2011년도 대한전기학회 하계학술대회 논문집 2011. 7. 20 - 22

계단모양 소속 함수 근사를 이용한 구간 2형 퍼지 시스템의 관측기 기반 제어기 설계

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Design of Observer-based Controller for Interval Type-2 Fuzzy System Using Staircase Membership Function Approximation

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Abstract - This paper presents observer-based controller design for interval type-2 fuzzy system with staircase membership approximation. In type-2 fuzzy case, membership function is itself fuzzy set itself. Thus, type-2 fuzzy system can deal with parametric uncertainties of nonlinear system by capturing the uncertainties in membership function. Likewise, stabilization condition of type-2 fuzzy system is derived from quadratic Lyapunov function, and it goes to linear matrix inequality. Furthermore, in this paper, to relax the conservativeness of stabilization condition, staircase membership function approximating method is applied. Observer-based control method is adopted to control system which has some unmeasurable states. To prove suitability of our proposed method, numerical example is presented.

1. INTRODUCTION

Recently, various researches have been processed on type-1 Takagi-Sugeno (T-S) fuzzy system[2]. T-S fuzzy system can represents nonlinear system in fuzzy blending of local linear systems and type-1 fuzzy sets. Thus by using T-S fuzzy systems, nonlinear system can be stabilized by various linear control methods. One of the control method is observer-based control. If some states of the nonlinear system cannot be measured, then state feedback control method cannot be applied to the T-S fuzzy system. In this cases, observer-based control method can be applied to the system.

As the membership functions for the type-1 fuzzy sets contain no uncertainty information, existing control method[2] cannot stabilize nonlinear systems with parametric uncertainties. To stabilize parametric uncertain nonlinear system, robust control method for type-1 fuzzy system was researched. This method manipulates parametric uncertainties. Recently, type-2 fuzzy logic systems was proposed to deal with parametric uncertain system. However, type-2 fuzzy logic systems are more complex and computationally intensive than type-1 T-S fuzzy systems. Thus type-2 fuzzy logic systems is still emerging area. To catch the simplicity of T-S fuzzy system and stabilizing problem for parametric uncertain nonlinear system, [1] proposed stabilization method for type-2 T-S fuzzy system. However, in previous results, it is assumed that all states of nonlinear system is measurable. Furthermore, in [2], to relax the conservativeness of the stabilization condition, new stability analysis method which is approximating membership function as staircase was proposed.

To overcome restriction of [1] which is all states must be measurable and relax the stabilization condition, this paper propose observer-based control method for type-2 T-S fuzzy system using staircase membership function approximation. Nevertheless, we still have assumption that all states in premise part of IF-THEN rules must be measurable. Likewise type-1 fuzzy system, stabilization condition of type-2 fuzzy system is derived from quadratic Lyapunov function, and this condition can be converted into linear matrix inequalities (LMIs). Thus, stabilization condition of our proposed method is similar to the type-1 T-S fuzzy system case, and proposed method can stabilize parameter uncertain nonlinear system when some states cannot be measured. To the best of our knowledge, this paper represents the first attempt to apply staircase membership function approximation in interval type-2 observer-based controlled fuzzy system.

감사의 글 : 2010년도 지식경제부의 재원으로 한국에너지 기술평가원(KETEP)의 지원을 받아 수행한 연구과제입니다. (NO. 20104010100590)

2. PRELIMINARIES

2.1 INTERVAL TYPE-2 FUZZY MODEL

In this section, a nonlinear plant subject to parametric uncertainties is represented by an interval type-2 fuzzy mode. Interval type-2 fuzzy model has following fuzzy IF-THEN rules [1].

Rule i:

IF $z_1(t)$ is $\widetilde{M}_{i1}$ and $\cdots$ and $z_p(t)$ is $\widetilde{M}_{ip}$

THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$ (1)

where $\widetilde{M}_{i\alpha}$ is an interval type-2 fuzzy set of rule i corresponding to the premise variable $z_\alpha(t)$, $(\alpha = 1,2,\cdots,p,\ i = 1,2,\cdots,r)$. $A_i \in \boldsymbol{R}^{n\times n}$ and $B_i \in \boldsymbol{R}^{n\times m}$ are known constant system and input matrices, respectively. $x(t) \in \boldsymbol{R}^{n\times 1}$ and $u(t) \in \boldsymbol{R}^{m\times 1}$ are state and input vectors, respectively.

The inferred output of type-2 fuzzy model is as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \quad (2)$$

where

$$h_i(z(t)) = h_i^L(z(t))\underline{v}_i(z(t)) + h_i^U(z(t))\overline{v}_i(z(t)) \in [0,\ 1]$$

$$h_i^L(z(t)) = \underline{\mu}_{\widetilde{M}_{i1}}(z(t)) \times \cdots \times \underline{\mu}_{\widetilde{M}_{ip}}(z(t)) \geq 0,$$

$$\sum_{i=1}^{r} h_i(z(t)) = 1, \quad ,$$

$$h_i^U(z(t)) = \overline{\mu}_{\widetilde{M}_{i1}}(z(t)) \times \cdots \times \overline{\mu}_{\widetilde{M}_{ip}}(z(t)) \geq 0$$

in which $\underline{\mu}_{\widetilde{M}_{i\alpha}}(z(t))$ and $\overline{\mu}_{\widetilde{M}_{i\alpha}}(z(t))$ are lower and upper membership function of interval type-2 fuzzy set, respectively. Further informations about interval type-2 fuzzy set for the fuzzy-model-based control system is available in [1]. Since membership function is itself fuzzy set, interval type-2 fuzzy model can capture the parametric uncertainties of nonlinear system.

2.2 INTERVAL TYPE-2 FUZZY OBSERVER-BASED CONTROLLER

Interval type-2 fuzzy model (2) can be stabilized by using following interval type-2 fuzzy observer-based controller, even if some states cannot be measured.

In the following, premise part of IF-THEN rule and membership function of observer and controller is same with interval type-2 fuzzy model. With the above knowledges, inferred output of observer is as follows:

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))\left\{A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\right\} \quad (3)$$

$$y(t) = \sum_{i=1}^{r} h_i C_i x(t), \quad \hat{y}(t) = \sum_{i=1}^{r} h_i C_i \hat{x}(t) \quad (4)$$

where $\hat{x}(t)$ is estimated states, $y(t)$ and $\hat{y}(t)$ are output and estimated output vector, respectively, and L_i are the observer gains.

Likewise, inferred output of observer-based controller is as follows:

$$u(t)=\sum_{i=1}^{r}(\underline{h}_i(z(t))+\overline{h}_i(z(t))K_i\hat{x}(t) \quad (5)$$

where

$$\underline{h}_i(z(t))=\frac{h_i^L(z(t)}{\Sigma_{l=1}^{r}(h_l^L(z(t))+h_l^U(z(t))}\geq 0,$$

$\overline{h}_i(z(t))=\frac{h_i^U(z(t)}{\Sigma_{l=1}^{r}(h_l^L(z(t))+h_l^U(z(t))}\geq 0$, K_i are controller gains, and C_i are output matrices.

In [1], (5) is called as normalized central (NC) fuzzy controller, because type reduction for interval type-2 fuzzy controller is characterized by the average normalized membership grades of the lower and upper membership functions. From now on, for lighten the notation, we will omit the time t and $z(t)$, i.e $h_i(z(t))=h_i$.

Let $e=x-\hat{x}$, than augmented system can be represented as follows:

$$\dot{x}_a=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\underline{h}_i+\overline{h}_i)G_{ij}x_a \quad (6)$$

where $x_a=[\hat{x}\ \ e]^T$, $G_{ij}=\begin{bmatrix}A_i-B_iK_j & B_iK_j\\ 0 & A_i-L_iC_j\end{bmatrix}$.

3. MAIN RESULT

In this chapter, observer-based controller design method for interval type-2 fuzzy system is proposed. To relax the conservativeness of the stabilization condition, we will introduce the following scalar and matrix.

γ_{ij} is the scalar such that

$$h_i(\underline{h}_j+\overline{h}_j)-\tilde{h}_i(\tilde{\underline{h}}_j+\tilde{\overline{h}}_j)-\gamma_{ij}\geq 0,$$

M is the arbitrary matrix such that

$$\sum_{i=1}^{r}\sum_{j=1}^{r}(h_i(\underline{h}_j+\overline{h}_j)-\tilde{h}_i(\tilde{\underline{h}}_j+\tilde{\overline{h}}_j))M=0.$$

Stabilization condition of interval type-2 fuzzy observer-based control system is presented in the following theorem.

Theorem 1: The augmented system (6) is globally asymptotically stabilizable when there exit a symmetric positive definite matrix X_1, X_2, and W_{ij} and pre-described scalar γ_{ij} satisfying following LMIs.

$$X=\begin{bmatrix}X_1 & 0\\ 0 & X_2\end{bmatrix}>0 \quad (7)$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\left\{(\tilde{h}_i(\tilde{\underline{h}}_j+\tilde{\overline{h}}_j)+\gamma_{ij})(Q_{ij}+W_{ij})+\gamma_{ij}M\right\}<0 \quad (8)$$

$$Q_{ij}+W_{ij}+M<0 \quad (9)$$

where

$$Q_{ij}=\begin{bmatrix}X_1A_i^T+A_iX_1-M_j^TB_i-B_iM_j & -B_iM_j\\ -M_j^TB_i^T & A_i^TX_2+X_2A_i-C_j^TX_2-X_2C_j\end{bmatrix},$$

$X_1=P_1^{-1}$, $X_2=P_2$, and $\tilde{h}_i$ means staircase approximation version of h_i. If (7-9) hold, controller and observer gains are defined as $M_j=K_jX_1$, $N_i=X_2L_i$, $(i,j=1,2,\cdots,r)$

proof: The proof is omitted due to the lack of the page. ■

The LMI (8) must be determined for the whole value of staircase membership functions. Furthermore, approximated membership function is used in only stabilization conditions, so quantization error is not exists.

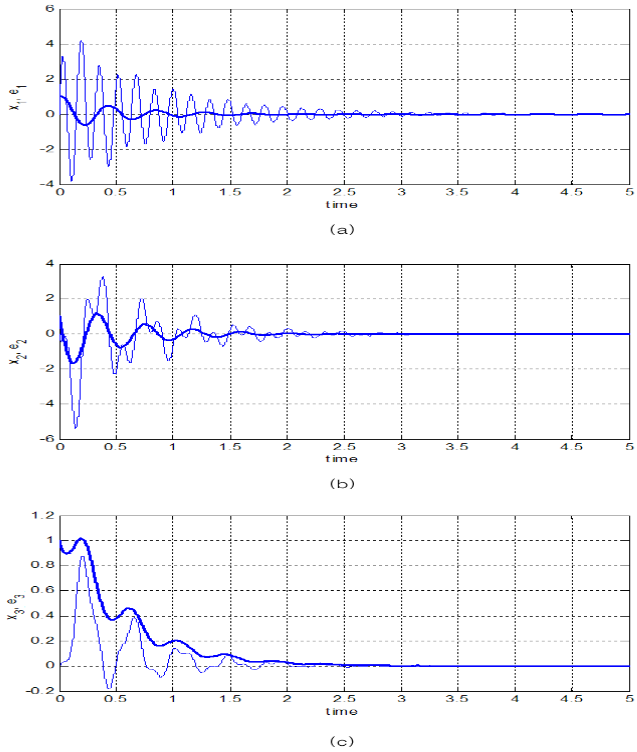
4. NUMERICAL EXAMPLE

To prove suitability of proposed method, numerical example is presented. Interval type-2 fuzzy system is given as follows:

$$A_1=\begin{bmatrix}-10 & 10 & 0\\ 28 & -1 & -m_1\\ 0 & m_1 & -8/3\end{bmatrix},\ A_2=\begin{bmatrix}-10 & 10 & 0\\ 28 & -1 & -m_2\\ 0 & m_2 & -8/3\end{bmatrix},$$

$$B_1=B_2=\begin{bmatrix}1\\0\\0\end{bmatrix},\ C_1=C_2=[1\ 0\ 0],\ m_1=-21,\ m_2=29$$

$h_1^L(x_1)=0.95-(0.925/(1+e^{-((x+3.5)/8)}))$,



<Fig. 1> System and error state responses. (a) x_1 (solid line) and e_1 (bold solid line). (b) x_2 (solid line) and e_2 (bold solid line). (c) x_3 (solid line) and e_3 (bold solid line)

$h_2^L(x_1)=0.025-(0.925/(1+e^{-((x-3.5)/8)}))$,
$h_1^U(x_1)=0.95-(0.925/(1+e^{-((x+4.5)/8)}))$,
$h_2^U(x_1)=0.025-(0.925/(1+e^{-((x-4.5)/8)}))$.

From theorem 1, controller and observer gains are given as follows: $K_1=[-7.7030\ \ 117.3319\ \ 0.6887]$, $K_2=[-7.7475\ \ 117.3285\ \ -0.4752]$, $L_1=[-6.0761\ \ 70.2792\ \ -4.5954]^T$, $L_2=[-5.7093\ \ 72.5267\ \ 6.3517]^T$.

Fig.1 shows system and error state responses. It can be seen from Fig.1 that the proposed method can stabilize system state and properly estimates the system states.

5. CONCLUSION

An interval type-2 fuzzy system and its observer-based controller design with approximated staircase membership function have been proposed to control uncertain and partly unmeasurable nonlinear system. Interval type-2 fuzzy system has upper and lower membership function, parametric uncertainties of nonlinear system can be effectively captured. To relax the conservative of the stabilization conditions, staircase membership function approximating method has been applied in proposed method. Similar to type-1 fuzzy system, stabilization condition of proposed model were derived from Lyapunov function and it was transformed into the LMIs.

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