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유연구조물의 준좌표계 운동 방정식에 대한 섭동법의 적용

Application of the Perturbation Method to the Equations of Motion for a Flexible Body using Quasi-coordinates

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1. Introduction

In dealing with the dynamics of flexible body, the rigid-body motions and elastic vibrations are analyzed separately. However, the rigid-body motions cause vibrations and elastic vibrations also affect rigid-body motions in turn, which indicates that the rigid-body motions and elastic vibrations are coupled in nature. The coupled equations of motion for a flexible body are derived by means of Lagrange's equations in terms of quasi-coordinates. The resulting equations of motion are hybrid and nonlinear. In this paper, we propose the unified approach for the equations of motion for a flexible body based on the perturbation method and the Lagrange's equations of motion in terms of quasi-coordinates. The resulting equations consist of zero-order equations of motion which depict the rigid-body motions and first-order equations which depict the perturbed rigid-body motions and elastic vibration.

2. Perturbation approach to the equations of motion of flexible body

To describe the motion of a flexible body in space, we introduce a set of inertial axes XYZ with the origin at O, a set of body axes xyz coinciding with the principal axes of the flexible body and with the origin at the mass center o of the body. The body axes are shown in Figure 1. We express the elastic displacements as linear combinations of space-dependent admissible functions multiplied by time-dependent generalized coordinates:

$$\mathbf{u} = \Phi \mathbf{q} \tag{1}$$

Where $\Phi = [\phi_1 \phi_2 \ldots \phi_n]$ is a matrix of admissible functions and $\mathbf{q} = [q_1 q_2 \ldots q_n]^T$ is vector of generalized coordinates.

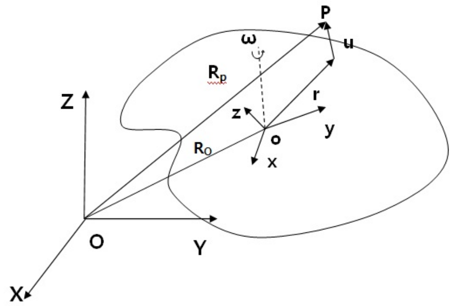

Fig.1 Coordinates for the motion of flexible structure in space.

The set of equations of motion for the flexible body can be obtained by means of Lagrange's equations in terms of quasi-coordinates.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \mathbf{V}_o}\right) + \tilde{\boldsymbol{\omega}}\left(\frac{\partial L}{\partial \mathbf{V}_o}\right) = \mathbf{F} \tag{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \boldsymbol{\omega}}\right) + \tilde{\mathbf{V}}_O\left(\frac{\partial L}{\partial \mathbf{V}_O}\right) + \tilde{\boldsymbol{\omega}}\left(\frac{\partial L}{\partial \boldsymbol{\omega}}\right) = \mathbf{M} \tag{3}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \mathbf{p}}\right) - \frac{\partial L}{\partial \mathbf{q}} = \mathbf{Q} \tag{4}$$

The equations (2), (3) and (4) can be combined into the integrated equation of motion in matrix form.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \mathbf{x}}\right) + \mathbf{G}\left(\frac{\partial L}{\partial \mathbf{x}}\right) - \mathbf{R}\left(\frac{\partial L}{\partial \mathbf{y}}\right) = \mathbf{H} \tag{5}$$

The perturbation approach is based on the simple observation that rigid-body motions tend to be large compared to the elastic motions. Consistent with this, we propose that $\mathbf{p}_0 = \mathbf{q}_0 = 0$ and the external force is in the form:

$$\mathbf{H} = \mathbf{H}_0 - \mathbf{d} + \varepsilon\left(\mathbf{H}_1 + \frac{1}{\varepsilon}\mathbf{d}\right) \tag{6}$$

where $\mathbf{d}$ is introduced to reflect the inertia force caused by the rigid body motion into the flexible

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motions, and ε is a small perturbation parameter. In addition, the following notations are introduced:

$$L_0 = T_0 - V_0 \,,\, L_1 = T_1 - V_1 \tag{7a,b}$$

$$\mathbf{G}_0 = \mathbf{G}(\mathbf{x}_0) \,,\, \mathbf{G}_1 = \mathbf{G}(\mathbf{x}_1) \tag{8a,b}$$

$$\mathbf{x} = \mathbf{x}_0 + \varepsilon \mathbf{x}_1 \,,\, \mathbf{y} = \mathbf{y}_0 + \varepsilon \mathbf{y}_1 \tag{9a,b}$$

$$\mathbf{M}(\mathbf{y}) = \mathbf{M}_0(\mathbf{y}_0) + \varepsilon \mathbf{M}_1(\mathbf{y}_0, \mathbf{y}_1) \tag{10}$$

The kinetic energy and the potential energy are written as below:

$$T_0 = \frac{1}{2} \mathbf{x}_0{}^T \mathbf{M}_0(\mathbf{y}_0)\mathbf{x}_0 \tag{11a}$$

$$T_1 = \mathbf{x}_0{}^T \mathbf{M}_0(\mathbf{y}_0)\mathbf{x}_1 + \frac{1}{2}\mathbf{x}_0{}^T \mathbf{M}_1(\mathbf{y}_0, \mathbf{y}_1)\mathbf{x}_0 \tag{11b}$$

$$V_0 = \frac{1}{2}\mathbf{y}_0{}^T \mathbf{K}\mathbf{y}_0 \,, V_1 = \mathbf{y}_0{}^T \mathbf{K}\mathbf{y}_1 \tag{12a,b}$$

Substituting equations (6) - (12) into equation (5) for the equations of motion:

$$\begin{aligned} &\mathbf{M}_0\dot{\mathbf{x}}_0 + \mathbf{G}_0\mathbf{M}_0\mathbf{x}_0 - \frac{1}{2}\mathbf{R}\frac{\partial}{\partial \mathbf{y}_0}\left(\mathbf{x}_0{}^T\mathbf{M}_0\mathbf{x}_0\right) \\ &\quad + \mathbf{R}\mathbf{K}\mathbf{y}_0 = \mathbf{H}_0 - \mathbf{d} \end{aligned} \tag{13}$$

$$\begin{aligned} &\mathbf{M}_0\dot{\mathbf{x}}_1 + \mathbf{M}_1\mathbf{x}_0 + \dot{\mathbf{M}}_1\mathbf{x}_0 - \mathbf{G}_0\mathbf{M}_0\mathbf{x}_1 + \mathbf{G}_0\mathbf{M}_1\mathbf{x}_0 \\ &+ \mathbf{G}_1\mathbf{M}_0\mathbf{x}_0 - \mathbf{R}\frac{\partial}{\partial \mathbf{y}_0}\left(\mathbf{x}_0{}^T\mathbf{M}_0\mathbf{x}_1\right) \\ &- \frac{1}{2}\mathbf{R}\frac{\partial}{\partial \mathbf{y}_0}\left(\mathbf{x}_0{}^T\mathbf{M}_1\mathbf{x}_0\right) + \mathbf{R}\mathbf{K}\mathbf{y}_1 = \mathbf{H}_1 + \frac{1}{\varepsilon}\mathbf{d} \end{aligned} \tag{14}$$

Because of $\mathbf{p}_0 = \mathbf{q}_0 = 0$, after reducing equations (13) and (14) we get the zero-order and the first-order equations of motion in simple matrix form:

$$\begin{bmatrix} m\mathbf{I} & \tilde{\mathbf{S}}_b^T \\ \tilde{\mathbf{S}}_b & \mathbf{J}_b \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{V}}_0 \\ \dot{\boldsymbol{\omega}}_0 \end{Bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{\omega}}_0 & 0 \\ \tilde{\mathbf{V}}_0 & \tilde{\boldsymbol{\omega}}_0 \end{bmatrix} \begin{Bmatrix} \mathbf{V}_0 \\ \boldsymbol{\omega}_0 \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_{\mathbf{V}_0} \\ \mathbf{M}_{\boldsymbol{\omega}_0} \end{Bmatrix} \tag{15}$$

$$\hat{\mathbf{M}}_0\dot{\mathbf{x}}_1 + \mathbf{G}_0\hat{\mathbf{M}}_0\mathbf{x}_1 + \mathbf{G}_1\hat{\mathbf{M}}_0\mathbf{x}_0 + \tilde{\mathbf{G}}_{(\mathbf{x}_0)}\mathbf{x}_1 + \hat{\mathbf{K}}\mathbf{y}_1 = \mathbf{H}_1 + \frac{1}{\varepsilon}\mathbf{d} \tag{16}$$

3. Numerical Study

The perturbation method was applied to the motion of planar free floating beam in space as shown in figure 2.

The zero-order and the first-order equations of motion of two-dimensional free-free beam can be derived by using the proposed method.

$$m\ddot{R}_{x0} + S_o s\theta_0\ddot{\theta}_0 - S_o c\theta_0\dot{\theta}_0^2 = F_{x0} \tag{17}$$

$$m\ddot{R}_{y0} + S_o c\theta_0\ddot{\theta}_0 - S_o s\theta_0\dot{\theta}_0^2 = F_{y0} \tag{18}$$

$$-S_o s\theta_0\ddot{R}_{x0} + S_o c\theta_0\ddot{R}_{y0} + I_o\ddot{\theta}_0 = M_{o0} \tag{19}$$

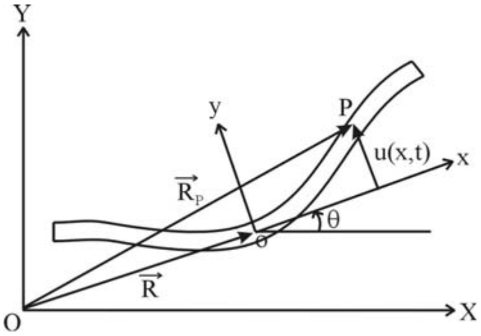

Fig.2 Free-Free flexible beam

$$\begin{aligned} &m\ddot{R}_{x1} - S_o s\theta_0\ddot{\theta}_1 - s\theta_0\bar{\Phi}\ddot{q} + 2S_o c\theta_0\dot{\theta}_0\dot{\theta}_1 - 2c\theta_0\dot{\theta}_0\bar{\Phi}\dot{q} \\ &+S_o\left(-c\theta_0\ddot{\theta}_0 + s\theta_0\dot{\theta}_0{}^2\right)\theta_1 + \left(\dot{\theta}_0{}^2 s\theta_0 - \ddot{\theta}_0 c\theta_0\right)\bar{\Phi}q = F_{x1} \end{aligned} \tag{20}$$

$$\begin{aligned} &m\ddot{R}_{y1} + S_o c\theta_0\ddot{\theta}_1 - S_o s\theta_0\ddot{\theta}_0\theta_1 - 2S_o s\theta_0\dot{\theta}_0\dot{\theta}_1 - S_o c\theta_0\dot{\theta}_0{}^2\theta_1 \\ &+c\theta_0 S_o\bar{\Phi}\ddot{q} - 2\dot{\theta}_0 s\theta_0\bar{\Phi}\dot{q} - \left(\dot{\theta}_0{}^2 c\theta_0 + \ddot{\theta}_0 s\theta_0\right)\bar{\Phi}q = F_{y1} \end{aligned} \tag{21}$$

$$\begin{aligned} &S_o s\theta_0\ddot{R}_{x1} + S_o c\theta_0\ddot{R}_{y1} + I_o\ddot{\theta}_1 + \tilde{\Phi}\ddot{q} \\ &\quad +\left[\bar{M}_{o0} - S_o\left(c\theta_0\ddot{R}_{x0} + s\theta_0\ddot{R}_{y0}\right)\right]\theta_1 \\ &\quad +\left[\tilde{M}_{o0} - \left(c\theta_0\ddot{R}_{x0} + s\theta_0\ddot{R}_{y0}\right)\bar{\Phi}\right]q = M_{o1} \end{aligned} \tag{22}$$

$$\begin{aligned} &-\bar{\Phi}^T s\theta_0\ddot{R}_{x1} + \bar{\Phi}^T c\theta_0\ddot{R}_{y1} + \tilde{\Phi}^T\ddot{\theta}_1 + M\ddot{q} \\ &\quad +\left(\bar{Q} - \bar{\Phi}^T c\theta_0\ddot{R}_{x0} - \bar{\Phi}^T s\theta_0\ddot{R}_{y0}\right)\theta_1 \\ &\quad +\left(K - \dot{\theta}_0{}^2 M\right)q = Q_0 + Q_1 - \tilde{\Phi}^T\ddot{\theta}_0 \\ &\quad +\bar{\Phi}^T\ddot{\theta}_0 + \bar{\Phi}^T s\theta_0\ddot{R}_{x0} - \bar{\Phi}^T c\theta_0\ddot{R}_{y0} \end{aligned} \tag{23}$$

Equation (17) through (19) are the zero-order equations of motion and equation (20) though (23) are the first-order equations of motion.

4. Conclusion

This paper presents a new approach to solving the equations of motion for a flexible body by using the unified method based on the perturbation method. Using this approach, the rigid-body motions and the elastic motions can be divided into low-dimensional set of nonlinear zero-order equations and high-dimensional set of linear first-order equations, respectively. The planar free-floating beam problem was analyzed using this method.

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