한국소음진동공학회 2010년 추계학술대회논문집, pp.271~272

Load Displacement Prediction for a Conical Disk Spring using Energy Method

오재웅† · Noor Fawazi* · 윤지현* 이정윤

1. Introduction

A Conical disk spring only possesses a coned disk segment with no lever arms segment. Based on Almens assumptions in the developed numerical formulation, only displacement due to rigid motion is considered. In this study, we attempt to propose energy method based on developed Almen formulation to predict the non-linear load displacement curve for the same conical disk spring. Since the proposed energy method is fundamentally derived from Almen equations, similar nonlinear load displacement curve to Almen results are obtained. To support the 2 numerical formulations, SAE formulation for conical disk spring is also presented which has similar formulation to Almen but with different dimensioning parameters. Due to limitation of 2-dimensional Almen assumptions, large error occurs in the increasing nonlinear curve once 3-dimensional analysis result is compared to Almen, SAE and the proposed energy method.

2. Conical Disk Spring Dimensioning Parameters

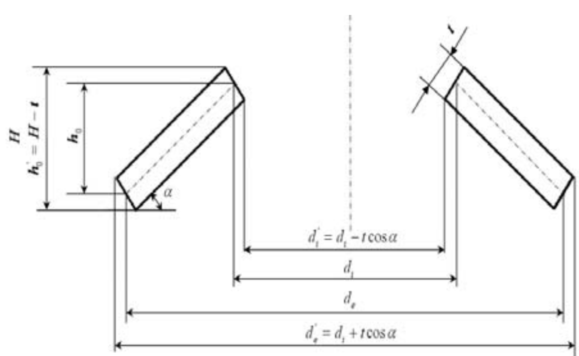


Fig. 1 Geometric Property of a Conical Disk Spring (Non-Slotted Disk Spring)

Theoretical Almen formulation expresses the geometric parameters by referring the central neutral line which is different to SAE geometric dimensioning parameters. Since our proposed energy method is based on Almen formulation, similar referred dimensioning parameters to Almen is employed. Fig. 1 shows the different geometric parameters expression for both conical disk spring formulations. All SAE dimensions are denoted with an apostrophe sign such as d_e' (outer diameter) and d_i' (inner diameter) while Almen dimensions are denoted without the sign. This is also implied to a straight and non-straight slotted disk spring dimensions.

Based on Fig. 1, the parameter expression for free height h_0 can be employed directly from Almen geometric parameter h_0. However, for SAE geometric parameter, the free height h_0 is simply defined as

$$h_0' = H - t \qquad (1)$$

where the expression for SAE free height dimension is denoted as h_0' (approximated free height). Numerically, theoretical Almen formulation is more accurate to define the free height h_0 in comparison to SAE formulation. This is because Almen formulation strictly defines the free height h_0 based on a central line. SAE formulation only approximates the free height h_0 by simply deducting the total height H with thickness t as it is convenient for real industrial measurement. It is therefore, once different referred dimensioning parameters are used in a same numerical formulation, a slight different can be clear seen.

3. Energy Method FEM Analysis

For a conical disk spring, the required energy to perform vertical displacement is simply defined from the spring non-linear load displacement function that can be plotted using Almen formulation.

Since the numerical formulations are defined based on non-linear elastic behavior, the FEM analysis is also simulated using non-linear elastic analysis with large deflection. Unlike in the previous research, elastic plastic analysis is not used here since plastic deformation or

† 오재웅; 한양대학교 기계공학부
E-mail : jeoh@hanyang.ac.kr
* 한양대학원 기계공학과
** 경기대학교 기계시스템디자인 공학부

material nonlinearity is not considered in the compared numerical formulation. It is impractical to compare both numerical formulation results using elastic plastic analysis such comparison permit dissimilar input variables. From a practical point of view, it is

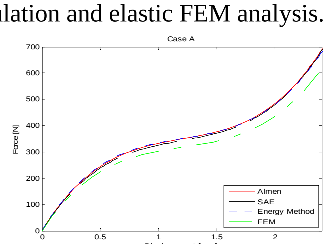
Case (s)	d_e	d_i	t	h_0	H
Case A	94.372mm	75.628mm	1.0mm	1.204mm	2.2mm
Case B	98.887mm	70.613mm	1.0mm	1.606mm	2.6mm
Case C	118.884mm	85.616mm	1.25mm	1.555mm	2.8mm

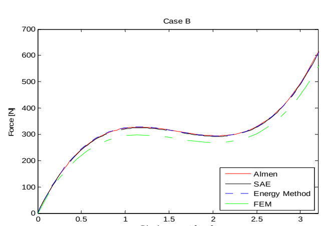
unreasonable to compare the elastic numerical simulation to an elastic plastic analysis.

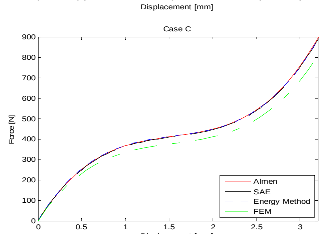
4. Example and Results

Table 1 Geometric Properties of Conical Disk Springs

The detail geometric and material properties of three examples of conical disk spring (non- slotted disk spring are shown in Table 1. Constant Elastic Modulus 200 000 N/mm^2 and Poisson ratio 0.3 are defined in this computation. By employing our proposed numerical formulation, the predicted non-linear load displacement functions are plotted and compared to Almen, SAE formulation and elastic FEM analysis.







The calculated load displacement results for the three cases are shown. It can be clearly seen that the Almen results tend to be similar to our proposed energy method. This is the consequence of initially deriving the proposed energy method from Almen formulation besides keeping all the given Almen's assumptions. In the other hand, there is a slight difference between Almen and SAE formulation results. This can be clearly explained by the different dimensioning parameters between Almen and SAE as mentioned in the previous section. Since our proposed energy method applies the same assumptions as Almen, it is reasonable to employ SAE formulation as a comparison to unavailable Almen slotted disk spring in the next section.

From the same figure, it is observed that FEM analysis results almost approach the three numerical formulations in the early quarter displacement but tends to contribute to large error in the following displacement region. This pattern is similar to results obtained in previous research [3] that compares to Almen results. One possible explanation is that, the 3-dimensional FEM model allows the deflection in the radial direction. However, the 2-dimensional numerical formulations completely ignored such deflection as assumed in Almen formulation. This is considered as a factor that contributes to such error.

5. Conclusion

The comparison between the employed numerical formulations to FEM especially in the increasing displacement region follows the trend as discussed in previous research. It is therefore, the occurred error is not because of the weakness of proposed energy method, but it is because the limitations of Almen assumptions. Such limitations are identified as factors to the weakness of proposed energy method which affects the prediction once the obtained results are compared to FEM analysis.

8. References

[1] Almen, J. O., and Laszlo, A.,1936, "The Uniform Section Disc Spring," Trans. ASME, **58**(4), pp. 305-314

[2] Society of Automotive Engineers, 1982 "Design and Manufacture of Coned Disc Springs (Belleville Springs) and Spring Washer" SAE HS-1582.

[3] Noboru Yahata, and Masaaki Watanabe, 1993, "Analysis of Coned Disk Spring by Finite Element Method," The Japan Society of Mechanical Engineering, **59**,pp. 260-265