시뮬레이션 신뢰도 계산방법의 정확성에 대한 영향요인 연구

렌지얀, 장전해, 고창섭
전기전자 컴퓨터공학부, 충북대학교

Study on the Influence Factors for Accuracy of Simulation-based Reliability Calculation Methods

Ziyan Ren, Dianhai Zhang, and Chang Seop Koh
College of Electrical and Computer Engineering, Chungbuk National University

Abstract - Monte Calrlo Simulation (MCS) and Sensitivity-based MCS (SMCS) are studied and compared in this paper. Quantitative analysis of the influence factors for reliability calculation accuracy is performed through a analytic function and finally applied to TEAM problem 22.

1. Introduction

For the reliability calculation of a design, the Monte Carlo Simulation method is usually employed for obtaining the benchmark results. However, the main drawback is time-consuming, and it is not suitable to being embedded in engineering optimization design processes, where the performance function are obtained by using Finite Element Method (FEM).

Considering the higher accuracy and lower efficiency of MCS, one sensitivity-based MCS is proposed. In the following paper, the influence factors for the accuracy of reliability calculation is researched.

2. Sensitivity-based Reliability Calculation Algorithm

2.1 Monte Carlo Simulation

In the MCS, in order to evaluate the reliability of a design X with respect to the constraint g(X)>0, N different samples are generated in a certain confidence interval $[\mu_i - k\sigma_i, \mu_i + k\sigma_i]\ i = 1,2,...M.$ If n samples satisfy the constraint g(X)>0, then the reliability of design X can be obtained as:

$$R(g(X) \geq 0) = n/N. \tag{1}$$

2.2 Sensitivity-based Monte Carlo Simulation

In SMCS, the sensitivity analysis is applied to construct an approximated analytic function especially for the nonlinear performance constraints.

For a given determinant design $\mathbf{X}_0$, the constraint functions in uncertain regions are approximated to linear ones using their gradient vectors as follows:

$$g(\boldsymbol{X}) \cong g(\boldsymbol{X}_0) + \nabla g(\boldsymbol{X}_0) \cdot (\boldsymbol{X} - \boldsymbol{X}_0). \tag{2}$$

The gradient vector is calculated as follows [1]:

$$\nabla g(\boldsymbol{X}_0) = \frac{\partial g}{\partial [\boldsymbol{X}]^T}\bigg|_{A=C.} - [\lambda]^T \left(\frac{\partial [R]}{\partial [\boldsymbol{X}]^T}\bigg|_{\nu = C.} - \frac{\partial [R]}{\partial \nu}\frac{\partial \nu}{\partial B^2}\frac{\partial B^2}{\partial [\boldsymbol{X}]^T} \right) \tag{3}$$

$$[K + \overline{K}]^T [\lambda] = \frac{\partial g}{\partial [\boldsymbol{A}]} \tag{4}$$

where **R** is the residual vector in Galerkin's approximation; other symbols have their usual meanings in FEM. Once the sensitivity information is obtained from (3), the reliability analysis can be performed to the approximated analytic function.

3. Numerical Examples and Conclusions

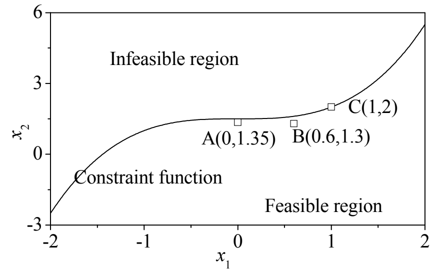
3.1 Analytic Test Function

The following constraint function, shown in Fig.1, is taken as an example, where the two uncertain design variables following the independent identically Gaussian distribution (IID), $x_i \sim N(\mu_i, \sigma_i)\ i = 1,2.$ During the design ranges of $-2 \leq x_1 \leq 2, -3 \leq x_2 \leq 6$, three testing points are selected as: A(0, 1.35), B(0.6, 1.3) and C(1, 2).

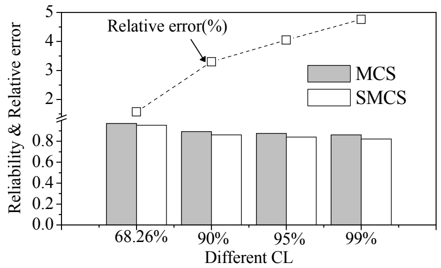
$$g(x) = x_1^3 - 2x_2 + 3 \geq 0 \tag{5}$$

The sensitivity vector is calculated as follows:

$$\nabla g(x) = \left\{3x_1^2, -2\right\} \tag{6}$$



<Fig. 1> Constraint function and testing points



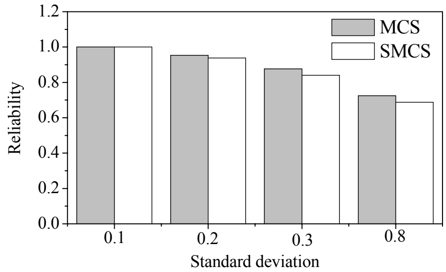
<Fig. 2> Reliability comparison and relative error for point B

A. Influence from Confidence Level

The deviation level of the uncertainty in design variable with respect to nominal value is expressed by confidence level (CL). Here, different CLs are applied to sensitivity analysis and the standard deviation is fixed as 0.3. The calculation result is shown in Table 1. The reliability of point B and relative error are compared in Fig.2. It shows that the reliability calculated from MCS and SMCS will decrease as the confidence level becomes bigger, at the same time, the relative error of SMCS will be bigger. In order to simulate the real uncertainties, the CL of 95% is used in the subsequent study.

B. Influence from Standard Deviation

In order to check the accuracy of SMCS, reliability analysis is performed under different standard deviations and results are listed in Table. 2. Figure 3 gives the intuitionstic comparison for point B. From the comparisons, it is obvious that the SMCS is much more efficient than the original MCS under smaller standard deviations. Even for bigger σ, the relative error (<5%) can be acceptable.



<Fig. 3> Reliability comparison under different standard deviations

3.2 TEAM Problem 22

For the electromagnetic application, the MCS and SMCS are applied to TEAM problem 22[2], where the deterministic design variables are selected as diameter of outside coil D_2($0.1m \le D_2 \le 0.3m$) and height of outside coil, H_2($0.1m \le H_2/2 \le 1.8m$);the uncertain design variables are current densities of inside and outside coils, (J_1,J_2), which follows Gaussian distribution: $J_1 \sim N(\mu=16.78MA/m^2, \sigma=0.179MA/m^2)$,$J_2 \sim N(\mu=-15.51MA/m^2, \sigma=0.179MA/m^2)$. In order to guarantee the superconductivity of the coil, the performance constraint for quench condition should be satisfied as:

$$g_i(\mathbf{d}) = 54 - |J_i| - 6.4 |B_{\max,i}(\mathbf{d})| \ge 0, \quad i = 1,2 \tag{7}$$

where $B_{max,i}$ (i=1,2) is the maximum magnetic flux density in the coil. The sensitivity vector is calculated as follows:

$$\nabla g_1 = \left\{ \frac{\partial g_1}{\partial J_1}, \frac{\partial g_1}{\partial J_2} \right\} = \left\{ -1 - \frac{6.4 \partial B_{\max,1}}{\partial J_1}, - \frac{6.4 \partial B_{\max,1}}{\partial J_2} \right\} \tag{8}$$

$$\nabla g_2 = \left\{ \frac{\partial g_2}{\partial J_1}, \frac{\partial g_2}{\partial J_2} \right\} = \left\{ - \frac{6.4 \partial B_{\max,2}}{\partial J_1}, \quad 1 - \frac{6.4 \partial B_{\max,2}}{\partial J_2} \right\}$$

In Eq.(8), the sensitivity parts can be calculated by using FEM [1]:

$$\frac{dB_{\max,i}}{d[\mathbf{J}]^T} = \frac{\partial B_{\max,i}}{\partial [\mathbf{J}]^T}\bigg|_{A=const} + [\boldsymbol{\lambda}]^T \cdot \frac{\partial \{\mathbf{Q}\}}{\partial [\mathbf{J}]^T}, (i = 1,2) \tag{9}$$

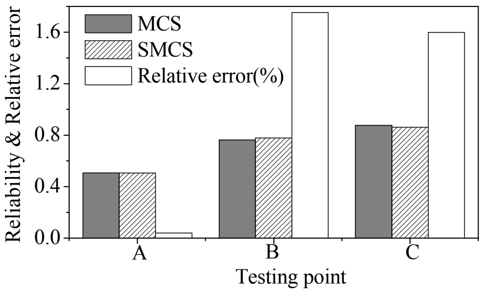
$$[\mathbf{K}]\{\boldsymbol{\lambda}\} = \frac{\partial B_{\max,i}}{\partial [\mathbf{A}]^T}, (i = 1,2) \tag{10}$$

Three testing points are selected. The calculation results are shown in Table.3 and Fig.4, where the trials are 1 million and reliabilities for g_2 are 1.

It can be seen that the SMCS method can obtain nearly same accuracy with the MCS. What is more, due to the design sensitivity analysis, the SMCS needs only 1 time FEM calls of the reliability calculation for a given design, however, the MCS needs 1,000,000 times of FEM calls. Therefore, in the view point of calculation time, the proposed method is much more efficient than the MCS. Therefore, under the certain condition, the SMCS is more helpful for reliability analysis in engineering application.

<Table 3> Testing points and reliability

Testing points	$R(g_1 \ge 0)$		Relative Error (%)
	MCS	SMCS	
A(0.244, 1.49)	0.5068	0.5066	3.9463E-2
B(0.25, 1.5)	0.7644	0.7778	1.753
C(0.253,1.482)	0.8759	0.8619	1.598



<Fig. 4> Comparisonofreliabilityforconstraint1

[Reference]

[1]J. S. Ryu, Y. Yao, C. S. Koh, S. Yoon, and D. S .Kim, "Optimal shape design of 3-D nonlinear electromagnetic devices using parameterized design sensitivity analysis," *IEEE Trans. Magn.,* vol.41, no.5, pp.1792-1795, May, 2005.

[2]G. Steiner, A. Weber, and C. Magele, "Managing uncertainties in electromagnetic design problems with robust optimization," *IEEE Trans. Magn.,* vol.40, no.2, pp.1094-1099, March, 2004.

<Table 1> Reliabilities under different confidence levels

CL	68.26%		90%		95%		99%	
	MCS	SMCS	MCS	SMCS	MCS	SMCS	MCS	SMCS
A	0.78007	0.78009	0.71228	0.71277	0.70057	0.70201	0.69320	0.69301
B	0.96989	0.95469	0.89208	0.86261	0.87601	0.84052	0.86302	0.82194
C	0.52085	0.50118	0.52933	0.50068	0.53298	0.49998	0.53226	0.50017

<Table 2> Reliabilities under different standard deviations

σ	0.1		0.2		0.3		0.8	
	MCS	SMCS	MCS	SMCS	MCS	SMCS	MCS	SMCS
A	0.95589	0.95600	0.78767	0.78782	0.70057	0.70201	0.57031	0.57817
B	1.00	1.00	0.95364	0.93784	0.87601	0.84052	0.72457	0.68740
C	0.50910	0.50037	0.52163	0.49902	0.53298	0.49998	0.54907	0.45418