



LBM을 이용한 Dam Breaking 수치해석연구

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COMPUTATIONAL STUDY ON TWO DIMENSINAL DAM BREAKING SIMULATION USING LATTICE BOLTZMANN METHOD

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In this paper we present an algorithm about how to simulate two dimensional dam breaking with lattice Boltzmann method (LBM). LBM considers a typical volume element of fluid to be composed of a collection of particles that represented by a particle velocity distribution function for each fluid component at each grid point. We use the modified Lattice Boltzmann Method for incompressible fluid. This paper will represent detailed information on single phase flow which considers only the water instead of both air and water. Interface treatment and conservation of mass are the most important things in simulating free surface where the Interface is treated by mass exchange with the water region. We consider the surface tension on the interface and also bounce back boundary condition for the treatment of solid obstacles. We will compare the results of the simulation with some methods and experimental results.

Keywords : Dam breaking, free surface, Lattice Boltzmann Method, Surface tension, Bounce back boundary condition.

1. INTRODUCTION

One of the important applications of free surface is breaking dam as it is directly related to free surface flow. Free surface actually is a boundary between two fluids, for example air and water. This means free surface flow is defined as a multiphase flow. But in case of particular application such as dam breaking, the air part can be neglected and so the multiphase is converted to single phase. In order to simulate the free surface, one has to be very careful in handling the interface as the proper treatment of interface gives good simulation. In this paper we use the Lattice Boltzmann Method for incompressible flow. We present the mathematical presentation of interface movement by mass exchange in the interface. Interface advection is done with the

help of the fluid fraction, which is updated by recording the inflow and outflow of mass via distribution functions. We apply the second order bounce back boundary conditions for the treatment of solid. The surface tension effect is also taken into account in the interface.

2. LATTICE BOLTZMANN METHOD

Lattice Boltzmann Method (LBM) works with Lattice Boltzmann equation(LBE). The LBE was derived from the lattice gas methods and it can be regarded as the first order discretization of the Boltzmann equation in the phase space[6]. The LBE looks like below:

$$f_i(x+e_i\Delta t, t+\Delta t) - f_i(x, t) = -1/\tau [f_i(x, t) - f_i^{eq}(x, t)] \quad (1)$$

In these equations f_i indicates the distribution function that means the probability of finding a particle in a phase space at

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particular velocity (e_i), position(x) and timestep(t). f_{ieq} represents equilibrium distribution function and τ is the relaxation time. This model is explained in more detail in e.g.[1]. It is commonly called LBGK model due to the simplification of the particle collisions with a single relaxation time[3,4].

For the LBE, the velocity space of the molecules is discretized. So depending on the dimension and the number of velocity directions, there are different models that we can use. Here we have used two dimensional D2Q9 model. This means the number of velocity direction is nine and it is two dimensional square lattice. The equilibrium function for the LBE for incompressible fluid flow can be written as below[1]:

$$f_{ieq} = w_i [\rho + (3/c^2) e_i \cdot u + (9/2c^4) (e_i \cdot u)^2 - (3/2c^2) u^2] \quad (2)$$

Where $c = \Delta x / \Delta t$ and called the basic speed on the lattice and counted as 1 lattice unit per second and w_i is called the weight of the lattice. The weights w_i are: $w_0 = 4/9$, $w_{1-4} = 1/9$, $w_{5-8} = 1/36$. The definition of velocity direction is:

$$\begin{aligned} e_i &= (0,0) \text{ for } i = 0; \\ e_i &= [\sqrt{2} \cos\{0.25(2i-9)\pi\}, \sqrt{2} \sin\{0.25(2i-9)\pi\}]c, \text{ for } i = 5-8 \\ e_i &= [\cos\{0.5(i-1)\pi\}, \sin\{0.5(i-1)\pi\}]c \text{ for } i=1-4 \end{aligned} \quad (3)$$

The microscopic variables velocity and density are directly calculated from the distribution functions as follows:

$$\rho = \sum f_i \text{ and } u = \sum e_i f_i \quad (4)$$

The relaxation time (τ) is directly related to viscosity of the fluid (ν). The relaxation time controls the rate at which the equilibrium is approached or another way we can call it as the viscosity of the fluid.

$\nu = (\tau - 1/2) c s^2 \Delta t$, where $cs = c/\sqrt{3}$, called the speed of sound of the fluid.

3. FREE SURFACE TREATMENT

In the governing equation we have added the gravitational acceleration (g) as the gravity is the only force working on the fluid[7]. The force term is also weighted according to each velocity direction.

$$f_i(x + e_i \Delta t, t + \Delta t) - f_i(x, t) = -1/\tau [f_i(x, t) - f_{ieq}(x, t)] + w_i \cdot p \cdot e_i \cdot g \quad (5)$$

3.1 Interface advection

For the movement of interface we consider three types of cells: fluid cells, interface cells and empty cells. Interface advection is done through the mass exchange between interface cells that is directly computed from the distribution functions [2,5]. We initialize the fluid fraction (ε) value for every cell that is computed with mass (m) dividing by the multiplication of density (ρ) and volume (v) of each cell ($\varepsilon = m/(\rho \cdot v)$). For the fluid cells the fluid fraction is equal to one and for empty cell its value is zero whereas we have fraction value of fluid fraction for interface cells [7, 2]. The mass exchange between interfaces is done as below.

$$\Delta m_i(x, t + \Delta t) = [f_i(x + \tilde{e}_i, t) - f_i(x, t)] \cdot (\varepsilon(x + \tilde{e}_i, t) + \varepsilon(x, t)) / 2 \quad (6)$$

Where $\tilde{e}_i = -e_i$. For example, in the computational domain if we have an interface at (i, j) and another interface at $(i+1, j)$ then the mass exchange between these interface will be follows:

$$\Delta m(i, j, 1) = [f(i+1, j, 3) - f(i, j, 1)] \cdot (\varepsilon(i+1, j) + \varepsilon(i, j)) / 2 \quad (7)$$

The empty cells are never taken into account for the mass exchange as we are neglecting the air part of the domain.

After computing the mass exchange for every interface cell, this exchanged mass is added to the previous mass of that interface cell to get the new mass in the current time steps.

$m^{n+1}(i, j) = m^n(i, j) + \Delta m(i, j)$, where n is previous time step and $n+1$ is the new time step.

3.2 Boundary condition

Two types of boundary conditions are considered in this paper: One for the free surface and another one for the solid boundary. During streaming the distribution functions from fluid cells and interface cells can be streamed normally, while the distribution functions (Dfs) that are coming from empty cells need to be reconstructed. It is assumed that the fluid has a much lower kinematic viscosity and the gas at the interface moves in the same way that a fluid does. For example, if there is an interface cell at (i, j) and an empty cell at $(i+1, j)$ then the Df coming from $(i+1, j)$ is reconstructed by the following equation.

$$f^{n+1}(i, j, 3) = [f_{eq}(i, j, 1) + f_{eq}(i, j, 3)] \cdot (1 + \alpha k(i, j)) - f^n(i, j) \quad (8)$$



Where σ is the surface tension in the interface and k is curvature. The curvature is calculated from the fluid fraction values in the interface with the finite difference scheme.

$$k = \nabla \cdot \mathbf{n} \quad (9)$$

Where $\mathbf{n} = \nabla \phi / |\nabla \phi|$ and \mathbf{n} is normal vector on the interface.

For the treatment of solid boundary, we consider the second order bounce back boundary condition. According to bounce back boundary method [8, 9], the solid boundary reflects back the distribution function that is coming to the boundary. This is governed by the following equation.

$$f_i(x+e_i \Delta t, t+\Delta t) = f_i(x, t) - 1/\tau^* [f_i(x, t) - f_{ieq}(x, t)] \quad (10)$$

3.3 Treatment of excess mass

After getting all the Dfs for an interface cell, the density and velocity are calculated. Then the new mass and new density is used to calculate to find out the new fluid fraction of each cell. Sometimes it is found that the fluid fraction is greater than one or less than zero for the interface cells. This means the interface has some positive or negative excess mass that are needed to be distributed to the surrounding cells in order to keep the conservation of mass. If $\epsilon > 1$ the cell is called filled cell and if $\epsilon < 0$ it is called emptied cells. For the filled cells the excess mass is calculated as $m_{ex} = m - (\rho \cdot v)$ and for the emptied cells $m_{ex} = m$.

The excess masses for the filled cells are distributed to the surrounding interface and empty cells. In order to do that first we have to make the empty cell into interface and then the excess mass is distributed evenly to the surrounding cells. In case of emptied cells, the excess mass is distributed evenly among the interface and fluid cells.

4. Results

We set up a computational domain of 100x100 and the physical length is assumed 1mm. So we have a physical spatial step $\Delta x = 0.00001m$. We have taken the compressibility restriction $g = 0.005[2]$ and depending on this value we found out the physical time step. Parameters like viscosity, density and surface tension have been made non-dimensional using this physical spatial and time step. The viscosity is used to find out the non-dimensional relaxation parameter ($\tau = 1$). Here we have

given simulation result of early time step. The results show the free surface contour and velocity vector field at the early steps. The velocity vector proves that the fluid is going downward direction according to gravitational acceleration and the contour shows the advection of breaking time as time goes by.

5. Conclusion

In this paper we have presented the procedure about how to handle the single phase free surface with LBM. Lattice Boltzmann Method is one of the powerful tools for the computational fluid dynamics. Ease of implementation of code, extensibility and computational efficiency are the major reasons for the growing field of LBM application and increasing popularity. The LBM differs from methods which are directly based on Navier-Stokes equations in various ways. The boundary condition can easily be implemented in this method. In our simulation result, we can see the velocity on the boundary getting smaller because of the proper implementation of bounce back boundary condition. The important thing is to maintain the mass conservation during the movement of interface and we have done it correctly. In future we will try to make more good and accurate results in order to simulate the free surface.

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