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층 구조로 이루어진 압전 복합 재료에서의 전단파 거동

Propagation behavior of SH waves in layered piezoelectric plates

강연준† · 손명섭* · 주경훈** · 전현철**

1. Introduction

In this study, the propagation of SH waves in a coupled plate consisting of a piezoelectric layer and an elastic layer is analytically investigated. The piezoelectric material is polarized in the z-axis direction and perfectly bonded to an elastic layer. The mathematical model of the SH wave propagation in this plate is based on the type of surface wave solution. Dispersion relations with respect to phase velocity are obtained for electrically open and mechanically free. Numerical examples are provided to illustrate graphically and compare the variations of the phase and group velocities versus the wave number for the different layers. The thickness ratio and the properties of the two layers have a significant effect on the propagation of SH waves. The conclusions are meaningful both theoretically and practically for the design of high-performance surface acoustic wave (SAW) devices.

2. Formulation of the problem

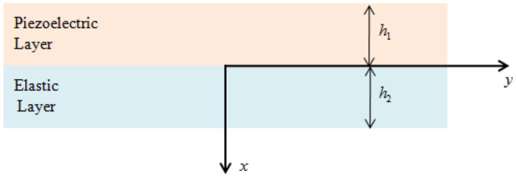

Fig. 1. Geometry of the layered piezoelectric plates.

Consider an elastic layer of thickness h2 with its surface bonded perfectly by a layer of piezoelectric material with thickness h1, as illustrated in Fig. 1. The piezoelectric material is polarized along the z-axis direction, perpendicular to the x-y plane. Taking transverse surface wave propagation in such a layered piezoelectric structure into account, it is assumed that the wave propagation is in the y-axis direction. From the boundary conditions and the continuity conditions, we can obtain the dispersive characteristic equations for the piezoelectric coupled plate to determine the unknown constants $A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2$:

$$
\begin{aligned}
&\lambda_2 \overline{c_{44}}[A_1 \sin\lambda_2 h_1 + A_2 \cos\lambda_2 h_1] - k e_{15}[B_1 e^{kh_1} - B_2 e^{-kh_1}] = 0,\\
&B_1 e^{kh_1} - B_2 e^{-kh_1} = 0,\\
&D_1 e^{-\lambda_1 h_2} - D_2 e^{\lambda_1 h_2} = 0,\\
&C_1 e^{-kh_2} - C_2 e^{kh_2} = 0,\\
&A_1 = D_1 + D_2,\\
&\lambda_2 \overline{c_{44}} A_2 - k e_{15}(B_1 - B_2) = -c_{44}^m \lambda_1 (D_1 - D_2),\\
&B_1 + B_2 + \frac{e_{15}}{\varepsilon_{11}} A_1 = C_1 + C_2,\\
&\varepsilon_{11} k (B_1 - B_2) = \varepsilon_{11}^m k (C_1 - C_2),
\end{aligned} \tag{1}
$$

For nontrivial solutions of the unknown constants, the determinant of the coefficient matrix of linear algebraic must equal zero, so we can obtain the dispersive relation

$$
\begin{aligned}
&[\overline{c_{44}}\sqrt{\frac{c^2}{c_{sh}^2}-1}\tan(kh_1\sqrt{\frac{c^2}{c_{sh}^2}-1}) - c_{44}^m\sqrt{1-\frac{c^2}{(c_{sh}^m)^2}}\tanh(kh_2\sqrt{1-\frac{c^2}{(c_{sh}^m)^2}})]\\
&= \frac{e_{15}^2 \varepsilon_{11}^m \tanh(kh_2)\tanh(kh_1)}{\varepsilon_{11}[\varepsilon_{11}^m \tanh(kh_2) + \varepsilon_{11}\tanh(kh_1)]},
\end{aligned} \tag{2}
$$

For example, when $e_{15}, \varepsilon_{11}, \varepsilon_{11}^m$ vanish and h_2 approaches infinity, Eq. (2) reduces to

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† 교신저자; 정회원, 서울대학교 기계항공공학부
E-mail : yeonjune@snu.ac.kr
Tel : +82-2-880-9203, Fax : +82-2-888-5950

* 서울대학교 자동차협동과정

** 서울대학교 기계항공공학부

$$c_{44}\sqrt{\frac{c^2}{c_{sh}^2}-1}\,\tan(kh_1\sqrt{\frac{c^2}{c_{sh}^2}-1})-c_{44}^m\sqrt{1-\frac{c^2}{(c_{sh}^m)^2}}=0, \tag{3}$$

3. Numerical simulations and discussions

An analytical equation of the phase velocity has been obtained for the propagation of SH waves in layered piezoelectric plates. In this section, based on the dispersion relation equation, numerical examples are plotted to illustrate the dispersion behaviors of SH waves. Values of the material properties used in numerical calculation are listed in Table 1, where PZT-4, PZT-5H, and PZT-7 are the piezoelectric materials and SiO_2 is the elastic material.

Table 1. Material properties used in the computation.

Material constants	PZT-4	PZT-5H	PZT-7	SiO_2
$c_{44}(10^9 N/m^2)$	25.6	23	25	31.2
$\rho(10^3 kg/m^3)$	7.5	7.5	7.8	2.2
$\varepsilon_{11}(10^{-9} C^2/Nm^2)$	6.46	27.7	17.1	3.36
$e_{15}(C/m^2)$	12.7	17	13.5	0.0
$c_{sh}(m/s)$	2596.6	2111.3	2138.1	3765.9

Also, to illustrate the dispersion relation, the group velocity c_g which expresses the rate at which energy is transported is introduced. The group velocity can be calculated by the following formula:

$$c_g=\frac{dw}{dk}=c+k\frac{dc}{dk}, \tag{4}$$

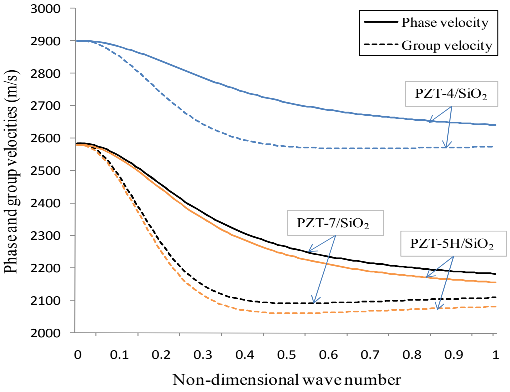

Fig. 2. Phase and group velocities of a three different types of piezoelectric layer, PZT-4, PZT-5H, and PZT-7, on a SiO2 plate for the first mode.

4. Conclusions

This paper investigates the propagation behavior of SH waves in layered piezoelectric plates. A general dispersion equation of the wave is derived and the numerical simulations are carried out. The numerical results show that the phase and group velocities approach the shear wave velocity of the piezoelectric layer. The dispersion curves for difference thickness ratios of the piezoelectric layer are also discussed. The thickness ratios have a great effect on the phase and group velocities when the wave number is lower, and all the curves converge as the wave number increases. Group velocity monotonously decreases as the wave number increases at smaller thickness ratios. These results provide a theoretical foundation and a basic model for the design and analysis of surface acoustic wave devices.

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