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Mathematical Tests that Examine Student's Capability for Future Mathematical Education[1]

Dvorkin, Mikhail*
Lyceum "Physical-Technical High School", dom 8, korpus 3, Hlopina str.,
Saint-Petersburg 194021, Russia; Email: mikhail.dvorkin@gmail.com

Ryzhik, Valery
Lyceum "Physical-Technical High School", dom 8, korpus 3, Hlopina str.,
Saint-Petersburg, 194021, Russia; Email: rvi@inbox.ru

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Any test given to a high school student is a diagnostics for certain properties of the student. That could be knowledge or problem solving skills. We stick to an integral property – capability for future mathematical education. Clearly, this "capability" stands for something larger than just having some set of knowledge and skills.

It is known that many perspective students that have successfully passed an entrance exam to the university do have serious troubles at the beginning of their higher education. Similar phenomenon is present in the high schools with advanced mathematics level: the best students from "usual" schools after moving to the "advanced" school just lag behind at first time.

So what is the "capability for future education"? We pick out several skills that are doubtless components of the "capability":

1) confirmation or disproof of a given statement (using any valid arguments);
2) analysis of the problem statement for unambiguity (possibility to give the unique answer) and correctness (consistency of the statement);
3) ascertainment of connection or lack of connection between two statements;
4) analysis of the logical structure of a statement;
5) understanding a concept in its general, abstract form;
6) translation from an analytic dependence to a pictorial rendition and backwards;

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* Corresponding author

7) reflection, i.e., differentiation between knowing and not knowing;
8) certain level of "logical culture".

Also important components of the "capability" are abilities to successfully carry out certain mathematical practices. To those we will attribute the following:

1) checking the existence (or non-existence) of an object;
2) checking the uniqueness (or non-uniqueness) of an object;
3) concluding properties of an object from its definition;
4) concluding properties from other properties or pure logical reasoning;
5) indentifying relationships between elements of a set (equality, congruence, etc.);
6) concluding properties (or breach of properties) of an object that is a result of a transformation of another object;
7) manipulating values (finding a value, estimating a value, comparing values). The list can be continued.

The tasks selected for the tests are chosen not arbitrarily, they are based on a tremendous experience of teaching mathematics in Russian high schools.

Each test consists of 5 statements (not questions!) They can relate to different parts of mathematics; such tests allow to check a wider range of knowledge and skills than the "American-style" tests.

The possible answers are:

- "+" stands for "Yes" – the student agrees with the statement;
- "–" stands for "No" – the student disagrees with the statement;
- "0" stands for "Don't know" – the student can't decide;
- "!" stands for "The task is ill-defined" – the situation described by the statement doesn't exist;
- "?" stands for "Can't be determined" – there's no unique answer.

The student's answer sheet can look like this: "+ + − 0 !".

Why such system? We don't see didactic basis for the tests in which the students choose between five proposed variants, one of which is correct. Where do the other four variants come from? In the best case, they could represent the four most probable mistakes of students, but this is hardly possible even theoretically. Also, it seems more fair if the student rather says "I don't know" than guesses at random. The "I don't know" answer is favorable, since it shows student's ability for reflection.

Moreover, the proposed form of tests resembles a dialogue. As if somebody says something to the student and he is to react. On the contrary, the situation where the student has to answer even if he doesn't know the answer seems psychologically aggressive.

As for contradictory and unambiguous statements in the tests, the reaction of the students to such statements seems as valuable as to the traditional statements. In fact, we

habituate the students to conceive the given information. Psychologists always say that such conception is important.

In the actual tests (being conducted for about 20 years), a correct answer scores +1, an incorrect answer scores −1, and the "I don't know" answer scores 0. Thus, the final score of a student can be smaller than the number of his correct answers, it can even be negative. This encourages the student to give only the answers he is sure about.

But we don't mind if teachers try other rules for result evaluation, e.g. giving more than +1 for correct answer, or less than −1 for the incorrect one.