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Euler Parameter 와 준 좌표계를 이용한 유연체의 동적 거동 해석

Dynamic Analysis of Flexible Body Using Euler Parameter and Quasi-Coordinate

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1. Introduction

In dealing with the dynamics of flexible body, the rigid-body motions and elastic vibrations are analyzed separately. However, the rigid-body motions cause vibrations and elastic vibrations also affect rigid-body motions in turn, which indicates that the rigid-body motions and elastic vibrations are coupled in nature. The coupled equations of motion for a flexible body are derived by means of Lagrange's equations in terms of quasi-coordinates. The resulting equations of motion are hybrid and nonlinear. Recently, the unified approach for the equations of motion for a flexible body based on the perturbation method and the Lagrange's equations of motion in terms of quasi-coordinates was proposed. The resulting equations consist of zero-order equations of motion which depict the rigid-body motions and first-order equations which depict the perturbed rigid-body motions and elastic vibration. However, Euler angles used in the previous researches have singularity in representing rigid-body rotations. In this paper, we propose a new method that utilizes Euler parameters and quasi-coordinates. Numerical results show the effectiveness of the new algorithm developed in this paper.

2. Unified equations of motion in terms of quasi-coordinates

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Let us consider a flexible body in space as shown in Fig. 1.

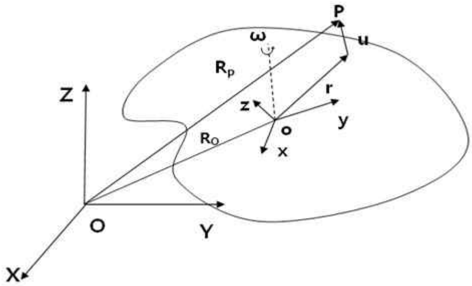

Fig.1 Coordinates for the motion of flexible structure in space

The kinetic and potential energies can be expressed as:

$$T = \frac{1}{2}\mathbf{x}^{T}\mathbf{M}(\mathbf{y})\mathbf{x}\ ,\ \ V = \frac{1}{2}\mathbf{y}^{T}\mathbf{K}\,\mathbf{y} \quad \text{(1a,b)}$$

where the vectors

$$\mathbf{x} = [\mathbf{V}^{T}\ \ \boldsymbol{\omega}^{T}\ \ \mathbf{p}^{T}]^{T}, \mathbf{y} = [\mathbf{R}^{T}\ \ \eta\ \ \boldsymbol{\varepsilon}^{T}\ \ \mathbf{q}^{T}]^{T} \quad \text{(2a,b)}$$

where $\mathbf{V}$ is the velocity vector of o, $\boldsymbol{\omega}$ is the angular velocity vector, $\mathbf{R}$ is the radius vector from 0 to o, $\mathbf{q}$ is vector of generalized coordinates, $\mathbf{p} = \dot{\mathbf{q}}$, η and $\boldsymbol{\varepsilon} = \left[e_1\ e_2\ e_3\right]^{T}$ are Euler parameters. The velocity vector of o using the Euler parameters can be written in terms of components along xyz as

$$\dot{\mathbf{R}}_{o} = \mathbf{C}^{T}(\boldsymbol{\varepsilon},\eta)\mathbf{V} \quad (3)$$

and the time derivative of the Euler parameters are expressed by the angular velocity vector of axes xyz

$$\dot{\eta} = -\frac{1}{2}\boldsymbol{\varepsilon}^{T}\boldsymbol{\omega}\ , \dot{\boldsymbol{\varepsilon}} = \frac{1}{2}\left(\tilde{\boldsymbol{\varepsilon}} + \eta\mathbf{I}_{33}\right)\boldsymbol{\omega} \quad \text{(4a,b)}$$

Based on the perturbation technique, the Lagrangian, matrices and vectors are expressed as

$$L_0 = T_0 - V_0,\ L_1 = T_1 - V_1 \tag{5a,b}$$

$$\mathbf{x} = \mathbf{x}_0 + \varepsilon \mathbf{x}_1,\ \mathbf{y} = \mathbf{y}_0 + \varepsilon \mathbf{x}_1 \tag{6a,b}$$

$$\mathbf{M}(\mathbf{y}) = \mathbf{M}_0(\mathbf{y}_0) + \varepsilon \mathbf{M}_1(\mathbf{y}_0, \mathbf{y}_1) \tag{7}$$

Considering $\mathbf{p}_0 = \mathbf{q}_0 = 0$, the unified equations of motion for a flexible body can be divided into zero-order and first-order unified equations of motion.

$$\hat{\mathbf{M}}_0 \dot{\hat{\mathbf{x}}}_0 + \hat{\mathbf{G}}_0 \hat{\mathbf{M}}_0 \hat{\mathbf{x}}_0 = \hat{\mathbf{F}}_0 \tag{8}$$

$$\bar{\mathbf{M}}_0 \dot{\mathbf{x}}_1 + \left[\mathbf{G}_0 \bar{\mathbf{M}}_0 + \tilde{\mathbf{G}}_0\right] \mathbf{x}_1 + \bar{\mathbf{K}} \mathbf{y}_1 = \mathbf{F}_1^* + \mathbf{d}_0 \tag{9}$$

and kinematical equations:

$$\dot{\hat{\mathbf{y}}}_0 = \hat{\mathbf{T}}_0 \hat{\mathbf{x}}_0 \tag{10}$$

$$\dot{\mathbf{y}}_1 = \bar{\mathbf{S}}_0 \mathbf{y}_1 + \bar{\mathbf{T}}_0 \mathbf{x}_1 \tag{11}$$

3. Numerical example

Let us consider a beam in space as shown below. It is assumed that the beam undergoes elastic deflections in y and z directions and elongation in x direction is neglected.

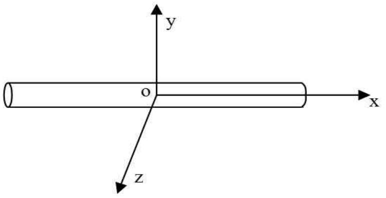

Fig. 2 3D free-free flexible beam

Apply bang-bang moment at the center of the beam, which is given by $M_z = 1Nm$ when $0s \le t < 0.5s$, $M_z = -1Nm$ when $0.5s \le t < 1s$, $M_y = 1Nm$ when $1s \le t < 1.5s$, and $M_y = -1Nm$ when $1.5s \le t < 2s$. In addition, $EI = 1$, $m = 1$, $\rho = 1$, $L = 1$.

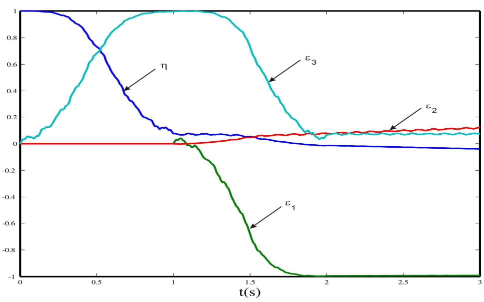

Fig. 3 Time histories of Euler Parameter

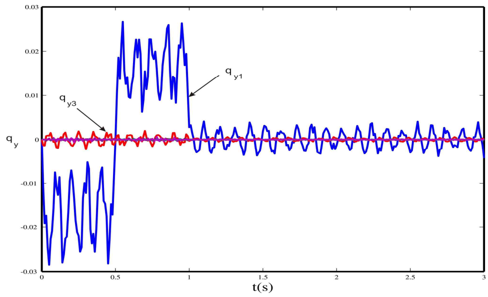

Fig.4 Time histories of generalized displacements q_y

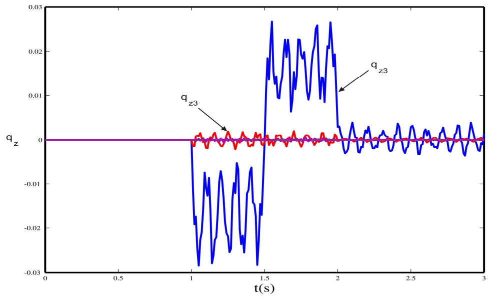

Fig.5 Time histories of generalized displacements q_z

Figures 3-5 show time histories of Euler parameters representing rigid-body rotation, and the elastic responses of the beam.

4. Discussion and Conclusions

This paper presents a new approach to solving the equations of motion for a flexible body by using perturbation method, Euler parameters and quasi-coordinates. The rigid-body motions and the elastic motions can be divided into low-dimensional set of nonlinear zero-order equations and high-dimensional set of linear first-order equations, respectively. The free-free beam problem was solved using the proposed method as a numerical example.

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