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수중에 부분 몰수된 외팔보의 고유진동 특성

Natural Vibration Characteristics of Cantilever Plate Partially Submerged into Water

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1. Abstract

The free flexural vibration of a cantilever plate partially submerged in a fluid is investigated. The fluid is assumed to be inviscid and irrotational. The virtual mass matrix is derived by solving the boundary-value problem related to the fluid motion using elliptical coordinates. The introduction of the elliptical coordinates naturally leads to the use of the Mathieu function. Hence, the virtual mass matrix which reflects the effect of the fluid on the natural vibration characteristics is expressed in analytical form in terms of the Mathieu functions. The virtual mass matrix is then combined with the dynamic model of a thin rectangular plate obtained by using the Rayleigh-Ritz method. This combination is used to analyze the natural vibration characteristics of a partially submerged cantilever plate qualitatively. Also, the non-dimensionalized added virtual mass incremental factors for a partially submerged cantilever plate are presented to facilitate the easy estimation of natural frequencies of a partially submerged cantilever plate. The numerical results validate the proposed approach.

2. Dynamic Modeling

Let us consider a rectangular plate with side lengths a in the X direction and b in the Z direction as shown in Fig. 1. By using the assumed mode method, the kinetic and potential energies of the rectangular plate can then be expressed as

$$T_p = \frac{\rho_p hab}{2}\dot{\mathbf{q}}^{\mathrm{T}}\bar{\mathbf{M}}_p\dot{\mathbf{q}}, V_p = \frac{Db}{2a^3}\mathbf{q}^{\mathrm{T}}\bar{\mathbf{K}}_p\mathbf{q} \quad (1)$$

where $\mathbf{q}(t) = [q_1\ q_2 \ldots q_m]^{\mathrm{T}}$ is a $m \times 1$ vector consisting of generalized coordinates, in which m is the number of admissible functions used for the approximation of the deflection, ρ_p the mass density, h the thickness, $D = Eh^3/12(1-\nu^2)$, E the Young's modulus, and ν the Poisson's ratio, $\bar{\mathbf{M}}_p$, $\bar{\mathbf{K}}_p$ are the non-dimensionalized mass and stiffness matrices, respectively. Refer to the reference(Kwak and Han, 2007) for the expression for those matrices.

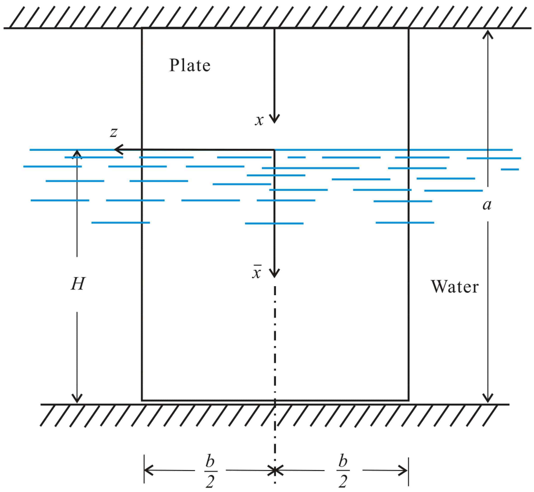

Fig. 1. Partially submerged cantilever plate.

3. Fluid−Structure Interaction

Let us consider a cantilever rectangular plate partially submerged in a fluid as shown in Fig. 1 and assume that the free end of a plate is close to the bottom so that the effect of the fluid motion between the plate and flat bottom can be neglected. The fluid motion is represented by the fluid's velocity potential function and the fluid is assumed to be inviscid and irrotational. The governing equation for the fluid is the Laplace equation

$$\nabla^2 \phi = 0 \tag{2}$$

where $\phi(\bar{x}, y, z, t)$ is the fluid potential. In this study, we consider the Mathieu and modified Mathieu functions of the integer order for the solutions of Eq. (2). In particular, the periodic and non-periodic solutions were considered in solving Eq. (2). Then, the fluid kinetic energy can be derived as

$$T_f = \frac{\rho_f a b^2}{2} \dot{\mathbf{q}}^{\mathrm{T}} \bar{\mathbf{M}}_f \dot{\mathbf{q}} \tag{3}$$

where

$$\bar{\mathbf{M}}_f = \frac{1}{2\pi\delta} \sum_{k=1}^{\infty} \sum_{r=1}^{\infty} c_{kr} \mathbf{I}_{kr}^{\mathrm{T}} \mathbf{I}_{kr} \tag{4}$$

represents the non-dimensionalized fluid mass matrix, in which

$$c_{kr} = \frac{\text{-Ge}_r(0, -q_k)}{\text{Ge}_r'(0, -q_k)} \tag{5}$$

where $\text{Ge}_r(\rho, -q_k)$ is the non-periodic even Mathieu function.

Considering the kinetic and potential energy expressions given by Eqs. (1) and (3), the free vibrations of the partially submerged cantilever plate can be derived as follows:

$$\rho_p hab \left(\bar{\mathbf{M}}_p + \gamma \bar{\mathbf{M}}_f \right) \ddot{\mathbf{q}} + \frac{Db}{a^3} \bar{\mathbf{K}}_p \mathbf{q} = 0 \tag{6}$$

where $\gamma = \rho_f b / \rho_p h$ is the non-dimensional constant representing the ratio of the fluid density to the plate's mass density times the ratio of the plate's length to its thickness. By solving the free vibration problem of Eq. (6), we can obtain natural frequencies and mode shapes.

4. Numerical Study

The strip method (Lindholm, 1965) gives the fixed NAVMI factors but they are valid only when the aspect ratio is very large. The NAVMI factors for the partially submerged cantilever plate versus the aspect ratio are not available. We computed the NAVMI factors for a fully submerged cantilever plate and compared them with those computed based on the Lindholm's experimental data as shown in Fig. 2. It can be seen from Fig. 2 that the theoretical predictions are in good agreement with the experimental results. It can be also observed in Fig. 2 that the theoretical NAVMI factors for the beam bending modes are higher than the experimental NAVMI factors.

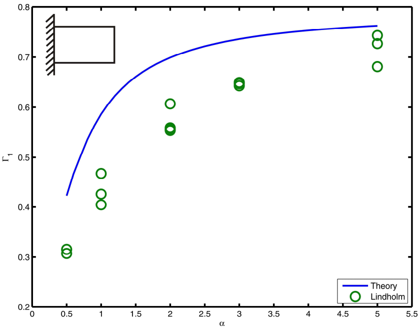

Fig. 2. NAVMI factors vs. aspect ratio for 1st mode

3. Conclusions

The mathematical formulation allows the derivation of the expression for the change of the natural frequency based on the NAVMI factors. The NAVMI factors for a partially submerged cantilever plate, which have never been obtained, were presented as graphs in terms of the aspect and draught ratios. It was found that theoretical predictions in this study were in good agreement with both experimental and numerical results, thus validating the proposed approach.

The numerical investigation showed that some natural modes do change by draught but differences between the dry modes and the wet modes become smaller as the draught increases. This seems evident since the virtual mass due to the water covers more surface as the draught increases. The fundamental natural frequency of a cantilever plate decreases rapidly as the draught increases but the decreasing rate slows down as the draught becomes more than half of the plate length.

The NAVMI factors obtained in this study can be effectively used to predict the change of the natural frequencies of the partially submerged cantilever plate due to the presence of the fluid.

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