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Nonlinear Vibration Analysis of a Spinning Beam with Deployment

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1. Introduction

This study of a spinning beam with deployment is related to a spinning beam which axially deploys from a fixed hub. The investigations on the vibration analysis of a spinning beam with deployment can be widely applied to the vibration behaviors of structures with spinning mechanism such as robot system, space vehicles, and satellite system.

There are relatively few researches on the dynamic behaviors of a spinning beam with axial motion. Bauer have studied the vibration behaviors of a spinning beam with translating motion including overall boundary conditions. Lee analyzed a pretwisted spinning and axially moving beam and investigated time response with various conditions. All above studies neglect the torsional vibration since the vibration behaviors of lateral direction is much more significant than torsional vibration. Also, it is reasonable that with doubly symmetric cross-section for which the shear center and centroid are coincident so the torsional vibration will be uncoupled with the flexural vibration.

On the other hand, when dealing with the vibration analysis of a spinning beam with axially motion, previous studies also neglect the axial displacement since in their study it is assumed to be small compared to lateral displacements. However, axial displacement cannot be neglected under some geometry parameters and motion conditions since it will cause a rather significant nonlinear effect on the lateral displacements. So this present study will include the nonlinear effects caused by axial displacement to lateral displacements.

The investigation procedure of present study is carried out in following steps. First, modeling of a spinning beam with deployment is given and governing differential equations of motion are derived by Hamilton's Principle. Then, weak form and discretized equations are derived by Galerkin method. Finally, dynamic time response will be given by Newmark method.

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2. Derivation of Governing Equation

2.1 Modeling

Including axial displacement u and displacements v, w in two lateral directions, a spinning deploying beam with uniform circular cross-section, total length L and deploying length l(t) is modeled. As mentioned above, it is reasonable that torsional vibration can be neglected since doubly symmetric cross-section. The beam is extruded with axial moving velocity V(t) and constant angular velocity Ω by an external force F(t) which is applied at the left end. It is assumed that the friction force between the hub and beam is neglected. X,Y,Z is a set of global space fixed coordinate system with corresponding unit vectors **I**, **J**, **K** and x,y,z is a set of local body fixed coordinate system with corresponding unit vectors **i**, **j**, **k**.

The position vector of a point on the centerline outside the fixed hub can be expressed in terms of the axial and two lateral displacements while the point inside the hub does not have lateral displacements, so the position vector of centerline outside the hub can be given as

$$\mathbf{r} = (X + x + u)\mathbf{i} + v\mathbf{j} + w\mathbf{k} \tag{1}$$

Velocity vector can be solved by vector differentiation based on above position vector. The velocity vector

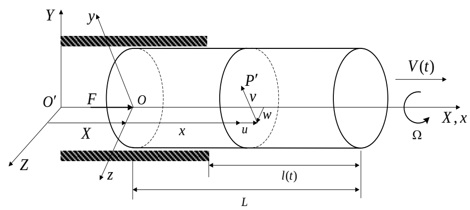

Fig.1.Modelling of a spinning beam with deployment

outside the hub can written as

$$\mathbf{v}=\left(V+\frac{\partial u}{\partial t}\right)\mathbf{i}+\left(\frac{\partial v}{\partial t}-\Omega w\right)\mathbf{j}+\left(\frac{\partial w}{\partial t}+\Omega v\right)\mathbf{k} \tag{2}$$

The beam is assumed to be slender enough although it looks like stubby as shown in figure for which geometry parameters can be clearly figured. Euler-Bernoulli beam theory and Von-Karman strain theory are adopted to get the nonlinear strain and linearized strain. The nonlinear strain outside the hub is given as

$$\varepsilon_x=\frac{\partial u}{\partial x}-y\frac{\partial^2 v}{\partial x^2}+z\frac{\partial^2 w}{\partial x^2}+\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{3}$$

2.1 Derivation of governing equation

In order to derive the governing equation, velocity vector is used to get the kinetic energy; linearized strain and nonlin ear stress are applied to derive the potential energy.

$$T=\frac{1}{2}\int_0^L\int_A \rho\mathbf{v}\cdot\mathbf{v}dAdx \tag{4}$$

$$U=\int_0^L\int_A \sigma_x\varepsilon_x^L dAdx \tag{5}$$

The governing equations of motion are derived by Hamilton's principle

$$\int_{t_1}^{t_2}(\delta T-\delta U+\delta W)dt=0 \tag{6}$$

According to above equations, governing equations of motion can be derived as

$$\rho A\frac{\partial^2 u}{\partial t^2}-EA\frac{\partial^2 u}{\partial x^2}=-\rho A\dot{V} \tag{7}$$

$$\rho A\left(\frac{\partial^2 v}{\partial t^2}-2\Omega\frac{\partial w}{\partial t}-\Omega^2 v\right)-EA\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}\right)+EI_{zz}\frac{\partial^4 v}{\partial x^4}=0 \tag{8}$$

$$\rho A\left(\frac{\partial^2 w}{\partial t^2}+2\Omega\frac{\partial v}{\partial t}-\Omega^2 w\right)-EA\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\right)+EI_{yy}\frac{\partial^4 w}{\partial x^4}=0 \tag{9}$$

As shown above, the axial equation is coupled with two lateral equations respectively that cause nonlinear effects and the two lateral equations are coupled with each other.

3. Discretization

In order to get time response of above nonlinear co upled equations, Galerkin method is applied to get the weak form of above governing equations. The discretiz ed equations can be obtained by introducing the trial f unction and weighting function into weak form. Based on boundary conditions, it is easy to get the admissible functions for the axial equation and comparison functi ons for lateral equations, by which, the components of discretized equations can be computed. The discretized equations are given as

$$\sum_{j=1}^{J}\left(m_{ij}\ddot{T}_j^u+Ak_{ij}T_j^u\right)=-\dot{V}f_i^a+\dot{V}f_i^b \tag{10}$$

$$\sum_{n=1}^{N}\sum_{q=1}^{Q}\Big[m_{mn}\ddot{T}_n^v+2g_{mn}\dot{T}_n^v+\Big(k_{mn}^a-\Omega^2k_{mn}^b+k_{mn}^c+A\sum_{j=1}^{J}\alpha_{jmn}T_j^u\Big)T_n^v-2\Omega g_{mq}\dot{T}_q^w-2\Omega k_{mq}T_q^w\Big]=0 \tag{11}$$

$$\sum_{q=1}^{Q}\sum_{n=1}^{N}\Big[m_{pq}\ddot{T}_q^w+2g_{pq}\dot{T}_q^w+\Big(k_{pq}^a-\Omega^2k_{pq}^b+k_{pq}^c+A\sum_{j=1}^{J}\alpha_{jpq}T_j^u\Big)T_q^w+2\Omega g_{pn}\dot{T}_n^v+2\Omega k_{pn}T_n^v\Big]=0 \tag{12}$$

4. Dynamic Time Response

Based on above discretized equations, dynamic time response can be computed by Newmark time integration method. The nonlinear dynamic time response of lateral w direction is given as in figure 2.

5. Conclusion

Considering the nonlinear effects caused by axial displacement to two lateral displacements and coupling effects between two lateral directions, the equations of motion and dynamics time response have been obtained. The nonlinear effects caused by axial displacement to two lateral displacements cannot be neglected under some geometry parameters and motion conditions.

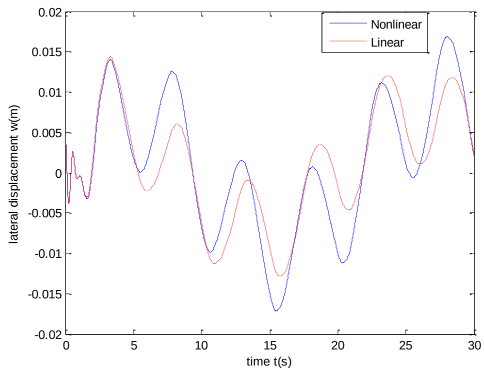

Fig.2. Lateral displacement w at the end of the beam