Numerical investigation of the gravity effect on the shape of natural supercavity

Hassan Pouraria[1] , Warn-Gyu Park[2]

The objective of this paper is to investigate the gravity effect on the shape characters of natural supercavity. A finite difference solver along with an implicit, dual time, preconditioned, three- dimensional algorithm has been used to solve the two- phase Navier Stokes equations. Numerical solutions were performed for natural supercavitating flow past a disk for different cavitation and Froud numbers. The numerical results were compared with corresponding analytical results in quantitative manner and it was found that the shape of supercavity was reasonably predicted. Numerical results indicated that the gravity effect can induce the asymmetry of supercavity. The asymmetry was apparent when the froud number was smaller so that for constant cavitation number when we reduced the froud number the offset of the axis of supercavity increased. Moreover, for specific froud number a decrease in cavitation number resulted in an increase in the offset of the supercavity. Numerical results revealed that for froud number greater than 25 the gravity effect is negligible.

keywords: *Numerical simulation, Natural supercavitation, Gravity effect, Shape characters.*

1.Introduction

Cavitation is the phase change of a fluid from liquid into the vapor when the static pressure drops below the corresponding saturation pressure. The importance of understanding cavitating flows is related to their occurrence in various technical applications such as, pumps, turbines, ship propellers and fuel injection systems, as well as in medical sciences such as lithotripsy treatment and the flow through artificial heart valves. The basic similarity parameter of cavitating flows is the cavitation number(σ). This parameter was first introduced by Thoma in 1923[1]

$$\sigma = \frac{2(p_\infty - p_c)}{\rho v^2} \qquad (1)$$

where P_∞ is the pressure at infinity, p_c is the cavity pressure, ρ is the water density and V_∞ is the mainstream velocity. When a body moves horizontally on the immersion depth H, we have $P_\infty=\rho gH$,where g is the gravity acceleration. When cavitation number decreases, the cavitation and its region extend. If the cavitation region, also called cavity, is developed enough, it can surround the body completely. This flow regime is called natural supercavitation. Owing to the existence of supercavity, body does not have any contact with liquid.

Moreover, in result of interface region, flow stream-lines are changed like flow around aerodynamic shapes. Hence, the total drag including friction and pressure drags decreases. Natural supercavitation is attained when cavitation number is less than 0.1 and velocity is very high V_∞>50 m/s. On the other hand experimental studies on supercavitation are usually carried out in Hydro-tunnels,

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where the ambient pressure is reduced artificially. For such condition supercavitation occurs at lower velocities. In some cases, by blowing the air or other gases past a cavitator it's

easy to attain supercavity at significantly lower velocity. Such supercavitation regime is called ventilated supercavitation. According to the similarity theory of hydrodynamic flows, for specific cavitation number the shape and dimensions of abovementioned cavities must be the same. However, in reality the full similarity does not fulfill. For cavitating flows Froud number is defined as follows:

$$Fr=\frac{V_{\infty}}{\sqrt{gd}} \quad (2)$$

Where d is the diameter of cavitator and V_{∞} is the mainstream velocity. Experimental studies show that for supercavitating flows with low froud number gravity effect becomes important and the supercavity moves upward to the body centerline as a result of buoyancy. Owing to lower velocities in ventilated supercavities, gravity effect in this case is more important and it also has a great effect on the closure character and type of gas leakage from the cavity. It can be concluded that , the Froud number and cavitation number are the basic similarity parameters for cavitating flows so that for certain cavitation and froud number the shape and dimensions of all cavities must be the same[2]. Thus far, many investigations have been carried out to study the gravity effect on the shape of supercavities[3,4,5]. most of investigations used potential flow theory and experiment. Following advancements in CFD methods, cavitation models based on Navier-stokes equations emerged. However, studying the influence of gravity using these CFD techniques was reported considerably less. In present study, A finite difference solver along with an implicit, dual time, preconditioned, three- dimensional algorithm has been used to solve the two- phase Navier Stokes equations for cavitating flow past a disk.

2.Governing equations and numerical solution

The governing equations have been normalizes with the liquid density, free stream velocity and the characteristic length of the body. The equations have been written in generalized curvilinear coordinates as follows:

$$\Gamma_e \frac{\partial \hat{Q}}{\partial t} + \Gamma \frac{d\hat{Q}}{d\tau} + \frac{\partial(\hat{E}-\hat{E}^V)}{\partial \zeta} + \frac{\partial(\hat{F}-\hat{F}^V)}{\partial \eta} + \frac{\partial(\hat{G}-\hat{G}^V)}{\partial \varsigma} = \hat{S} \quad (3)$$

where

$$\hat{Q}=\frac{1}{J}\begin{pmatrix} p \\ u \\ v \\ w \\ \alpha_l \end{pmatrix} \quad (4)$$

The convective terms are

$$\hat{E}=\frac{1}{J}\begin{pmatrix} U \\ \rho_m uU + \zeta_x p \\ \rho_m vU + \zeta_y p \\ \rho_m wU + \zeta_z p \\ \alpha_l U \end{pmatrix};\ \hat{F}=\frac{1}{J}\begin{pmatrix} V \\ \rho_m uV + \eta_x p \\ \rho_m vV + \eta_y p \\ \rho_m wV + \eta_z p \\ \alpha_l V \end{pmatrix} \quad (5)$$

$$\hat{G}=\frac{1}{J}\begin{pmatrix} W \\ \rho_m uW + \varsigma_x p \\ \rho_m vW + \varsigma_y p \\ \rho_m wW + \varsigma_z p \\ \alpha_l W \end{pmatrix}$$

The contravariant velocities are given by

$$\begin{aligned} U&=\zeta_x u + \zeta_y v + \zeta_z w \\ V&=\eta_x u + \eta_y v + \eta_z w \\ W&=\varsigma_x u + \varsigma_y v + \varsigma_z w \end{aligned} \quad (6)$$

the source termis as follows:

$$\hat{S}=\frac{1}{J}\{(\dot{m}^{+}+\dot{m}^{-})\left(\frac{1}{\rho_l}-\frac{1}{\rho_v}\right),\rho_m g_x,\rho_m g_y,\rho_m g_z,(\dot{m}^{+}+\dot{m}^{-})\frac{1}{\rho_l}\}^T \quad (7)$$

The matrix Γ and are as follows:

$$\Gamma=\begin{bmatrix}\frac{1}{\beta} & 0 & 0 & 0 & 0\\ 0 & \rho_m & 0 & 0 & u\Delta\rho_l\\ 0 & 0 & \rho_m & 0 & v\Delta\rho_l\\ 0 & 0 & 0 & \rho_m & w\Delta\rho_l\\ \frac{\alpha_l}{\beta} & 0 & 0 & 0 & 1\end{bmatrix}$$

(8)

$$\Gamma_e=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0 & \rho_m & 0 & 0 & u\Delta\rho_l\\ 0 & 0 & \rho_m & 0 & v\Delta\rho_l\\ 0 & 0 & 0 & \rho_m & w\Delta\rho_l\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

In this study, cavitation model proposed by Merkle et al[6] . was used to model mass transfer between two phases. In this model evaporation and condensation rates are given as follows:

$$\dot{m}^{-}=-k_v\frac{\rho_v\alpha_l}{t_\infty}\min\{1,\max(\frac{(p_v-p)}{k_p p_v},0)\} \quad (9)$$

$$\dot{m}^{+}=-k_l\frac{\rho_v\alpha_v}{t_\infty}\min\{1,\max(\frac{(p-p_v)}{k_p p_v},0)\} \quad (10)$$

In this model, a ramping function is defined as

$$f=\min\{1,\max(\frac{(p_v-p)}{k_p p_v},0)\} \quad (11)$$

which is only to ensure the stability of the numerical scheme. Hence, the factor should be as small as possible so that the scaling constants are the only main parameters which control phase changes.

In current study the fluid was assumed to be newtonian and laminar. All computations were conducted for supercavitation regimes where the cavities close in the liquid. For such phenomenon, turbulence effects are not important. Because of high Reynols number in the supercavitation regime, most reported theories for describing this phenomenon are based on potential flow. turbulence intensity is increased at the end of the cavity where the reentrant jet is formed, therefore this region is turbulent. The flow inside the cavity is however laminar. Flow turbulence mainly affects the transient behavior of partial cavitation and the inception of cavitation[7,8]. in studying the steady behavior of supercavities that closes inside the liquid region rather than on a solid wall the turbulence is only a second order effect that will not change the numerical solution in general. A manifestation of this point is the good agreement that will be shown later in the paper between the numerical and analytical results.

3. Results and discussion

3.1Geometry and boundary conditions

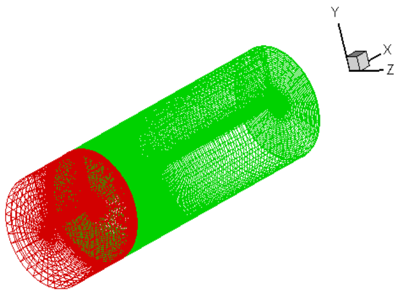
Due to practical importance of circular disk, computations were performed for this kind of cavitator. The used grid and geometry to study cavitating flow have been presented in Figure 1.The disk has diameter D and thickness of 0.2D. The boundaries of upstream and downstream were located at 10D and 40D. The upper and lower boundaries were located at 8D from the center of the disk. At the inlet, the velocity and liquid fraction were imposed. Pressure at the upper, lower and downstream boundaries were computed according to the depth from the reference point($P=P_{ref}+\rho gh$). However, the pressure at the upstream was extrapolated from inner points of the grid. At the wall velocity was zero while the other variables were extrapolated from the inner points. Two block structured grid consisting 1,175,157 cells was used to simulate cavitating flow around the disk. Simulations were performed for different cavitation and Froud numbers by changing the velocity and reference pressure.

3.2Results and discussion

The structure of cavities in a heavy fluid , in a gravity acceleration g, differs somewhat from cavities in a weightless fluid. A cavity with volume V_c should have a buoyancy lift $B_y = \rho g V_c$. On the other hand from the bernoullis equation, it is found that due to the existence of gravity field, the velocity at the top and bottom of the cavity should be different so that the velocity at upper side is less than its magnitude at lower side of the cavity. Distribution of velocity around the cavity results in a net circulation[9]. Lift force of this circulation can be determined by Kutta-Joukowski's theorem. This force acts downward on the cavity and is balanced by the buoyancy force[Fig. 2].

$$\rho \ \Gamma \ V_\infty \ b = \rho \ g \ V_c$$

From what mentioned above, it's obvious that the effect of gravity is due to the difference in hydrostatic pressure along the depth. Considering that the velocity at the inlet is uniform , it is expected that the transient behavior of development of supercavity at the upper and lower side should be somewhat different.



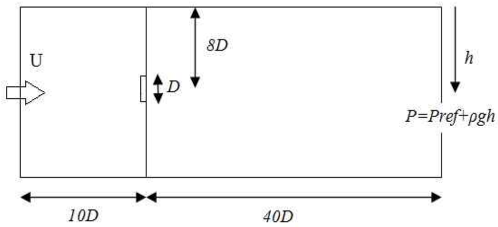


Fig. 1. The used multi block grid and 2D sketch of the geometry.

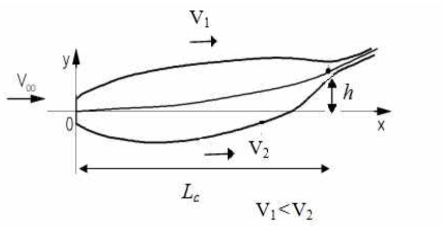


Fig. 2.scheme of a supercavity.

Since the corresponding cavitation number at upper side is less than lower side, supercavity develops faster in this region. Figure 3 and 4 show the development of supercavities for two different cases. In figure 3 the gravity effect has been neglected. Hence, there is no hydrostatic pressure gradient in normal direction. As seen the upper and lower sides of the cavity develop with same rate. On the other hand, in figure 4 the gravity effect was considered and it was observed that due to the gravity effect the upper side of the cavity develops faster than the lower side. However, when the supercavity reaches the steady state , velocity at upper side becomes less than lower side thereby the length of the cavity at upper and lower sides will become the same.

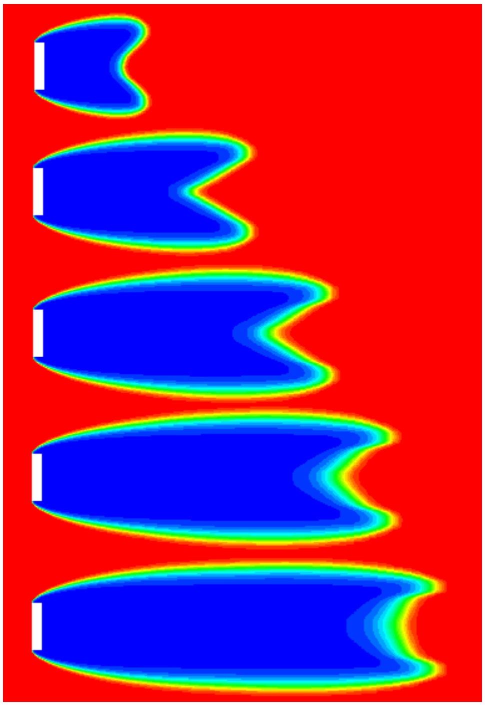


Fig. 3. development of natural supercavity when gravity effect is not important.

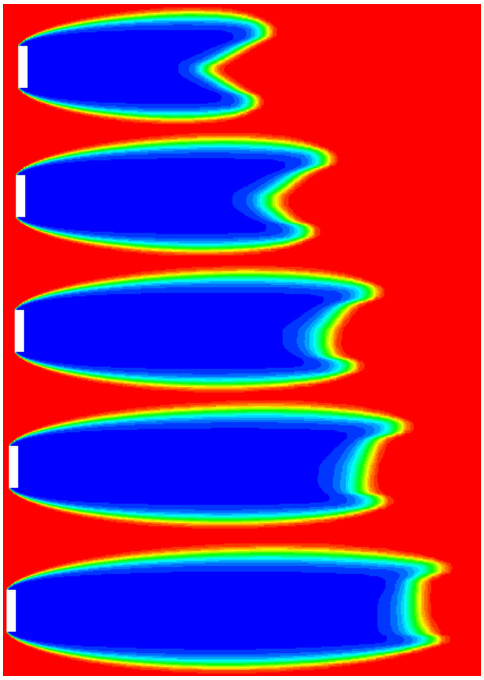


Fig. 4. development of natural supercavity when gravity effect is taken into account(Fr=15.2,cavitation number=0.08).

Figure 5 and 6 show the numerical results of the shape of steady supercavities for with an without gravity cases. As seen, for without gravity case the axis of the supercavity and cavitator are identical. However, when gravity is taken into account, the shape of supercavity changes. From figure 6 it is obvious that the gravity destroys the axis-symmerty of the natural supercavity.

Based on experimental results, L. A. Epshtein proposed a formula to determine the offset of the axis(h) of supercavity at Fr=15.2 [5].

$$\bar{h}=0.34(\bar{L}/Fr)^2-0.016(\bar{L}/Fr)^4 , \quad \bar{h}=h/D, \quad \bar{L}=L/D \tag{12}$$

In order to validate obtained results, numerical solution was carried out for Fr=15.2 and two different cavitation numbers. Table 1 shows a comparison between the results of numerical solution and formula proposed by epshtein. This comparison verifies the accuracy of numerical results. Figure 7 illustrates the effect of Froud number on the shape of natural supercavities. According to this figure, when froud number decreases, the gravity effect becomes stronger, and the asymmetry higher. Moreover, this figure illustrates a comparison between numerical results and analytical formula proposed by Yu. Zguravlev[10].

$$\bar{h}=0.33, \quad \bar{h}=h/D \tag{13}$$

As seen, there is satisfactory agreement between these results. This figure also indicates that for constant Froud number a decrease in cavitation number results in an increase in the offset of the axis of supercavity. In fact a decrease in cavitation number results in an increase in the size of supercavity, which, in turn, leads to increase in buoyancy force and the offset of the axis of supercavity.

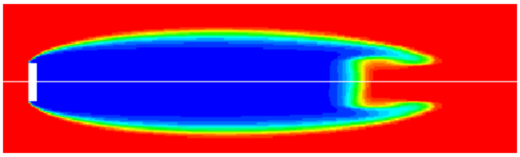


Fig. 5. Numerical result for the shape of steady supercavity for Fr=∞ and cavitation number=0.1.

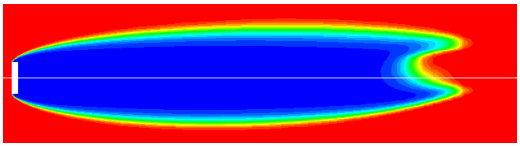


Fig. 6. Numerical result for the shape of steady supercavity for Fr=15.2 and cavitation number=0.08.

Cavitation number	0.08	0.1
Offset of the axis of supercavity predicted by Epshtein's formula	0.29	0.23
Offset of the axis of supercavity predicted by Numerical solution	0.335	0.21

Table. 1. Comparison between the offset of the axis of supercavity predicted by numerical results and formula proposed by L.A. Epshtein.

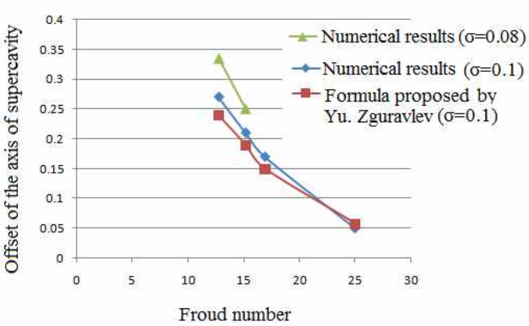


Fig. 7. Comparison between numerical results and formula proposed by Yu. Zuravlev.

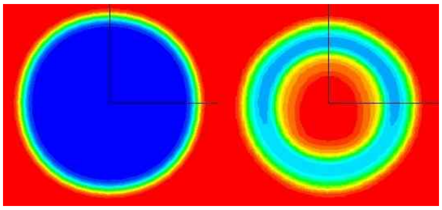


Fig. 8. shape of supercavity at different sections when gravity effect is not important.

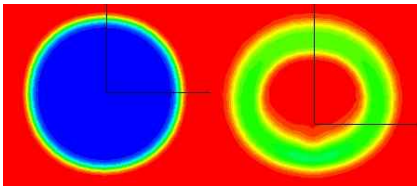


Fig. 9. shape of supercavity at different sections when gravity effect becomes important.

Figure 8 and figure 9 show the shape of supercavities at different sections. In these figures, pictures at left and right hand sides shows the shape of supercavity near the cavitator and at closure region respectively. In figure 8 the gravity effect was neglected in numerical solution. As seen in both regions the shape of supercavity is symmetric. However, because of considering the gravity effect in figure 9 the shape of supercavity changed so that near the cavitator it is almost symmetric and at the closure region supercavity moved upward to the cavitator centerline.

4.Conclusion

Numerical solutions were performed for natural supercavitating flow past a disk for different cavitation and Froud numbers. The numerical results were compared with corresponding analytical results in quantitative manner and it was found that the shape of supercavity was reasonably predicted. Numerical results indicated that the gravity effect can induce the asymmetry of supercavity. The asymmetry was apparent when the froud number was smaller so that for constant cavitation number when we reduced the froud number the offset of the axis of supercavity increased. Moreover, for specific froud number a decrease in cavitation number resulted in an increase in the offset of the supercavity.

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