Relay Selection Based on Rank-One Decomposition of MSE Matrix in Multi-Relay Networks

Youngtaek Bae and Jungwoo Lee
School of Electrical Engineering and INMC
Seoul National University, Seoul 151-744
Email: ytbae@wmspl.snu.ac.kr and junglee@snu.ac.kr

Abstract

Multiple-input multiple-output (MIMO) systems assisted by multi-relays with single antenna are considered. Signal transmission consists of two hops. In the first hop, the source node broadcasts the vector symbols to all relays, then all relays forward the received signals multiplied by each power gain to the destination simultaneously. Unlike the case of full cooperation between relays such as single relay with multiple antennas, in our case there is no closed form solution for optimal relay power gain with respect to minimum mean square error (MMSE). Thus we propose an alternative approach in which we use an approximation of the cost function based on rank-one matrix decomposition. As a cost function, we choose the trace of MSE matrix. We give several simulation results to validate that our proposed method obtains a negligible performance loss compared to optimal solution obtained by exhaustive search.

I. Introduction

Cooperative relay schemes for the wireless networks have received considerable attention because they give us a cheap coverage extension and increase the reliability of transmission through diversity increasing [1]–[3]. However the most previous works focus on the system model in which each node has single antenna. Well known techniques, multiple-input multiple-output (MIMO) [4], have been recently combined with relay networks in the distributed manner in order to increase the throughput and/or reliability further [5] [6].

As a first coupling between two techniques, many papers considered three node scenario for which each node has multiple antennas. In [7] [8], authors consider the optimization problems for relay processing matrix to maximize the overall capacity. On the other hand, relay strategies have been studied in terms of mean square error (MSE) in [9] [10]. In [11], the systems for which each node has single antenna but there are multiple relay nodes were investigated. Then as a further extension, the network with multiple relay nodes having multiple antennas was considered in [12], [13].

One point to notice in previously mentioned papers is that authors assumed that each relay node can know the received signal of all the other relays even though there are multiple relay nodes. This means that relay nodes can work in a fully cooperative manner. In fact, this is the same as single relay case with the number of antennas which are the sum of all relay antennas. To the best of our knowledge, there are no related results for separate multiple relays with single antenna while the source and destination have multiple antennas. This is a more practical scenario since the relay nodes should be as possible as simple. Therefore in this paper we consider this system setup. We will show that there is no closed form solution with respect to the cost function such as MSE. Then we will propose an alternative approach based on rank-one matrix decomposition.

The remainder of the paper is organized as follows. In Section II, we describe the problem formulation with respect to MSE, then propose an alternative relay selection scheme instead of exhaustive search for optimal relay power gain. Simulation results are given in Section III to validate our proposed scheme. Finally we provide the conclusion in Section IV. Notation: The complex transposition of a matrix is denoted by $(\cdot)^*$. Also the $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ are the trace and determinant of a matrix.

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II. Problem Formulation & Proposed Approach

The source node and destination node have the M transmit antennas and N receive antennas respectively. In network there are R relays with single antenna. First the source broadcasts the M symbols to the all relays, then each relay multiplies the received noisy signal with a powr gain. After that, all relays transmit these signals to the destination simultaneously. Equivalent discrete time signal model can be summarized as follows,

$$\begin{aligned} y_s &= \sqrt{\frac{E_s}{M}} H_s s + v_s \\ y_t &= \sqrt{\frac{E_s}{M}} H_t F H_s s + H_t F v_s + v_t \triangleq H s + n \end{aligned} \tag{1}$$

where s is $(M \times 1)$ the transmitted vector signal from source node with covariance matrix R_s, and y_s $(R \times 1)$ and y_t $(N \times 1)$ denote the received signal at the relays and destination. E_s is the average transmit power, and H_s and H_t are the size $(R \times M)$ and $(N \times R)$ channel coefficient matrices from the source to the relay and from the relay to the destination, which have independent and complex Gaussian distributed elements with zero mean and unit variance, i.e., Rayleigh fading. v_s and v_t are additive complex Gaussian noise with covariance matrices R_{v_s} and R_{v_t} and F is diagonal matrix with each relay power gain. $H = \sqrt{E_s/M} H_t F H_s$ is the equivalent channel matrix, $n = H_t F v_s + v_t$ is compounded noise vector with $R_n = R_{v_t} + H_t F R_{v_s} F^* H_t^*$.

Problem Formulation:

$$\begin{aligned} &\text{minimize} && \mathrm{MSE}(F) \\ &\text{subject to} && ||F y_s||^2 \le E_r \end{aligned} \tag{2}$$

where optimization parameter is F and cost function is 'MSE'. Inequality constraint limits the total power of all relays to E_r.

We assume that the destination knows the full CSI and solves this optimization problem, then feeds back the gain parameters to the relays.

If F is once assigned, MMSE equalizer at the destination is given as follows

$$G_{\text{mmse}} = (R_s^{-1} + H^* R_n^{-1} H)^{-1} H^* R_n^{-1} \tag{3}$$

By using orthogonality principle between error signal $(s - \hat{s})$ and input signal to equalizer (y_t), MSE matrix K is obtained as follows

$$\begin{aligned} K &= (R_s^{-1} + H^* R_n^{-1} H)^{-1} \\ &= \left(R_s^{-1} + \frac{E_s}{M} H_s^* F^* H_t^* (R_{v_t} + H_t F R_{v_s} F^* H_t^*)^{-1} H_t F H_s \right)^{-1} \end{aligned} \tag{4}$$

Therefore the cost function in terms of MSE is $\text{tr}(K)$ and the inequality constraint is $\text{tr}\left(\frac{E_s}{M} F H_s R_s H_s^* F^* + F R_{v_s} F^*\right) \leq E_r$.

We can use this trace function as an upperbound of bit error ratio (BER) as will be shown. Since BER is completely nonlinear function and depends on the decoding schemes, the optimization in terms of BER is known to be usually difficult. Thus we can also use MSE as error rate performance criterion instead of BER.

When we use MMSE equalizer as a demodulation step, signal-to-interference and noise ratio (SINR) of i-th transmitted stream is related with MSE matrix K as $SINR_i = \frac{1}{K_{ii}} - 1$ where K_{ii} denotes the i-th diagonal element which is a positive real due to the positive definite property of K matrix. Let error rate function at $SINR_i$ to $P_e(SINR_i)$. Then the upperbound of overall error rate can be given by

$$\begin{aligned} P_e &= \frac{1}{M} \sum_{i=1}^{M} P_e(SINR_i) = \frac{1}{M} \sum_{i=1}^{M} P_e\left(\frac{1}{K_{ii}} - 1\right) \\ &\leq P_e\left(\frac{1}{\text{tr}(K)} - 1\right) \end{aligned} \tag{5}$$

where we use $K_{ii} \leq \text{tr}(K)$ and monotonically decreasing property of error rate function as the SINR increases.

Minimizing the MSE over F turns out to be really nonlinear optimization problem which should be conquered by only numerical search. To solve this barrier, we will use an alternative approach. From now we use the assumption which is usually used in the literature. Each source signal is uncorrelated and normalized, i.e., $R_s = I_M$, and noise variance matrices are also normalized, i.e., $R_{v_s} = I_R$ and $R_{v_t} = I_N$. A positive semidefinite matrix $H^* R_n^{-1} H$ has nonnegative real eigenvalues. Thus $K = (I + H^* R_n^{-1} H)^{-1}$ is a positive definite matrix and has positive real eigenvalues. This means that the trace is also real and positive for any complex F.

We decompose the matrix products in K matrix into the sum of rank-one matrices. That is given by (6) (See the top of next page) where $A = \sum_{i=1}^{L} f_i h_{ti} h_{si}$ and $B = \sum_{i=1}^{L} f_i^2 h_{ti} h_{ti}^*$ for $L < R$ assuming f_i for $L+1 \leq i \leq R$ are very small and f_i is a i-th diagonal element of F, h_{ti} is i-th *column* of H_t and h_{si} is i-th *row* of H_s. $\varepsilon_1 E_1$ and $\varepsilon_2 E_2$ denote the sum of remainder rank-one matrices. In this case we can approximate the original cost function as follows

$$\text{tr}(K) \simeq \text{tr}\left(\left(I + \frac{E_s}{M} A^* (I + B)^{-1} A \right)^{-1} \right) \tag{7}$$

Now we will use this approximation as the cost function. In fact, this is the more constrained minimization problem for which the additional constraints are $f_i = 0$ for $L+1 \leq i \leq R$. In this case, f_i for $1 \leq i \leq L$ should be readjusted to fully satisfy the original constraint. However this has still the same difficulty as the original optimization even though the searching dimensions for solution are reduced. That is, we should also resort to numerical method in order to find a solution of new problem. Instead, we choose the alternative approach which searches the number of relays to approximate the original cost function together with simply computable method for the relay power gains. As a simple method, we will use the conventional method for the relay gains which only depend on backward channel (from the source to relays). This reduces the amount of feedback information since we need only feed back the 1-bit information which relays are selected to each relay instead of the gain values itself. Using this alternative approach,

$$\begin{aligned} &\min_{f_i \in \mathcal{R}} \text{tr}(K) \lesssim \min_{1 \leq L \leq R} \text{tr}\left(\left(I + \frac{E_s}{M} A'^* (I + B')^{-1} A' \right)^{-1} \right) \\ &\text{where} \quad A' = \sum_{i=1}^{L} g_i h_{ti} h_{si}, B' = \sum_{i=1}^{L} g_i^2 h_{ti} h_{ti}^*, g_i = \sqrt{\frac{E_r/L}{||h_{si}||^2 + 1}} \end{aligned} \tag{8}$$

where g_i for $1 \leq i \leq L$ are selected to satisfy the constraint with equality. As will be shown in simulation results, even if we use this alternative approach on the optimum cost values, the proposed method can achieve the very similar performance compared to optimal solution case.

III. Simulation Results

In this section we verify our derivation with numerical methods. For the comparison, we include the performance of conventional relay power gain. Legend 'Conventional' denotes that all the relays are used and the relay gain matrix

$$F_{\text{Conv}} = \sqrt{\frac{E_r}{R}} \Big(\text{diag}(H_s H_s^*) + I \Big)^{-\frac{1}{2}} \tag{9}$$

'Exhaustive Search' means that optimal power gains are founded by numerical method. 'Exhaustive Search' require the power gains of all relays to be fed back which are very large amount. 'Conventional' need not feed back any information. Even if the proposed method is required to feed back the information which relays are selected, the total amount of feedback information is less than R bits which are very small compared with 'Exhaustive Search'. In the case of 'Exhaustive Search', if we use X bits for quantization of each real power gain value, the total bits are RX which may be very large when using fine quantization, i.e., large X. Additionally these may also include the quantization error in the practical system. In all simulations, we assume that the total power of all relays is the same as $E_r = E_s$, the number of relays and transmit antennas are eight and two respectively.

Fig. 1 and Fig. 2 show the MSE performances as the SNR increases. SNR is defined as E_s/N_0 with $N_0 = 1$ for convenience. We can notice that our proposed methods are working very well compared to optimal solution. In fact the proposed methods are using power gain similar to 'Conventional' but only subset of relays are used. Therefore we can know from this results that the performance gap between optimal 'Exhaustive Search' and 'Conventional' shrinks by selection process. When we increase the number of destination antennas, this performance gap shrinks more and the proposed method is almost same as optimal case. This phenomenon shows that the larger number of receiver antennas there are,

$$K = \left(I + \frac{E_s}{M} H_s^* F^* H_t^* (I + H_t F F^* H_t^*)^{-1} H_t F H_s\right)^{-1}$$
$$= \left(I + \frac{E_s}{M}\left(\sum_{i=1}^{R} f_i h_{ti} h_{si}\right)^* \left(I + \sum_{i=1}^{R} f_i^2 h_{ti} h_{ti}^*\right)^{-1} \left(\sum_{i=1}^{R} f_i h_{ti} h_{si}\right)\right)^{-1} = \left(I + \frac{E_s}{M}(A + \varepsilon_1 E_1)^* (I + B + \varepsilon_2 E_2)^{-1} (A + \varepsilon_1 E_1)\right)^{-1} \quad (6)$$

the more compensation of loss can be achieved by using the proposed method.

Fig.3 shows the comparison of BER performances. Solid lines denotes $N = 4$ and dashed lines denotes $N = 2$. For BER performance, we use MMSE receiver and previous relay power gains. As mentioned, this is not BER optimal but can give upperbound of BER for MMSE receiver. From figure, we can notice that the proposed methods significantly increase the performance than 'Conventional' using (9).

IV. Conclusions

In this paper we propose an alternative approach for relay power gains in the network of the multiple relays with single antenna by using the approximation of cost function, i.e., trace of MSE matrix. Through several simulations, we can know that the proposed method for relay power gains works well in terms of MSE and BER. Our proposed method is based on the relay selection together with conventional simple power gain for each relay.

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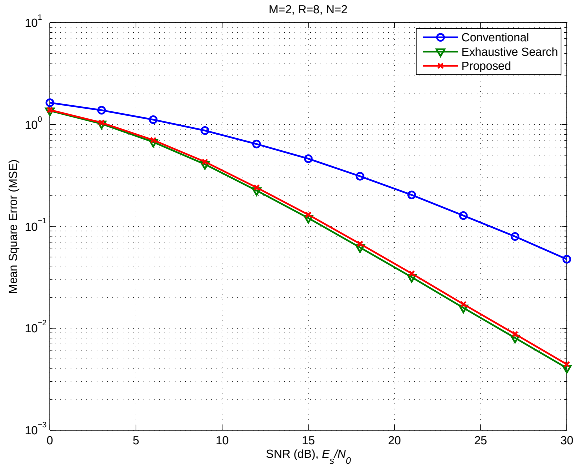

Fig. 1. Mean square error (MSE) vs. SNR, $M = 2$, $R = 8$, $N = 2$

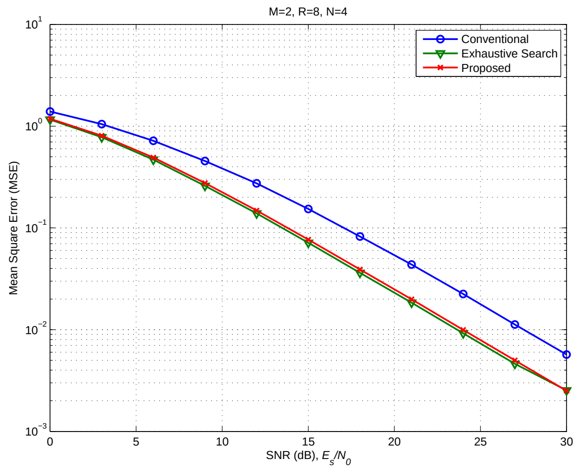

Fig. 2. Mean square error (MSE) vs. SNR, $M = 2$, $R = 8$, $N = 4$

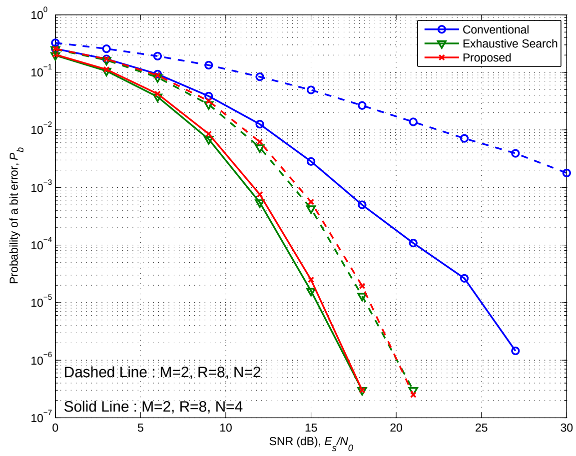

Fig. 3. Probability of a bit error vs. SNR