구면상의 지역에서의 퍼텐셜마당에 대하여 2차원 푸리에변환을 이용한 위로 또는 아래로의 연속

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Upward and Downward Continuation of Potential Field on Spherical Patch Area using 2-Dim DFT

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Two most important potential fields in geophysics are gravity field and stationary magnetic field. Sometimes we need to know the values of such field at a specified height, which is either higher or lower than original datum plane. Then downward or upward continuation of the field can be readily achieved by using two dimensional DFT (Discrete Fourier Transform), when the area is relatively small compared with the Earth's radius, In this study, we derive an algorithm for upward or downward continuation of gravity field on a spherical patch area. And this algorithm can be applied for any other potential field on spherical patch area. Gravity field vector $\vec{g}$ is expressed as the gradient of the gravity potential U, so that $\vec{g} = \vec{\nabla} U$. Usually vertical component $g = g_r = \frac{\partial U}{\partial r}$ only is considered.

Laplace equation for a gravity potential field U, like any other, in spherical coordinate is given as eq.(1).

$$\nabla^2 U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial U}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 U}{\partial\phi^2} = 0 \tag{1}$$

Suppose we have a set of potential field data acquired on a spherical patch area, which has range of $\theta_1 \le \theta \le \theta_2$ and $\phi_1 \le \phi \le \phi_2$ with grid spacing of $\triangle\theta$ and $\triangle\phi$. We denote $U(r, \theta_1 + k\triangle\theta, \phi_1 + l\triangle\phi)$ as $U(r,k,l)$ and its Fourier transform as $\widetilde{U}(r,m,n)$. So their relations can be written as the following.

$$\widetilde{U}(r,m,n) = \frac{1}{K}\sum_{k=0}^{K-1}\frac{1}{L}\sum_{l=0}^{L-1} U(r,k,l)\exp\left(-2\pi i\frac{km}{K}\right)\exp\left(-2\pi i\frac{ln}{L}\right) \tag{2a}$$

$$U(r,k,l)=\sum_{m=0}^{K-1}\sum_{n=0}^{L-1}\widetilde{U}(r,m,n)\exp(+2\pi i\frac{km}{K})\exp(+2\pi i\frac{ln}{L}) \tag{2b}$$

Then the discrete version of Laplace eq. in terms of $\widetilde{U}(r,m,n)$ is found as

$$\sum_{m=0}^{K-1}\sum_{n=0}^{K-1}\left(\begin{array}{c} r^2\dfrac{\partial^2\widetilde{U}}{\partial r^2}+2r\dfrac{\partial\widetilde{U}}{\partial r} \\ +(\triangle\theta)^{-2}(-4\pi^2\dfrac{m^2}{K^2})\widetilde{U}+(\triangle\theta)^{-1}(2\pi i\dfrac{m}{K})\cot(\theta_1+k\triangle\theta)\widetilde{U} \\ +(\triangle\phi)^{-2}(-4\pi^2\dfrac{n^2}{L^2})\sin^{-2}(\theta_1+k\triangle\theta)\widetilde{U}\end{array}\right)\exp(2\pi i\frac{km}{K})\exp(2\pi i\frac{ln}{L}) = 0$$

And it is claimed here that the coefficient of the summand vanishes, The resulting differential equation for $\widetilde{U}(r,m,n)$ is written as

$$r^2\frac{d^2\widetilde{U}}{dr^2}+2r\frac{d\widetilde{U}}{dr}+A\widetilde{U} = 0 \ , \tag{3a}$$

with $$A=-(\frac{2\pi m}{K\triangle\theta})^2+\frac{2\pi i m}{K\triangle\theta}\cot(\theta_1+k\triangle\theta)-(\frac{2\pi n}{L\triangle\phi\sin(\theta_1+k\triangle\theta)})^2 . \tag{3b}$$

Eq.(3a) belongs to Euler-Cauchy differential eq. and its solution is given as $\widetilde{U}(r,m,n)=C\,r^{\lambda}$, where $\lambda=\dfrac{-1\pm\sqrt{1-4A}}{2}$ ('-' should be taken here).

The continuation of gravity potential U can be done by the following procedure.

$$U(r_1,k,l) \ \Rightarrow \ \widetilde{U}(r_1,m,n) \ \Rightarrow \ \widetilde{U}(r_2,m,n) \ \Rightarrow \ U(r_2,k,l)$$

The three successive procedures are 1) Fourier transform, 2) $\widetilde{U}(r_2,m,n)=\widetilde{U}(r_1,m,n)(\dfrac{r_2}{r_1})^{\lambda}$, and 3) inverse Fourier transform.

In practice, usually the gravity field itself is processed. The continuation of gravity field g is attained similarly.

$$g(r_1,k,l) \ \Rightarrow \ \tilde{g}(r_1,m,n) \ \Rightarrow \ \tilde{g}(r_2,m,n) \ \Rightarrow \ g(r_2,k,l)$$

It should be noted that the exponent in the second procedure is replace by $\lambda-1$ due to the difference in radial dependence. Below are attached sample outputs.