation falling in a watershed intercepted by vegetal cover and other objects above the ground such as roofs. Most interception loss develops during the initial portion of the storm period and the rate of the loss rapidly reaches zero. An excellent description of the interception process has been given by Horton (1919). The interception within a rainstorm event can be expressed by Eq. (7):

$$I = C_4 + C_5 P_c^n \quad (7)$$

where P_c is amount of precipitation over the vegetal cover (Gray 1973; Bras 1990), and C_4, C_5 , and n are constant values related to vegetal coverings, as given by Horton (1919).

Composite loss rate combining the four losses

First, infiltration and interception are combined. The interception loss develops during the initial portion of the storm period and its rate rapidly reaches zero (Singh 1989). Therefore, the interception in Eq. (7) may be added to the initial infiltration rate f_0 in Eq. (1) of the Horton model as follows

$$l_{ii} = f_c + (C_4 + C_5 P_c^n + f_0 - f_c) \exp(-k_1 t) \quad (8)$$

in which l_{ii} means a loss combining the two losses of the infiltration and interception.

Adding the depression storage loss rate v in Eq. (3) and evapotranspiration loss rate E_t in Eq. (4) to the loss rate of l_{ii} in Eq. (8) combining two losses—infiltation and interception, the total loss l with time t can be described as

$$l = fn1 + fn2 + fn3 + l_c \quad (9)$$

where

$$fn1 = A_1 \exp(-k_1 t) \quad (10a)$$

$$fn2 = A_2 \exp(-k_2 t) \quad (10b)$$

$$fn3 = A_3 \exp(-k_3 t) \quad (10c)$$

where $A_1 = C_4 + C_5 P_c^n + f_0 - f_c$, $A_2 = k_2 S_d$ as in Eq. (3), $A_3 = C_3 f(u) (e_s - e_0)$ and $l_c = f_c + C_1$.

If the decay factors of k_1, k_2 , and k_3 are as small as decimals or centesimals, it can be proved that Eq. (9) is close to Eq. (11), as is explained by the Taylor's series described in the next section:

$$l = fn + l_c \quad (11)$$

$$\text{where } fn = A \exp(-kt) \quad (12)$$

in which k is a composite decay factor that is a function of k_1, k_2 , and k_3 as in Eq. (16b)

$$\text{and } A = A_1 + A_2 + A_3 = l_0 - l_c \quad (13)$$

$$\text{in which } l_0 = C_4 + C_5 P_c^n + C_v + C_3 f(u) (e_s - e_0) + f_0 + k_2 S_d \quad (14)$$

where l_c is the initial loss value associated with the interception, vegetal coverage, wind speed, potential vapor pressure in the air, initial infiltration capacity, solar radiation, and maximum depression

storage.

Simplification of combining exponential loss functions

Let us consider three exponential loss functions of time (t) such as Eqs. (10a)~(10c): $A_1 \exp(-k_1 t)$, $A_2 \exp(-k_2 t)$, and $A_3 \exp(-k_3 t)$ where A_1 , A_2 , A_3 , k_1 , k_2 , and k_3 are constants, and t and exp describe time and the exponential function, respectively. Let a function, fn in Eq. (12), combine the three functions as in Eqs. (10a)~(10c), in which A and k are composite constants.

The function, fn in Eq. (12), expanded into a power series using the Taylor's series theorem, can be closed to Eq. (15) by study (Yoo, 2012).

$$fns = A \exp(-kt) + R_t = fn + R_t \tag{15}$$

where

$$A = A_1 + A_2 + A_3 \tag{16a}$$

and

$$k = \frac{A_1 k_1 + A_2 k_2 + A_3 k_3}{A_1 + A_2 + A_3} \tag{16b}$$

where R_t is a redundant term treated as minor when $k_1, k_2,$ and k_3 together are small. To show that the composite function of fn in Eq. (12) approaches the summed function of fns in Eq. (15), the functions of fn1, fn2, fn3, fns, and fn are calculated with time when $A_1 = 10$, $A_2 = 4$, $A_3 = 1$, $k_1 = -0.02$, $k_2 = -0.008$, and $k_3 = -0.002$ in a parametric example study. In the parametric study, it is shown in Fig. 1 that with $A=15$ as per Eq. (16a) and $k=-0.0156$ as per Eq. (16b) the composite function values are approximately equal to the summed function values at the same time over $0 \leq t \leq 50$. In short, if the decay factors of $k_1, k_2,$ and k_3 are small when the loss elements—infiltration, depression storage, and evaporation—vary slightly in a storm, the summed function of fns in Eq. (15) can be simplified as the composite function of fn in Eq. (12).

Finally the derived equation, fn, for a composite loss rate model combining the four losses—infiltration, depression storage, evapotranspiration, and interception—can be derived as an exponential function with time in Eq. (12), and the cumulative loss is derived as in Eq. (17).

$$L = \int_0^t l \, d\tau = l_c t + \frac{1}{k} (l_0 - l_c) [1 - \exp(-kt)] \tag{17}$$

where t is the arbitrary time in a rain storm and dτ is a dummy variable for integration.

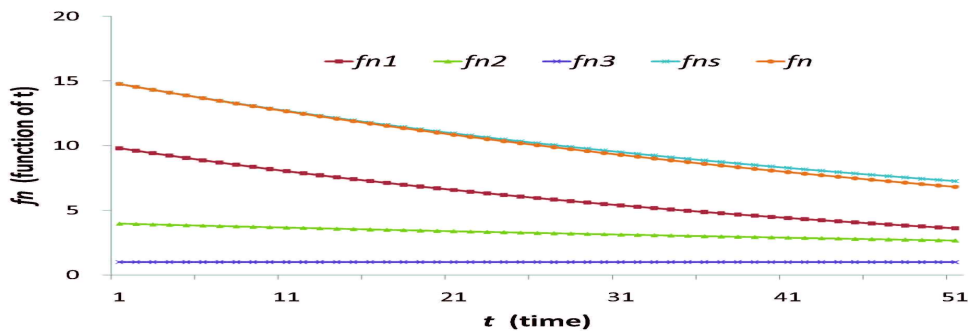


Fig. 1. Comparison of the composite exponential function $fn = 15 e^{-0.0156 t}$ with the summed exponential function ($fns = fn1 + fn2 + fn3$), where $fn1 = 10 e^{-0.02 t}$, $fn2 = 4 e^{-0.008 t}$, and $fn3 = e^{-0.002 t}$

3. Summary and conclusion

The primary results of this study may be summarized as follows: 1). A precipitation loss rate model is revealed by combining the four loss rates of infiltration, depression storage, evapotranspiration, and interception. 2) As a result, the model revealed in this study is similar to Horton's infiltration model in form, but it differs from the latter because it is a composite of four loss elements. 3) The study focuses on the derivation and formulation of a new model.

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