

Geometric Design of a Slotted Disc Spring Based on a Prescribed Load Displacement Function

오재응† · Noor Fawazi* · 윤지현* 이정윤*

1. Introduction

Since each nonlinear spring application requires a unique load-displacement function, a spring geometric parameters must be precisely custom designed. However, there is no specific algorithm available to calculate such geometric design parameterization. The aim of this study is to propose a generalized algorithm for a slotted disc spring geometric design that ensures the output design exhibits identical load-displacement function with any prescribed one.

2. Problem Statement

The main objective of this work is to propose an algorithm that supports the design process of a slotted disc spring. The design process is not only deal with the nonlinear behavior of load displacement characteristics, but also to possibly maintain the null slope in the intermediate region of the load displacement characteristics [2]. With a linear manner within initial and a final region of nonlinear load displacement characteristics, the null slope in the intermediate region is significantly important to ensure constant load with greater variations of displacement. Reference [2] clearly indicates this type of nonlinear load displacement of a slotted disc spring is the major optimum type in Belleville spring design. It is therefore, the proposed algorithm in study focuses on this type of nonlinear load displacement as a target design which possesses constant load with variations of displacement in the intermediate region. The proposed design limits the load displacement range based on a given target functions. These target functions are obtained by using available equations [1] which is described in the next section.

3. Construction of Prescribed Function

As a reference, prescribed function formulation was referred [1].

4. Algorithm Description

Step 1: Define Meeting Point (x_c^0, f_c)

Step 2: Define thickness t and Free Height h_0 . Vary $\delta = de/di$ (1.10~1.14, $\Delta 0.02$)

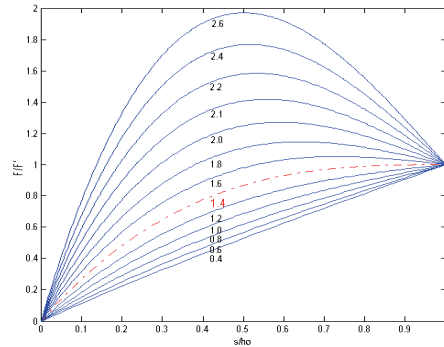


Fig. 1 Common Function (constant load-variation displacement $h_0/t=1.4$)

$$t = \left[\frac{1}{4} \frac{f_c (1 - \mu^2) K_1 d_e^2 x_c^0}{1.4^2 E} \right]^{0.2} \tag{1}$$

Step 3: Define Slot Segment Parameters

Step 4: Modify Meeting Point

$$x_c^{n+1} = x_c^0 - \delta^{n+1} \tag{2}$$

$$\delta_b = K \frac{4 f_c (r_i - r_o)^3}{E b_0 t^3 m} (1 - \mu^2) \tag{3}$$

Step 5: Recalculation (Repeat Step 1,2, and 3)

Step 6: Iterative Calculation (Repeat Step 1,2, 3and 4)

$$\left| \frac{x_c^{n+1} - x_c^n}{x_c^n} \right| < 0.001 \tag{4}$$

Step 7: Comparison Target Function-Predicted Function (LMS error)

$$LMS = \sqrt{\frac{\sum_{i=1}^r (f_i' - f_i)^2}{r}} \tag{5}$$

† 오재응; 한양대학교 기계공학부
E-mail : jeoh@hanyang.ac.kr

* 한양대학원 기계공학과

** 경기대학교 기계시스템디자인 공학부

Step 8: Search $\delta = \delta_{optimum}$ with the least LMS error

Step 9: Recalculation using $\delta = \delta_{optimum}$ (Repeat Step 1,2, 3 and 4)

Step 10: Produce Predicted Nonlinear Load Displacement and Geometric Design Parameters

5. Example

To assess the proposed algorithm explained in the previous section, an example is shown by comparing the results obtained between our proposed algorithm and the original geometric parameters used to plot the prescribed function.

In order to propose geometric parameters of a slotted disc spring that matched previous plotted prescribed function, a number of constant design variables were defined. Design variables such as Elastic Modulus $E=206\ 000\ N/mm^2$, Poisson ratio $\mu=0.3$, Outer Diameter $d_e=120.0mm$ and number of slot $m=10$ were initially defined as constant design variables. With these constant variables, designer can limit the maximum diameter of the spring and decide the number of slots to meet any design requirement.

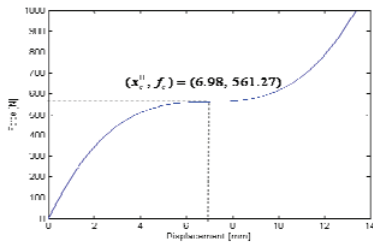


Fig. 2 Target Function

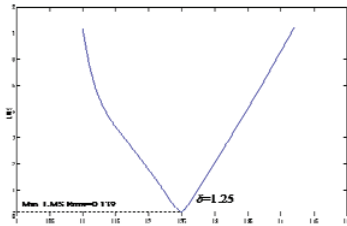


Fig. 3 Searching ratio δ with the minimum LMS

Iter. No.	Displacement Meeting Point	δ_b bending	t	d_e	d_f	d_o	h_o	H
0	$x_c^0 = 6.977$, $f_c^0 = 561.27$	-	1.61	120	96.00	45.80	2.26	6.98
1	$x_c^1 = 6.687$, $f_c^1 = 561.27$	δ_b^1 0.290	1.60	120	96.00	48.28	2.24	6.69
2	$x_c^2 = 6.724$, $f_c^2 = 561.27$	δ_b^2 0.253	1.60	120	96.00	47.97	2.24	6.72
3	$x_c^3 = 6.719$, $f_c^3 = 561.27$	δ_b^3 0.257	1.60	120	96.00	48.00	2.24	6.72

Table1 Changes of Geometric Parameters using Iterative Bending Penalization

Comparison Geometric Parameters	d_e	d_f	d_o	t	h_o	H
Target	120	96.0	48.0	1.60	2.24	6.72
Proposed	120	96.0	48.0	1.60	2.24	6.72
Error (%)	0.00	0.00	0.00	0.00	0.00	0.00

Table 2 Comparison Geometric Parameters between Target and Proposed Designs

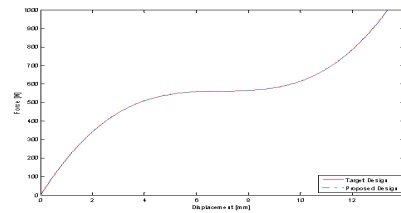


Fig. 4 Comparison Load Displacement Function between Target and Proposed Designs

6. Conclusion and Further Study

This study has presented a novel of new generalized algorithm to propose the geometric parameters of a slotted disc spring which are required to meet a prescribed function. The new proposed algorithm can be used to meet the industrial needs which are always looking for new design method to improve their production.

7. References

[1] Society of Automotive Engineers, 1982 "Design and Manufacture of Coned Disc Springs (Belleville Springs) and Spring Washer" SAE HS-1582.
 [2] La Rosa, G., and Messina, M., Stiffness of variable thickness Belleville Spring, Journal of Mechanical Design, 2001, 123, pp. 294-299