Dynamic Modeling and Control of Production/Inventory System

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ABSTRACT : This paper presents the system dynamics methodology for modeling and control the production/inventory system. Under system dynamics point of view, we can apply some production/inventory policies as if we use the control laws for dynamics systems, then the behavior of system is analyzed and evaluated to improve the performance of production/inventory system. We also utilize the hybrid modeling method for the dynamic of production system with the combination of Matlab/Simulink and Matlab/Sateflow. Finally, the numerical simulation results are carried out in Matlab/Simulink environment and compare with the results from other works. It is shown that our approach can obtain some good performances (such as operational cost, stability of inventory, customer service level).

KEYWORDS : Production system, system dynamics, dynamic modeling, SCM, hybrid system.

1. Introduction

Recently, due to the rapid development of marketplace and the large variability in products, customer demands, along with the increasing of competiveness of companies, the effective and efficient supply chain management (SCM) plays an important role in achieve high profit, lower costs and better customer satisfaction in the firm's business. Managing and planning for production/inventory system in SCM are one of the key issues to achieve business goals. However making the plan for controlling and managing production/inventory system in supply chain has met many challenges in the presence of mentioned above factors. To deal with the issues, various alternative approaches have been proposed for modeling and control of supply chain. Among those approaches system dynamics (dynamical system) provide a convenient and compact way to model and analyze supply chain system. A number of papers have dealt with application of control theory to dynamical supply chain and production inventory systems can be found in (Ortega and Lin, 2004) and (Sarimveis et al., 2008). To obtain the high revenue, low lost and better customer satisfaction, the optimal decision policies should be applied to supply chain management and optimal control is a suitable method to do this. Since the objective of optimal control is to find the control input (the number of orders to manufacturing plant or to supplier) to minimize the quadratic cost function while tracking and keeping the inventory close to target level.

In this paper, we will model the production inventory system as the differential equation with lead time modeled as first order model. The time-domain state space representation with a set of input, output and state variables were used to build the dynamic system. Then the linear quadratic optimal controller is presented to find the optimal polices for the production inventory system. Finally, we analyze the performance of system and compare with other control policy such as order-up-to level or proportional controller. It is shown that our approach can obtain some good performances (such as operational cost, stability of inventory, customer service level).

2. Dynamic modeling of production system

2.1 Modeling of lead time

In this section, we will consider the continuous-time dynamic modeling such kinds of lead times. The generic lead time model with two parameters was proposed by (J.Wikner, 1999) as the following continuous-time formulation.

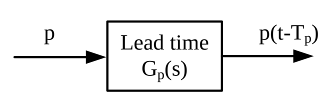

Fig. 1 Lead time model

2.2 Dynamic modeling of production/inventory system

The production inventory system in this study is the factory model with a single-stage, single-product, continuous production system. To produce products the factory need the setup time θ to prepare for production process and products will be finished after the production lead time L_1 (total lead time is $L = \theta + L_1$). In order to create a factory model realistically, we model the setup time delay and production lead time as pure delay and first order delay model respectively. The fluid analogy representation is used to model the factory model with differential equations.

The dynamic equation of inventory is as follow:

$$\dot{I}(t) = p_c(t) - d(t) \tag{1}$$

The dynamic equation of lead time model can be modeled as first-order delay model:

$$\dot{p}_c = -\frac{1}{\tau_p} p_c + \frac{1}{\tau_p} p_o \tag{2}$$

$I(t)$	Inventory level at factory warehouse
p_c	Amount of finished products received into the inventory after production lead time
p_o	Order production before lead time
$d(t)$	Customer demand signal
τ_p	Process lead time
θ	Setup time

To construct the state-space model, we define state variable $x = [p_c, I]^T$, control input $u = p_o$, disturbance: $w = d(t)$

Then the state space model is given as follow

$$(S): \begin{cases} \dot{x} = Ax + Bu + Dw \\ y = Cx \end{cases} \tag{3}$$

$$A = \begin{bmatrix} -\frac{1}{\tau_p} & 0 \\ 1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} \frac{1}{\tau_p} \\ 0 \end{bmatrix}, \; D = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \; \text{C = eye(2)}$$

3. Controller design

The objective of optimal trajectory tracking system is to find the control input $u(t)$ to make the output vector of system follow the desired trajectory vector $r(t)$ over a specified time interval $[t_0, T]$ while minimizing the finite-time quadratic cost function

$$J(t_0) = \frac{1}{2}[Cx(T) - r(T)]^T P[Cx(T) - r(T)] + \frac{1}{2}\int_{t_0}^{T} \{[Cx(t) - r(t)]^T Q[Cx(t) - r(t)] + u^T Ru\} dt \tag{4}$$

According to (F. L. Lewis, 1995), the solution for the LQ tracking with disturbance is given as below:

$$u(t) = -K(t)x(t) + R^{-1}B^T v(t) \tag{5}$$

where $K(t)$ is the control gain and $v(t)$ is the auxiliary function are given as:

$$K(t) = R^{-1}B^T S(t) \tag{6}$$

$$-\dot{v}(t) = (A - BK)^T v(t) + S(t)w(t) \tag{7}$$

And $S(t)$ is the solution of the Riccati equation:

$$-\dot{S}(t) = A^T S(t) + S(t)A - S(t)BR^{-1}B^T S(t) + Q \tag{8}$$

4. Main results and conclusion

To verify the proposed model and controller, the numerical simulation was carried out with the parameters: the step demand 30 units occurs from day 10th to 60th. The setup time 5 days, the production lead time is 6 days and the inventory target is 50 units. The simulation result is shown in fig. 2.

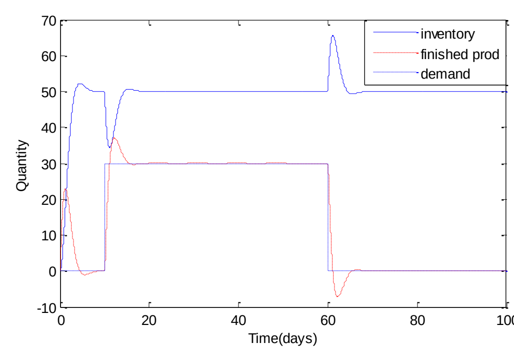

Fig. 2 Simulation results

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