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열탄성 감쇄를 갖는 불완전 원형 링의 성능지수 예측 모델

Quality factor estimation model for imperfect circular ring with thermoelastic damping

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1. Introduction

Circular ring has been studied by numerous researchers for the various applications in engineering fields such as rotating components, vibratory gyros, and etc. Especially, thermoelastic damping and various types of imperfections on the ring are studied for estimating the performance of ring resonator gyros. A relatively simple analysis can be performed by considering energy relations of the structure. As Z. F. hisaeva and M. Ostoja-Starzewsi[1] carried out specific study for thermoelastic damping on ring resonator gyro. Moreover, S. J Wong et. al. [2] presented analytical model for the gyro with thermoelastic damping estimating its performance. Further, the effects of imperfections on circular ring has analyzed by C.H.J. Fox[3] and Rouke et al. [4]. They investigated frequency split due to imperfections of the ring and presented trimming theory based on their model. In this paper, the natural frequency of the ring is presented considering combined effects due to thermoelastic damping and imperfections.

2. Formulations

In this section, mathematical model of effects of point masses on the ring is formulated. Also, thermoelastic damping is considered by applying heat conduction equations and thermal strain terms into energy relations. Eventually, The natural frequency of the system is determined by using Rayleigh' s energy method for bending mode of vibration.

First of all, the stress-strain relationship can be expressed as

$$\sigma_{11} = \frac{E}{1-\mu^2}(\varepsilon_{11} + \mu\varepsilon_{22}) - \frac{E\alpha T}{1-\mu}$$
$$\sigma_{22} = \frac{E}{1-\mu^2}(\varepsilon_{22} + \mu\varepsilon_{11}) - \frac{E\alpha T}{1-\mu} \qquad (1)$$

where E, μ and κ denote Young' s modulus, Poission' s ratio and curvature of the middle surface of the structure, respectively.

The energy relations can be written as

$$K_T = K_0 + K_M$$
$$U_T = U_0 + U_D \qquad (2)$$

where K and U denote kinetic and potential energy, and subscripts T, 0, M and D represent total energy, energy of perfect structure, energy of imperfection, and energy from thermal effects, respectively. Note that as the imperfection energy is only considered in kinetic energy relation and as thermal effect is in potential energy. The maximum energy terms for K_0, U_0 and K_M can be obtained directly by applying inextensional mode shape functions. The results are presented as

$$K_{0_{\max}} = \frac{1}{2}\omega^2 a^3 \rho h b D^2 \int_0^{2\pi} \left(\cos^2(n\theta) + n^2 \sin^2(n\theta)\right) d\theta$$

$$U_{0_{\max}} = \frac{\pi D^2 h^3 E}{6(1+\mu)a^2} n^2 \left(n^2-1\right)^2 \int_0^{\frac{\pi}{2}} \sin^{-3}\theta \tan^{2n}\frac{\phi}{2} d\theta \qquad (3)$$

$$K_{M_{\max}} = \frac{1}{2}\omega^2 a^2 D^2 m \left(\cos^2\left(n(\theta_1 - \zeta_j)\right) + n^2 \sin^2\left(n(\theta_1 - \zeta_j)\right)\right) \quad (j = H, L)$$

where subscript i denotes the $i-$th point mass in total point masses.

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denotes the number of point masses.

Then, the heat conduction equation is introduced for obtaining remaining energy term.

$$\frac{\partial^2 T}{\partial r^2} - \frac{1}{\chi}\frac{\partial T}{\partial t} = \frac{1}{\chi}\frac{\Delta_E^2}{\alpha^2}\frac{\partial}{\partial t}\varepsilon \qquad (4)$$

Eq. (5) can be solved by applying linearization and iterations as stated in Ref. [2]. The solution of Eq. (5) can be expressed as

$$T = T_0 e^{i\omega t}$$

$$T_0 = \frac{\Delta_E}{\alpha}\left\{\frac{D^2 n(n^2-1)\sin(n\theta)\tan^n\left(\frac{\phi}{2}\right)}{a\sin^2\phi}\right\}\left\{r - \frac{\sin(kr)}{k\cos\left(\frac{kh}{2}\right)}\right\} \qquad (5)$$

Using Eq. (7), the potential energy term representing thermal strain can be determined and then, the natural frequency of the ring can be determined by applying Rayleigh' s energy method as

$$\omega_{n,H} = \omega_0\sqrt{(1+n^2)/\{(1+n^2)+(2m/m_0)\}}\sqrt{1+\Delta_E[1+f(\omega)]} \qquad (6)$$
$$\omega_{n,L} = \omega_0\sqrt{(1+n^2)/\{(1+n^2)+(2mn^2/m_0)\}}\sqrt{1+\Delta_E[1+f(\omega)]}$$

where the complex function f(w) is

$$f(\omega) = \frac{(1+\mu)}{(1-\mu)}\left\{\frac{1}{4} + \frac{3}{k^2h^2} - \frac{6\tan(\frac{kh}{2})}{k^3h^3}\right\} \qquad (7)$$

3. Results and discussions

The results are presented by Q-factor diagram for varying diameter and thickness of the ring for L-mode frequency. The diagram shows Q-factor difference compared to original Q-factor from perfect ring structure.

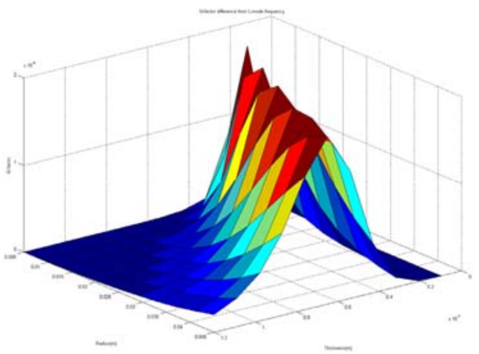

Fig. 1 Q-factor difference for L-mode

Fig. 1 shows Q-factor difference for L-mode which has greater difference than H-mode Q-factor. The region with higher value than rest of the area means greater influence of mass imperfection occurs within the region.

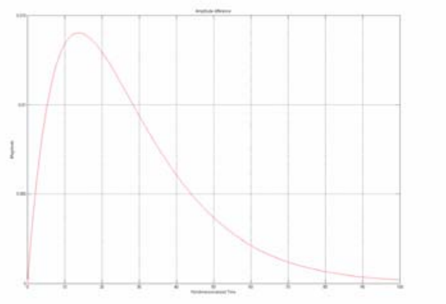

Fig. 2 Magnitude difference of free vibration

Further, Fig. 2 presents magnitude difference between perfect and imperfect ring, and the results implies that the imperfect ring has faster energy dissipation.

4. Conclusions

The mathematical model for estimating Q-factor considering imperfect circular ring is presented in the study. The study reveals that the mass imperfection can affect to Q-factor which means mass imbalance is inducing increased energy dissipation. Also, Q-factor decreased greatly when the mass of imperfection becomes larger, or the number imperfect mass increases.

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