A HEART MODEL IN THE CIRCULATORY SYSTEM

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We present a mathematical model of left heart governed by the partial differential equations. This heart is coupled with a lumped model of the whole circulatory system governed by the ordinary differential equations. The immersed boundary method is used to investigate the intracardiac blood flow and the cardiac valve motions of the normal circulation in humans. We investigate the intraventricular velocity field and the velocity curves over the mitral ring and across outflow tract. The pressure and flow are also measured in the left and right heart and the systemic and pulmonary arteries. The simulation results are comparable to the existing measurements.

Keywords: heart, immersed boundary method, lumped-parameter model, circulatory system

1. Introduction

We develop a new multiscale model of the blood circulation including the left heart. Compared with other multiscale methods, our method has an advantage that one can easily handle moving elastic materials such as heart and compliance vessels. This is because we use the IB method. Our main purpose in this work is to show the possibility of combining the IB method with other simplified ODE models for the circulation. We modify Adeler''s left heart model [1] and couple it with a multicompartment lumped-parameter model for the circulatory system. The left heart model is governed by a PDE system which describes the interaction between blood flow and elastic heart muscle and mitral valve. The multicompartment lumped-parameter model for the whole circulation is described by a ODE system for the time-dependent pressure waves in the compartments. The combination of these two models can be done through the pressure values in the left atrium and aorta where the two models are connected. Without these two pressure values, the lumped-parameter ODE system is incomplete. Using the pressure-flow relation and the linearity of the discretized Navier-Stokes equations, we can derive two more equations for the pressures in the left atrium and aorta and make the ODE system complete.

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2. Mathematical Model

2.1 Immersed boundary left heart model

Consider an incompressible viscous fluid in a two-dimensional square domain in which a compliant left heart (muscular wall and valve leaflets) is immersed. We call this an immersed boundary. In the IB computation, the fluid exists not only inside the heart but also outside the heart [2].

Let the fluid velocity be u(x, t), the fluid force be $f(x,t)$ and the fluid pressure be p(x, t), We use the capital letter for the elastic material (heart wall). A positive value of Q(t) represents a source and a negative value represents a sink. Blood coms into the left heart through the atrium (source) and leave out of the heart through the aortic outflow tract (sink). Then the governing equations read:

$$\mathbf{F} = -\frac{\partial E}{\partial \mathbf{X}},$$

$$\mathbf{f}(\mathbf{x}, t) = \int \mathbf{F}(s, t)\delta(\mathbf{x} - \mathbf{X}(s, t))ds,$$

$$\rho(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f},$$

$$\nabla \cdot \mathbf{u} = \sum_{j=0}^{2} Q_j(t)\psi_j(\mathbf{x}),$$

$$\begin{aligned} \frac{\partial \mathbf{X}}{\partial t}(s, t) &= \mathbf{u}(\mathbf{X}(s, t), t) \\ &= \int \mathbf{u}(\mathbf{x}, t)\delta(\mathbf{x} - \mathbf{X}(s, t))d\mathbf{x}. \end{aligned}$$

2.1 A multi-compartment lumped model for the whole circulation

Consider now the whole circulatory system including the systemic artery/vein (P3/P4) and the pulmonary artery/vein (P7/P8), right atrium and ventricle (P5/P6), and three heart valves (black arrows) (see Figure 1). The following governing equations can be derived from the volume conservation, Ohm's law and the definition of complinace:

$$\frac{d(C_1P_1)}{dt} = G_8(P_8 - P_1) - G_1^*(P_1 - P_2),$$
$$\frac{d(C_2P_2)}{dt} = G_1^*(P_1 - P_2) - G_2^*(P_2 - P_3),$$
$$C_3\frac{dP_3}{dt} = G_2^*(P_2 - P_3) - G_3(P_3 - P_4),$$
$$C_4\frac{dP_4}{dt} = G_3(P_3 - P_4) - G_4(P_4 - P_5),$$
$$\frac{d(C_5P_5)}{dt} = G_4(P_4 - P_5) - G_5^*(P_5 - P_6),$$
$$\frac{d(C_6P_6)}{dt} = G_5^*(P_5 - P_6) - G_6^*(P_6 - P_7),$$
$$C_7\frac{dP_7}{dt} = G_6^*(P_6 - P_7) - G_7(P_7 - P_8),$$
$$C_8\frac{dP_8}{dt} = G_7(P_7 - P_8) - G_1(P_8 - P_1),$$

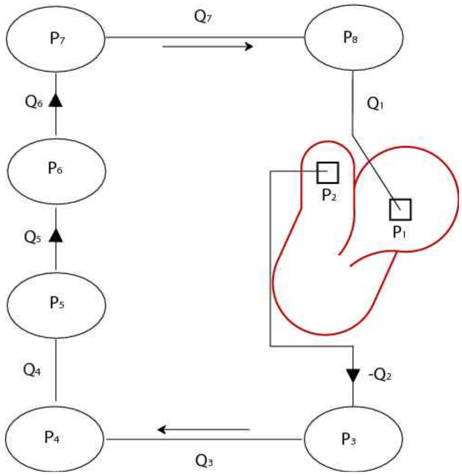

Figure 1. Initial configuration of a left PDE heart coupled with a lumped model for the whole cicculatory system.

For detaied numerical implimenation, see the reference [3].

3. Results and Discussion

In this section, we present the simulation results of the whole circulation model in a normal healthy person. The meshwidth of the computational domain for the left heart model, h = x = y = 16/256cm, is uniform and fixed in time, and the time duration is t = 0.000025sec. Since the period of the heartbeat is chosen to be 0.936sec, one heart cycle needs 37440 time steps. In most our simulations, we run 5 cardiac cycles. The blood density =1.05g/cm3 and the viscosity μ = 0.04g/(cm·sec) are used.

Figure 2 shows the pressures as functions of time in the left ventricle (dotted curve), the aortic outflow tract (P2, dashed curve), and the systemic artery (P3, solid curve) during the fourth and fifth cardiac cycles. We can see that the pressures in the left ventricle and the aortic outflow tract are almost the same. The systemic arterial pressure is higher than the pressure in the ventricle during diastole when the aortic valve closes, but the two pressures are almost the same during systole when the aortic valve opens. Top frame in Figure 3 is the pressures in the left heart from a literature. Our simulation results in the bottom frame shows a reasonable comparison with the data from the literature.

Figure 3 displays eight snapshots of the velocity vector fields during the fifth cardiac cycle from t=3.744sec to t=4.68sec. Each figure includes the interactive motions of the heart boundary and the mitral valve. The direction and size of the arrows represent the direction and magnitude of velocity field. We can clearly observe not only the inflow through the mitral valve and the outflow across the aortic outflow tract but also the vortex motions near the valve leaflets and inside the ventricle. During early mitral inflow period (t=3.978sec), a large anterior vortex is created from the tips of the value leaflets and stays in the ventricle and a wide inflow jet is observed. The inflow has increased during the last filling period.

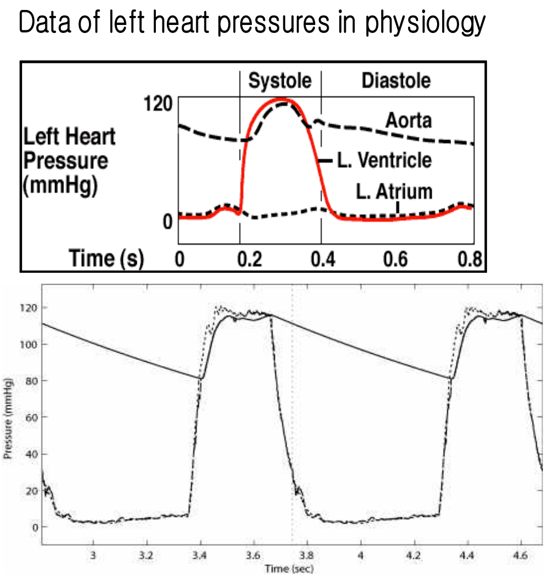

Figure 2. Pressures in the left heart and aorta.

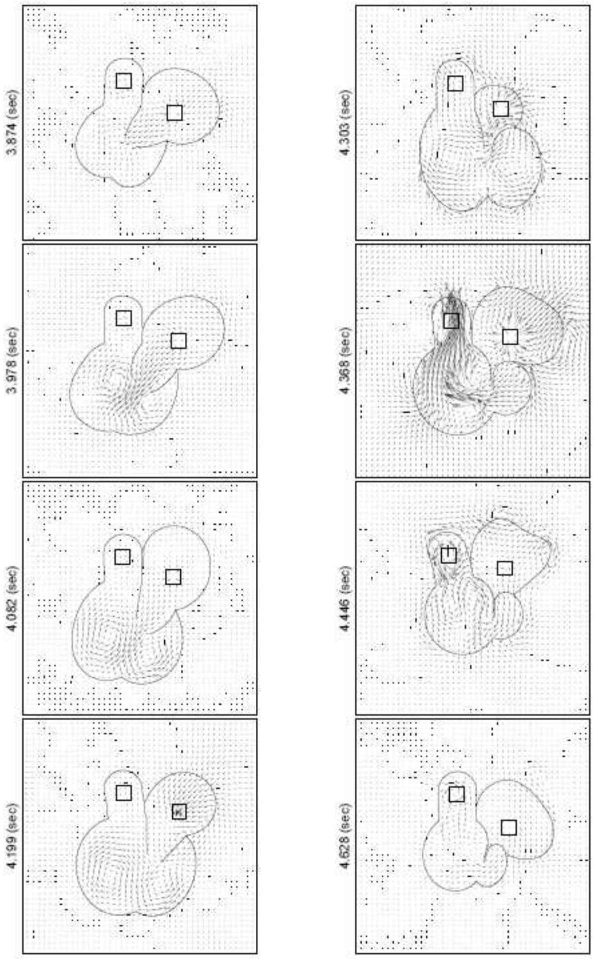

Figure 3. The velocity vector fields are displayed.

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