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도파관 유한 요소법을 이용한 철로 진동 해석
Railway vibration using waveguide finite element method

유정수†
Jungsoo Ryue

1. Introduction

This paper present a numerical analysis, called the waveguide finite element method, is applied to predict vibration of railway tracks. The waveguide FE (WFE) approach is used to calculate the vibration of an infinite, continuously supported rail excited by appoint force. The WFE method models just a 2 dimensional cross-section of the rail to simulate wave propagation along the rail. In this presentation, a brief description for the WFE equation will be given and it will be applied to predict dispersion relation, decay rate and point mobility of the waves in railway tracks.

2. Waveguide finite element method

Suppose that there is an elastic waveguide structure which is infinitely long in one direction, call it the x-direction, and its cross-section normal to the x-axis is uniform along x. Time harmonic displacements, (u,v,w), of the element in three directions of (x,y,z) can be expressed with separable variables as

$$\begin{aligned} u(x,y,z,t) &= \chi(y,z)\, e^{-j\kappa x} e^{j\omega t} \ , \\ v(x,y,z,t) &= \psi(y,z)\, e^{-j\kappa x} e^{j\omega t} \ , \\ w(x,y,z,t) &= \xi(y,z)\, e^{-j\kappa x} e^{j\omega t} \ , \end{aligned} \quad (1)$$

where t denotes time, y and z denote coordinates of the cross-section, χ, ψ and ξ define the displacements of the cross-section and κ is wavenumber along the x direction.

By using these wave solutions for the x direction in the finite element formulation, a two-dimensional finite element equation is made over a cross-sectional model, instead of a three-dimensional full FE model. The differential equation for a cross-sectional model is given by

$$\left\{\mathbf{K}_2 \frac{\partial^2}{\partial x^2} + \mathbf{K}_1 \frac{\partial}{\partial x} + \mathbf{K}_0 - \mathbf{M}\frac{\partial^2}{\partial t^2}\right\}\mathbf{U}(x,t) = 0 \quad (2)$$

where $\mathbf{K}_2$, $\mathbf{K}_1$ and $\mathbf{K}_0$ are stiffness matrices, $\mathbf{M}$ is the mass matrix of the cross-section and $\mathbf{U}(x,t)$ is the displacement vector.

Since $\mathbf{U}(x,t) = \tilde{\mathbf{U}} e^{j(\omega t - \kappa x)}$ as described in Eq.(1) the differential equation in Eq.(2) can be simplified to an eigenvalue problem,

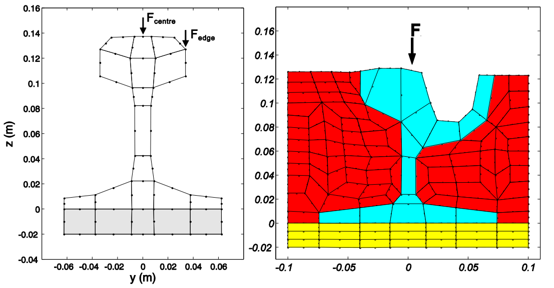
$$[\mathbf{K}_2(-i\kappa)^2 + \mathbf{K}_1(-i\kappa) + \mathbf{K}_0 - \omega^2\mathbf{M}]\tilde{\mathbf{U}} = \tilde{\mathbf{F}} \quad (3)$$

where $\tilde{\mathbf{U}}$ contains the displacements of the cross-section which define the deformation shapes of waves. Here κ and ω are unknown variables to be identified.

3. Rail models

The first rail profile used here is a standard CEN 40E1 section, which is shown in Fig. 1(a). Note that the pad thickness is exaggerated compared with a real pad. This is done to clarify figures showing displacements of the rail. However, to avoid non-physical standing waves within the pad a very low density is used. The second rail is a standard CEN 51Ri profile used for trams and the embedding is made to fill the space around this profile as shown in Fig. 1(b). For this rail, two different embedding materials are used; a softer ‘pad’(yellow) and a stiffer ‘fill’ material around the rail (red).

† 교신저자; 정회원, 울산대학교
E-mail : jsryue@ulsan.ac.kr
Tel : 052-259-2168, Fax : 052-259-2677



(a) (b)

Fig. 1. Model mesh of rails. (a) Open rail, CEN 40E1, (b) embedded rail, CEN 51Ri.

4. Results

Fig. 2 illustrates the dispersion relations of the waves propagating along the tracks. The different waves are distinguished with marks in Fig. 2. In addition, Fig. 3(a) shows the point mobility of the open rail for the two force positions indicated in Fig. 1(a). The main peak, around 240 Hz, is due to the resonance of the rail mass on the stiffness of the rail pad. For the

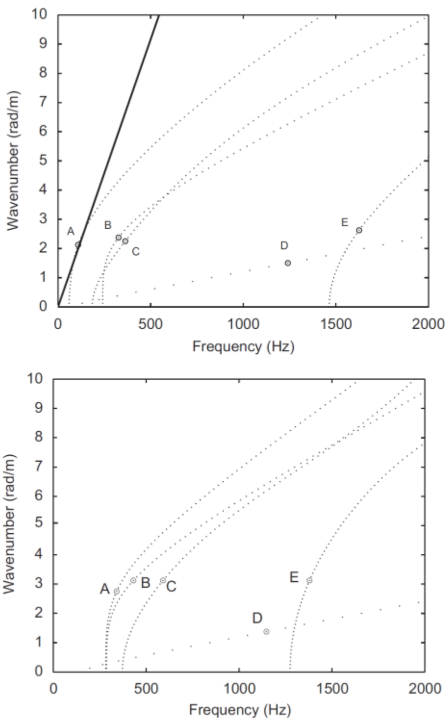


Fig. 2. Dispersion relations of the waves in (a) the open rail, CEN40E1, (b) the embedded rail, CEN51Ri.

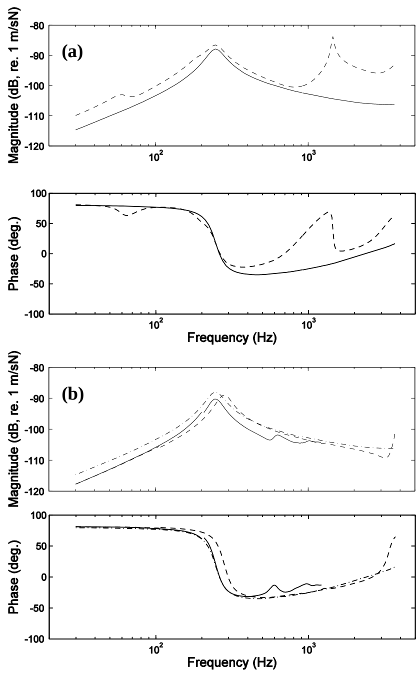


Fig. 3. Point mobilities of (a) the open rail (solid: centre excitation; dash line: edge excitation), CEN 40E1, (b) the embedded rail, CEN 51Ri (solid line: normal mass embedding; dashed line: reduced mass embedding; dash-dot line: open rail).

forcing point away from the centre of the rail, the response is higher. The point mobilities of the embedded rail are shown in Fig. 3(b). Since the curves for two different 'filling' materials are similar, waves propagating primarily in the embedding material have only limited effect on the mobility of the rail.

5. Conclusion

The WFE method to calculate the rail vibrations is described in this study. Two different railway tracks were considered as application examples; open rail and embedded rail. Dispersion relations and decay rates of the waves propagating along the railway tracks were easily identified in this method. In terms of the mobility, the railway tracks chosen have the similar behavior for the vertical excitation. The radiation from the tracks also can be predicted adding boundary elements on the surface of the rail, which will be discussed in the second presentation of the work.