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PROJECT SCHEDULING WITH START-TIME DEPENDENT COST AND IMPRECISE DURATION

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ABSTRACT: The goal of a project manager is generally to minimize the cost of the project and also to cope with uncertainty. This paper deals with the problem of project scheduling a set of activities satisfying precedence constraints in order to minimize the sum of the costs associated with the starting times of the activities in the network with imprecise activity durations, represented by means of interval or fuzzy numbers. So far this problem has been completely solved by several authors when the activities durations have crisp values. However, they do not consider the imprecision in activity durations in their models. Here the framework of possibility theory is proposed to solve this problem. In fuzzy arithmetic, usually, the interval calculations are used for the aim of complexity reduction and simplification. Thus the case of interval-valued durations is first addressed, and then extended to fuzzy intervals. A numerical example is used to illustrate the developed concept.

Keywords: Project scheduling, Activity networks, start-time dependent cost, Fuzzy intervals

1. INTRODUCTION

The CPM [24] is a network-based method designed to assist in the planning, scheduling and control of real world projects and CPM has become one of the tools that are most useful in practice. Its objective is to minimize the makespan of the project. The activity durations in the CPM are deterministic and known, although precise information about the durations of activities is seldom available. To deal quantitatively with imprecise data, the Program Evaluation and Review Technique (PERT) [27] and Monte Carlo simulation (e.g., [25], [37]) based on probability theory can be employed. So far, in the literature, hundreds of papers have used these stochastic approaches and search on this area is still carried out (e.g [13], [14] and [39]). However, these stochastic methods rely on statistical data or subjective probabilities, which are out of reach in many cases [26], and on dubious independence assumptions [6]. The detailed critiques of PERT can be found in the work of Shipley et al. [38].

Since the pioneering work of Zadeh [51], other researchers in this field have started to reject the stochastic approaches and recommend the use of fuzzy models for activity durations. The advocates of the fuzzy activity duration approach argue that probability distributions for the activity duration are unknown due to the lack of historical data. As activity duration has been estimated by human experts, often in a non-repetitive or even unique setting, project management is often confronted with judgmental statements that are imprecise. In these situations, the fuzzy set scheduling literature recommends the use of fuzzy numbers for modeling activity durations rather than stochastic variables. Instead of probability distributions, these quantities make use of possibility distributions [52].

In particular, the problems of computing the intervals of possible values of the latest starting times and floats of activities with imprecise durations represented by fuzzy or interval numbers have attracted attentions intensively and many solution methods have been proposed. Most of them are straightforward extensions of deterministic CPM ([9], [19], [28], [35]). These methods compute the possible values of the earliest starting times by means of a forward recursion procedure comparable to the one used in classical CPM problems. However, as pointed out by several authors [21], the backward recursion does not work for reasoning under uncertainty. When durations are described by means of fuzzy intervals, the backward recursion takes the imprecision of some duration twice into account [11]. Kaufmann and Gupta [23], Hapke et al. [20] and Rommelfanger [34] proposed a backward recursion that relies on the 'optimistic' fuzzy subtraction and they provided good results for particular networks but these methods fail to compute the fuzzy latest starting times and floats in the general networks. Nasution [33] resorts to symbolic computations on the variable duration times. McCahon and Lee [30] Mon et al. [31] and Yao and Lin [50] propose to go back to standard critical path methods via defuzzification of the fuzzy activity durations.

Zielinski [53] completely solved the problems of determining the possible values of the latest starting times of a given activity (see [42]). Yakhchali and Ghodsypour [45] proposed a simple polynomial algorithm for these problems which improves complexity by a constant factor. They also proposed algorithms for computing the latest starting times and maximal floats in a network with imprecise activity and time lag durations [48]. Dubois et al. [12] have proposed an efficient algorithm based on path enumeration to compute optimal intervals for latest starting times and floats. In practice it is often necessary to specify other than the finish-start precedence relations, so Yakhchali and Ghodsypour [44] proposed a polynomial algorithm for these problems in networks with generalized precedence relations. They suggested an algorithm for computing both the possible values of the latest starting times and the floats of all activities [46] and proposed a polynomial algorithm for computing the minimal latest starting times of all activities [47] in networks with generalized precedence relations and imprecise durations. Fortin et al. [18] have provided a complete solution to the problem of finding the maximal floats of activities and Yakhchali and Ghodsypour [43] have proposed a hybrid genetic algorithm for the problem of finding the minimal floats of activities.

Chanas and colleagues proposed a series of studies on the topic of the fuzzy CPM problem. For example, Chanas et al. [2] studied necessarily critical activities; Chanas and Zielinski [4] discussed the complexity of criticality; Chanas and Zielinski [3] proposed a natural generalization of the criticality concept for project networks with interval and fuzzy activity times, in which two methods of calculating the degree of possible criticality and some results are provided. The problems of the necessarily and possibly critical paths in the networks with imprecise activity and time lags durations have been discussed by Yakhchali et al. [40], [41] and the problems of the criticality of paths in the networks with generalized precedence relations and imprecise durations have been proposed in [49]. Chen [5] proposed a novel approach based on the extension principle and linear programming formulation to critical path analysis for a project network with activity times.

As already mentioned, the traditional objective, suggested by the CPM model, is to minimize the makespan of the project. Over the years, this assumption has been relaxed and many research efforts have been directed towards project scheduling with other objectives (see [22] for a review). One of the objectives, that generalizes many popular objective functions, is to minimize irregular starting time costs. The project scheduling problem with irregular starting time costs (PSIC) arises in several real-life applications of activity scheduling (e.g., in aircraft landing). Moreover, it arises in computing valid lower bounds to both the Resource Constrained Project Scheduling Problem and the Resource Availability Cost Problem, when the resource constraints are relaxed in a Lagrangean fashion [29].

Maniezzo and Mingozzi [29] develop a branch-and-bound procedure for this problem and identify special cases that are solvable in polynomial time. Mohring et al. [32] present a collection of previously established results which show that the general problem is solvable in polynomial time and review some related results for specializations and generalizations of the problem. Although the project scheduling problem with irregular starting time costs is discussed, this problem with imprecise activity durations was not addressed in the literature. So, we provide the full picture of this problem in the following.

2. The problem definition

The project scheduling problems to be dealt with throughout this paper in the project network can be stated as follows. A set *V={1,2, …, n}* of *activities* has to be executed where the dummy activities *1* and *n* represent the beginning and the termination of the project, respectively. Activity durations $i \in V$ are chosen from intervals $D_i = [\underline{d}_i, \overline{d}_i]$, $\underline{d}_i \geq 0$ which contains possible duration of $i \in V$. Activities can be represented by an *activity-on-node (AON)* network *G=<V, E >* with node set *V* and arc set *E*, |*E*|=*m*. We assume, without loss of generality, that the nodes are topologically numbered such that an arc always leads from a smaller to a higher node number.

The starting of activity $i \in V$ at time *t* involves a cost c_{it}. It is assumed that an upper bound *T* on the project duration is given such that *t=0,1,2, …, T.* The objective is to schedule the activities such that the start-time dependent costs are minimized.

The notation of *configuration* denoted by Ω has been defined by Buckley **Error! Reference source not found.** to relate the interval case to the deterministic case. A configuration is tuple Ω ($d_1, d_2, ..., d_n$) of activity durations such that $\forall i \in V$, $d_i \in D_i$. For a configuration Ω, $d_i(\Omega)$ will denote the duration of the activity *i.* $x_{it}(\Omega)$ denotes the zero-one variable which equals *1* when the activity *i* start at time *t* in the configuration Ω and $x_{it}(\Omega) = 0$ otherwise.

Thus, the minimum of the start-time dependent costs in the configuration Ω, denoted by *w*(Ω)*, can be calculated by the following integer programming formulation (**Error! Reference source not found.**, **Error! Reference source not found.**):

$$w^*(\Omega) = min\{\sum_{i=1}^{n}\sum_{t=1}^{T} c_{it} x_{it}(\Omega)\} \qquad (1)$$

Subject to:

$$\sum_{t=1}^{T} x_{it}(\Omega) = 1 \qquad \forall i \in V \qquad (2)$$

$$\sum_{q=t}^{T} x_{iq}(\Omega) + \sum_{q=0}^{t+d_i(\Omega)-1} x_{jq}(\Omega) \leq 1 \qquad (i,j) \in E, \ t = 0,1,...,T \qquad (3)$$

$$x_{it}(\Omega) \in \{0,1\} \qquad i \in V, t = 0,1,...,T \qquad (4)$$

The objection function in (1) is to minimize the start-time dependent costs in the configuration Ω . The constraints in (2) enforce a unique starting time for each activity. Inequality (3) denotes the precedence relations and $X(\Omega)$ denotes the optimal solution for formulas (1-4).

In the case of interval activity durations, the bounds of possible value of the minimum of project costs, denoted by $W = [\underline{W}, \overline{W}]$, are obtained by $\underline{W}$ *=min w*(Ω)* and $\overline{W}$ *=max w*(Ω)*, where min and max are taken over the set of possible configurations. Assume that ω denotes the set of possible configurations of activity durations, i.e. ω is the Cartesian product of corresponding intervals D_i, $i \in V$ and there are obviously $|\omega| = 2^{n-2}$ such configurations. Thus, $\overline{W} = max_{\Omega \in \omega} w^*(\Omega)$ and $\underline{W} = min_{\Omega \in \omega} w^*(\Omega)$. The main aim in this paper is to determine the values of $\overline{W}$ and $\underline{W}$.

The trivial problem of Figure 1 easily illustrates the difficulty of the problem. The precedence relations are the finish-start type. The number above each node represents the activity duration.

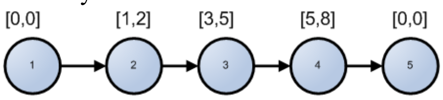

Figure 1: *A network with interval activity durations and start-time dependent costs*

The value of starting cost c_{it} is shown in Figure 2.

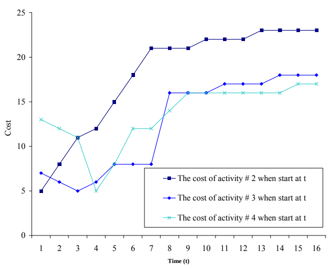

Figure 2: *The value of starting cost* c_{it} *for the activities in Figure 1.*

In Table 1, there exist 2^3 configurations as shown in Figure 1. The minimum cost of each configuration is computed, so $W = [19,24]$.

Table 1: *The minimum cost of each configuration in Figure 1.*

Activity #	Conf. (1)	Conf. (2)	Conf. (3)	Conf. (4)	Conf. (5)	Conf. (6)	Conf. (7)	Conf. (8)
1	0	0	0	0	0	0	0	0
2	1	1	1	1	2	2	2	2
3	3	3	5	5	3	3	5	5
4	5	8	5	8	5	8	5	8
5	0	0	0	0	0	0	0	0
Minimum Cost (W*)	19	19	23	23	22	22	24	24

3. Determination of the minimum cost in a network with interval activity durations

Lemma 1 and Lemma 2 are useful for determining the bound of the minimum cost, $W = [\underline{W}, \overline{W}]$.

Lemma 1: Assume that $k \in V$ is a distinguished activity and Ω, $\Omega \in \omega$ is a configuration that $d_k(\Omega) = \overline{d}_k$. If Ω' is a configuration that defines as formula (5) then $w^*(\Omega') \le w^*(\Omega)$.

$$d_i(\Omega') = \begin{cases} d_i(\Omega) & \text{for } i \ne k \\ \underline{d}_k & \text{for } i = k \end{cases} \tag{5}$$

Proof: We conduct an indirect proof. Assume on the contrary that $w^*(\Omega') > w^*(\Omega)$. Since $X(\Omega)$ is not a feasible solution for Ω' (otherwise $X(\Omega') = X(\Omega)$ and $w^*(\Omega') = w^*(\Omega)$).

All feasible solutions for formulas (1-4) for the configuration Ω are the feasible solution for the configuration Ω' because all the activities $i \ne k$, $d_i(\Omega') = d_i(\Omega)$ and $d_k(\Omega') \le d_k(\Omega)$. This contradicts that $X(\Omega)$ is not a feasible solution for Ω'. Thus $w^*(\Omega') \le w^*(\Omega)$.□

Lemma 2: Assume that $k \in V$ is a distinguished activity and Ω, $\Omega \in \omega$ is a configuration that $d_k(\Omega) = \underline{d}_k$. If Ω' is a configuration that defines as formula (6) then $w^*(\Omega') \ge w^*(\Omega)$.

$$d_i(\Omega') = \begin{cases} d_i(\Omega) & \text{for } i \ne k \\ \overline{d}_k & \text{for } i = k \end{cases} \tag{6}$$

Proof: The proof goes in the similar manner to the one of Lemma 1.□

The optimistic configuration, denoted by $\overline{\Omega}$, is a configuration $\overline{\Omega} \in \omega$ that $d_i(\overline{\Omega}) = \overline{d}_i$ for all $i \in A$. Similarly $\underline{\Omega}$, called the pessimistic configuration, is a configuration $\underline{\Omega} \in \omega$ that $d_i(\underline{\Omega}) = \underline{d}_i$ for all $i \in A$. Lemma 3 calculates the bound of the minimum cost based on these two lemmas.

Lemma 3: $\underline{W} = w^*(\underline{\Omega})$ and $\overline{W} = w^*(\overline{\Omega})$.

Proof: Let us assume that there exists a configuration $\Omega' \in \omega$ which $w^*(\Omega') = min_{\Omega \in \omega} w^*(\Omega)$ and $\underline{\Omega} \ne \Omega'$. Thus, there exists at least an activity $i \in V$, which $d_i(\Omega') = \overline{d}_i$ and obviously $d_i(\underline{\Omega}) = \underline{d}_i$. Based on Lemma 1, $w^*(\underline{\Omega}) \le w^*(\Omega')$. Since Ω' minimizes the minimum of the start-time dependent costs over the set of possible configurations, then $w^*(\underline{\Omega}) = w^*(\Omega')$, and we can deduce that $\underline{\Omega}$ minimizes the minimum of the start-time dependent costs too, $\underline{W} = w^*(\underline{\Omega})$.

Again assume that the configuration $\Omega'' \in \omega$ maximizes the minimum of the start-time dependent costs over the set of possible configurations $w^*(\Omega'') = max_{\Omega \in \omega} w^*(\Omega)$ and this configuration is not equal to the pessimistic configuration, $\overline{\Omega} \ne \Omega''$. There exist at last an activity $i \in V$, which $d_i(\Omega'') = \underline{d}_i$. Based on Lemma 2, $w^*(\overline{\Omega}) \ge w^*(\Omega'')$ and since maximizes the minimum of the start-time dependent costs, then $w^*(\overline{\Omega}) = w^*(\Omega'')$ and we can conclude that $\overline{W} = w^*(\overline{\Omega})$.□

The configuration number (1) and the configuration number (8) in Table 1are the optimistic and the pessimistic configuration, so based on Lemma 3, the bound of the minimum cost is $W = [19,24]$.

4. Determination of the minimum cost in a network with fuzzy activity durations

All the elements of the network *G* are the same as in the interval case except for the activity durations, which are determined by means of fuzzy numbers $\tilde{d}_i$, $i \in V$. A fuzzy number $\tilde{d}$ is a normal convex fuzzy set in the space of real number with an upper semi-continuous membership function $\mu_{\tilde{d}}$ **Error! Reference source not found.**. A fuzzy set $\tilde{d}$ is convex if and only if its membership function is quasiconcave, i.e., it fulfills the condition: $\mu_{\tilde{d}}(z) \ge min\{\mu_{\tilde{d}}(x), \mu_{\tilde{d}}(y)\}$ for each *x*, *y*, *z* such that $z \in [x, y]$.

Fuzzy number $\tilde{d}_i$ expresses uncertainty connected with the ill-known activity duration modeled by this number. It generates a possibility distribution **Error! Reference source not found.** for the sets of values containing the unknown activity duration. More formally, we say that the assertion of the form "*T* is $\tilde{d}_i$", where *T* is a variable and $\tilde{d}_i$ is a fuzzy number, generates the possibility distribution of *T* with respect to the following formula (see **Error! Reference source not found.**, **Error! Reference source not found.**):

$$Poss(T = x) = \mu_{\tilde{d}_i}(x), \quad x \in \Re_+ \tag{7}$$

Let Ω be a configuration of activity durations in the network with activity duration times $d_i \in \Re^+$, $i \in V$. Thus, the (joint) possibility distribution over configurations, denoted by $\pi(\Omega)$, is determined by the following formula:

$$\pi(\Omega) = min_{i \in V}\, \mu_{\tilde{d}_i}(d_i), \quad \Omega \in \Re_+^n \tag{8}$$

Hence, the possibility distribution describing possible values for the minimum of the start-time dependent costs is defined in the following way:

$$\mu_{\tilde{w}}(x) = Poss(w = x) = sup_{\Omega: x = w^*(\Omega)}\, \pi(\Omega), \quad x \in \Re_+ \tag{9}$$

where $w^*(\Omega)$ is the minimum of the start-time dependent costs of the project network in configuration Ω.

The possibility distributions in formula (9) can be determined via the use of α-cuts. This method in the interval case computes α-cuts, $\tilde{w}(\alpha)$, of each $\tilde{w}$ in a network with interval durations $\tilde{d}_i(\alpha) = [\underline{d}_i(\alpha), \overline{d}_i(\alpha)]$, $i \in V$. So, the possibility distributions, $\tilde{w}$, are reconstructed from their α-cuts. This approach makes sense since intervals $\tilde{w}(\alpha) = [\underline{w}(\alpha), \overline{w}(\alpha)]$ are nested. Such an approach for the problems of determining the possible values of the latest starting times and floats of activities has been proposed by **Error! Reference source not found.**,**Error! Reference source not found.**.

Hence, the main difficulty of determining the fuzzy minimum of the start-time dependent costs is the interval valued case and does not lie on the introduction of fuzzy sets. Thus, the fuzzy minimum of the start-time dependent costs can be determined by means of the proposed lemma (Lemma 3).

Additionally, Fortin and Dubois **Error! Reference source not found.** have shown that the algorithms in the interval-valued case can be adapted to fuzzy intervals considering them as crisp intervals of gradual numbers. The notions of gradual numbers are introduced by Fortin et al. **Error! Reference source not found.**.

5. Conclusions

This research was to introduce and solve the problem of project scheduling with irregular starting time costs in the network with imprecise activity durations. Imprecise data is represented by means of interval or fuzzy numbers. Several researchers study a project scheduling problem with irregular starting time costs in the network when the activities duration times are deterministic and known. We have modified the former presented approaches for this problem by assuming imprecise activity duration in the scheduling of the project.

In the interval case, the solution space grows exponentially when the numbers of activities in the network increase. The proposed lemmas determine the intervals of the possible values of the minimum of the start-time dependent costs in the polynomial time.

Intuitively, when the activity durations are fuzzy numbers, the minimum of the start-time dependent costs becomes fuzzy as well. On the basis of Zadeh's extension principle, a natural generalization of the problem in the network with fuzzy activity durations is given. The proposed approach can be used to solve the problem in network with fuzzy activity durations, by the usual decomposition of the fuzzy intervals into α-cuts.

The future research will extend the project scheduling problem under the objective of maximizing the net present value (NPV) of the project in the network with imprecise activity durations.

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