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# Vibration control of the spherical pendulum by dynamic vibration absorber moving in radial direction

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## 1. Introduction

A tuned-mass damper (TMD), or dynamic vibration absorber (DVA), is a very well-known vibration control device, which consists of a moving mass attached to the main structure through springs and dampers.

Though there are some studies on pendulum type of DVA, the primary structure is often modeled as a spring-mass system. However, other models also have high interest in research and engineering application. In particular, the pendulum type systems occurring as a model of solid body with a fixed fulcrum point can play an important role in many fields such as machinery, transportation and civil engineering. The pendulum has been used to illustrate some types of structures such as ropeway gondola, crane or floating structures (ships, tension leg platform).

As shown by some previous studies [1-4], using DVA is a mean for reducing swing of pendulum structures. There are two main types of DVA. The first one moving in the circumference direction (left and right) (as shown in Fig.1a) was investigated theoretically by Matsuhisa [1].

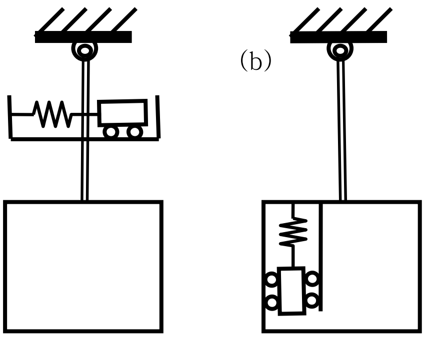


Fig. 1: Absorbers in a planar pendulum structure: (a) DVA moving in circumference direction, (b) DVA moving in radial direction

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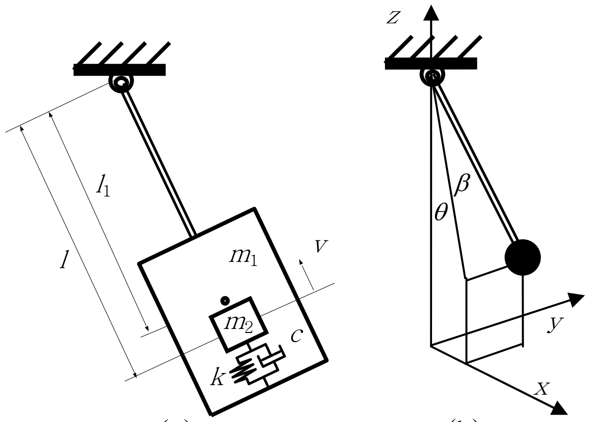
Since the first installation of the dynamic absorber on the ropeway chair lifts in 1995, dynamic absorbers have been installed on about 20 ropeways in Japan [4]. The more general study on DVA installed in the inverted pendulum type systems were also presented by Anh et al [5]. Matsuhisa et al [6] also propose the second type of DVA moving in the radial direction (up and down) (as shown in Fig.1b). This type of absorber produces Coriolis force as the damping force.

In many practical situations, however, the planar pendulum should be replaced by spherical pendulum in order to model the structures more precisely. In comparison between two types of DVA, it is easy to see that the second type (Fig. 1b) has the bidirectional nature while the first type can only reduce vibration in a plane. Therefore, in this paper, we study the vibration control problem of the spherical pendulum by using the DVA moving in the radial direction. The structure of paper is as follows. At first, the nonlinear motion equation is written in non-dimensional form. Then the second-order approximation is used to explore some important characteristics of the system. The optimal DVA' s parameters are chosen to minimize the system total potential energy. We specify a special vibration type, in which the DVA has little effect. However, when the spherical pendulum is subjected to random external excitation, this special vibration type does not occur and in most cases, the DVA still has good effect. The Monte Carlo simulations are used to verify that conclusion.

## 2. Equations of motion and some initial remarks

As shown in Fig. 2, the spherical pendulum has a concentrated mass $m_1$, $l_1$ is the length between the gravity center and the fulcrum (center of swing).




(a) (b)

Fig. 2: Geometrical description of the spherical pendulum: (a) geometrical parameters, (b) coordinate system and swing angles

The rotational angle (measured in the xz plane) is denoted by $\theta$, while $\beta$ is the angle of pendulum cable measured from $xz$ plane, $v$ is the DVA's displacement in radial direction measured from the static position, $l$ is the distance between the fulcrum and the DVA in the static condition, $g$ is the acceleration of gravity, $m_2$, $k$ and $c$ are mass, spring constant and damping coefficient of the DVA, respectively. The structural damping is denoted by $c_1$ and is assumed to be identical in all directions. The spherical pendulum system combining with DVA has three degrees of freedom including $\theta$, $\beta$ and $v$. By considering the coordinate system as shown in Fig. 2b, the positions of the structure ($x_1$, $y_1$, $z_1$) and the DVA ($x_2$, $y_2$, $z_2$) are obtained easily:

$$x_1 = l_1 \cos\beta \sin\theta \text{ , } y_1 = l_1 \sin\beta \text{ , } z_1 = l_1\left(1-\cos\beta\cos\theta\right) \quad (1)$$

$$\begin{aligned} &x_2 = \left(l-v\right)\cos\beta\sin\theta, y_2 = \left(l-v\right)\sin\beta, \\ &z_2 = l-\left(l-v\right)\left(1-\cos\beta\cos\theta\right) \end{aligned} \quad (2)$$

The kinetic energy $T$, the potential energy $V$ and the energy dissipation function $F$ are:

$$T = \frac{1}{2}m_1\left(\dot{x}_1^2+\dot{y}_1^2+\dot{z}_1^2\right)+\frac{1}{2}m_2\left(\dot{x}_2^2+\dot{y}_2^2+\dot{z}_2^2\right) \quad (3)$$

$$V = m_1 g z_1 + m_2 g\left(z_2 - v\right)+\frac{1}{2}kv^2 \quad (4)$$

$$F = \frac{1}{2}c_1\left(\dot{x}_1^2+\dot{y}_1^2+\dot{z}_1^2\right)+\frac{1}{2}c\dot{v}^2 \quad (5)$$

Let assume that the spherical pendulum is subjected to external forces $P_x(t)$ and $P_y(t)$ in $x$-direction and $y$-direction, respectively. The Lagrange motion equations become:

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial(T-V)}{\partial\dot{\theta}}\right)-\dfrac{\partial(T-V)}{\partial\theta}+\dfrac{\partial F}{\partial\dot{\theta}} = P_x\dfrac{\partial x_1}{\partial\theta}+P_y\dfrac{\partial y_1}{\partial\theta} \\ \dfrac{d}{dt}\left(\dfrac{\partial(T-V)}{\partial\dot{\beta}}\right)-\dfrac{\partial(T-V)}{\partial\beta}+\dfrac{\partial F}{\partial\dot{\beta}} = P_x\dfrac{\partial x_1}{\partial\beta}+P_y\dfrac{\partial y_1}{\partial\beta} \\ \dfrac{d}{dt}\left(\dfrac{\partial(T-V)}{\partial\dot{v}}\right)-\dfrac{\partial(T-V)}{\partial v}+\dfrac{\partial F}{\partial\dot{v}} = 0 \end{cases} \quad (6)$$

After some manipulations, the following nonlinear equations are obtained:

$$\begin{aligned} &\left(m_1l_1^2+m_2\left(l-v\right)^2\right)\left(\ddot{\theta}\cos^2\beta-2\dot{\theta}\dot{\beta}\cos\beta\sin\beta\right) \\ &\quad +c_1\dot{\theta}\cos^2\beta+g\left(m_1l_1+m_2\left(l-v\right)\right)\cos\beta\sin\theta \\ &\quad -2m_2\dot{\theta}\dot{v}\left(l-v\right)\cos^2\beta = l_1P_x\cos\beta\cos\theta \\ &\left(m_1l_1^2+m_2\left(l-v\right)^2\right)\left(\ddot{\beta}+\dot{\theta}^2\cos\beta\sin\beta\right)+c_1\dot{\beta} \\ &\quad +g\left(m_1l_1+m_2\left(l-v\right)\right)\sin\beta\cos\beta-2m_2\dot{\beta}\dot{v}\left(l-v\right) \\ &\quad = l_1P_x\sin\beta\sin\theta-l_1P_y\cos\beta \\ &m_2\ddot{v}+c\dot{v}+kv+m_2\left(\dot{\theta}^2\cos^2\beta+\dot{\beta}^2\right)\left(l-v\right) \\ &\quad -m_2g\left(1-\cos\theta\cos\beta\right) = 0 \end{aligned} \quad (7)$$

The following non-dimensional parameters are introduced:

$$\begin{aligned} &\mu = \frac{m_2}{m_1};\omega_s = \sqrt{\frac{g}{l_1}};\zeta_s = \frac{c_1}{2l_1^2m_1\omega_s};\alpha = \frac{\sqrt{k/m_2}}{\omega_s}; \\ &\zeta = \frac{c}{2m_2\omega_s};\gamma = \frac{l}{l_1};u = \frac{v}{l_1};\tau = \omega_s t; p_x = \frac{P_x}{m_1g}; p_y = \frac{P_y}{m_1g} \end{aligned} \quad (8)$$

In which, $\mu$ is the mass ratio, $\omega_s$ and $\zeta_s$ respectively are natural frequency and damping ratio of the main structure, $\alpha$ is the natural frequency ratio, $\zeta$ is the absorber's damping ratio, $\gamma$ is the location parameter specifies the position of the dynamic absorber, $u$ is the non-dimensional form of absorber's displacement, $\tau$ is the non-dimensional time with time scale $\omega_s^{-1}$, $p_x$ and $p_y$ are the non-dimensional forms of external excitations, respectively. The motion equations (7) are simplified and rearranged as following non-dimensional form:

$$\begin{aligned}
&\left(\ddot{\theta}\cos^2\beta-2\dot{\theta}\dot{\beta}\cos\beta\sin\beta\right)\left(1+\mu(\gamma-u)^2\right)\\
&\quad+2\zeta_s\dot{\theta}\cos^2\beta+\cos\beta\sin\theta\left(1+\mu(\gamma-u)\right)\\
&\quad-2\mu\dot{u}\dot{\theta}\cos^2\beta(\gamma-u)=p_x\cos\beta\cos\theta\\
&\left(\ddot{\beta}+\dot{\theta}^2\cos\beta\sin\beta\right)\left(1+\mu(\gamma-u)^2\right)+2\zeta_s\dot{\beta}\\
&\quad+\cos\theta\sin\beta\left(1+\mu(\gamma-u)\right)-2\mu\dot{u}\dot{\beta}(\gamma-u)\\
&\quad=p_x\sin\beta\sin\theta-p_y\cos\beta\\
&\ddot{u}+2\zeta\alpha\dot{u}+\alpha^2u-\left(1-\cos\theta\cos\beta\right)\\
&\quad+\left(\dot{\beta}^2+\dot{\theta}^2\cos^2\beta\right)(\gamma-u)=0
\end{aligned}\tag{9}$$

In the below sections, we consider the minimization of potential energy. Therefore, the potential energy $V$ described in (4) is rewritten in the following form:

$$\begin{aligned}
&V=m_1gl_1\left(1+\mu(\gamma-u)+\mu_e e\right)\left(1-\cos\theta\cos\beta\right)\\
&m_1gl_1\left(\mu\alpha^2\frac{u^2}{2}+\mu_e u_e\sin\theta+\mu_e\alpha_e^2\frac{u_e^2}{2}\right)
\end{aligned}\tag{10}$$

The non-dimensional nonlinear motion equations (9) will be used in numerical calculations. Before moving further, let us discuss some important characteristics of the motion equations. In the first two equations of (9), the Coriolis forces $-2\mu\dot{u}\dot{\theta}\cos^2\beta(\gamma-u)$ and $-2\mu\dot{u}\dot{\beta}(\gamma-u)$ act as the nonlinear damping forces to reduce the pendulum vibration angles $\theta$ and $\beta$. In order to clearly observe the interaction between the absorber and the spherical pendulum, let consider the harmonic vibrations of pendulum angles $\theta$ and $\beta$ with natural frequency as follows:

$$\theta=\theta_m\cos\tau\,,\quad \beta=\beta_m\cos(\tau+\varphi)\tag{11}$$

in which $\varphi$ is the phase shift between two angles. Moreover, let assume that the vibration angles are small enough to eliminate the third order and higher order terms, the third equation of (9) reduces to:

$$\ddot{u}+2\zeta\alpha\dot{u}+\alpha^2u\approx\frac{\theta^2+\beta^2}{2}-\gamma\left(\dot{\beta}^2+\dot{\theta}^2\right)$$

or

$$\begin{aligned}
&\ddot{u}+2\zeta\alpha\dot{u}+\alpha^2u\approx\\
&\frac{(2\gamma+1)}{4}\left(\theta_m^2\cos2\tau+\beta_m^2\cos(2\tau+2\varphi)\right)+\frac{\left(\theta_m^2+\beta_m^2\right)}{4}(1-2\gamma)
\end{aligned}\tag{12}$$

Some important remarks can be drawn from (12):

- In order to amplify the DVA’ s displacement $u$, the natural frequency ratio $\alpha$ should be chosen to be near 2 to produce resonance, and then the damping ratio $\zeta$ should be chosen an optimal value to produce maximum energy dissipation.
- The DVA’ s displacement is proportional to the location parameter $\gamma$. Therefore the larger location parameter $\gamma$ often gives the better effect.
- The DVA's displacement is proportional to the square of pendulum vibration angles. Therefore, this type of DVA has poor effect for small vibration.
- Let consider the special case when the following condition holds:

$$\theta_m^2\cos2\tau+\beta_m^2\cos(2\tau+2\varphi)=0\tag{13}$$

yields:

$$\begin{aligned}
&\left(\theta_m^2+\beta_m^2\cos(2\varphi)\right)\cos(2\tau)+\beta_m^2\sin(2\varphi)\sin(2\tau)=0\\
&\Rightarrow\begin{cases}\theta_m^2+\beta_m^2\cos(2\varphi)=0\\ \beta_m^2\sin(2\varphi)=0\end{cases}\Rightarrow\begin{cases}\theta_m=\beta_m\\ \varphi=\pi/2\end{cases}
\end{aligned}\tag{14}$$

When the condition (14) holds, the dynamic excitation term in equation (12) vanishes, the DVA can not be excited and therefore the DVA has no effect at all. In order to explain this phenomenon more clearly, the conditions (14) are substituted into (11), then into (1). By assuming the small vibration angles, we obtain:

$$x_1^2+y_1^2=l_1^2\theta_m^2\tag{15}$$

The equation (15) simply expresses the circular motion in the horizontal plane. This motion can not excite the motion in radial direction. It is also noted that the planar pendulum does not have this type of motion.

## 3. Optimal parameters of dynamic vibration absorber

In fact, the DVA is specified by four parameters including: mass ratio $\mu$, location parameter $\gamma$, natural frequency ratio $\alpha$ and damping ratio $\zeta$. The mass ratio $\mu$ and the location parameter $\gamma$ should be chosen to be as large as possible. The natural frequency ratio $\alpha$, as shown in the previous section, should be tuned close to 2. In this section, the numerical calculation is used to investigate the effects of parameters on the system responses. For simplicity, only planar free vibration with initial angle is considered. That means the following assumptions are applied to motion equations (9)

$$\theta(0)=\theta_0;\dot{\theta}(0)=\beta(0)=\dot{\beta}(0)=u(0)=\dot{u}(0)=p_x=p_y=0 \tag{16}$$

However, because of the homogenous nature of the spherical pendulum, the optimal parameters of DVA can also work in the three-dimensional case, which is shown in the next section.
The nonlinear equations (9) with the assumptions (16) are numerically solved. By using the potential energy V taken from (10), the following non-dimensional performance index is considered to be minimized:

$$J=\frac{1}{T_f}\int_0^{T_f}\frac{V}{m_1 g l_1}dt= \frac{1}{T_f}\int_0^{T_f}\left(\left(1+\mu(\gamma-u)\right)\left(1-\cos\theta\cos\beta\right)+\mu\alpha^2\frac{u^2}{2}\right)dt \tag{17}$$

in which, $T_f$ is the total time of simulation. The total potential energy described in (17) is preferred to the pendulum angle itself, because the performance index $J$ can take into account the DVA's displacement. When $\mu$=0, we obtain the performance index of pendulum structure without DVA. The total time of simulation $T_f$ is taken of 200s, the structural damping ratio $\zeta_s$ is taken of 0.5%. The DVA' s parameters $\mu$, $\gamma$, $\alpha$, $\zeta$ and the initial angle $\theta_0$ are changed to study their effects.

The plots of performance index versus the natural frequency ratio $\alpha$ for various parameters are shown in Figs. 3-6. In Figs. 3-6, $J$ and $J_u$ denote the performance indexes with and without DVA, respectively.

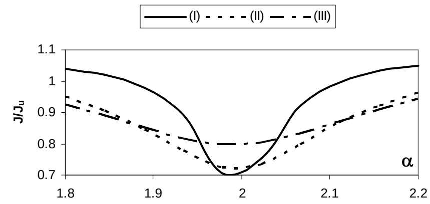



Fig. 3: $J$ as function of $\alpha$;
(I), (II), (III): $\zeta$=1%, 5% and 8%, respectively;
other parameters: $\mu$=0.05, $\gamma$=1, $\theta_0$=20°

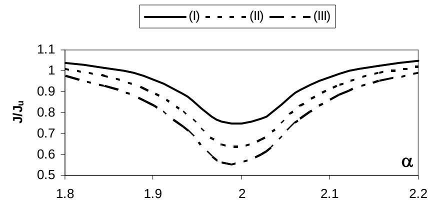



Fig. 4: $J$ as function of $\alpha$;
(I), (II), (III): $\theta_0$=15°, 20° and 25°, respectively;
other parameters: $\mu$=0.05, $\gamma$=1, $\zeta$=2%

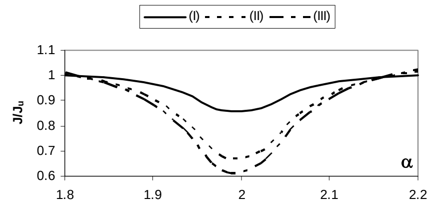



Fig. 5: $J$ as function of $\alpha$;
(I), (II), (III): $\mu$=1%, 4% and 6%, respectively;
other parameters: $\theta_0$=20°, $\gamma$=1, $\zeta$=2%

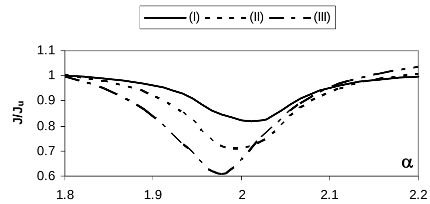



Fig. 6 : $J$ as function of $\alpha$;
(I), (II), (III): $\gamma$=0.5, 1 and 1.5, respectively;
other parameters: $\theta_0$=20°, $\mu$=3%, $\zeta$=2%

Some remarks can be drawn from the plots:
- In most cases, the optimal natural frequency ratio should be near to 2. However, in Fig. 6, when $\gamma$ is quite large, the optimal natural frequency ratio is slightly smaller than 2. This can be explained by observing the motion equations (9). When $\gamma$ increases, the equivalent mass $1+\mu(\gamma-u)^2$ increases faster than equivalent

stiffness $1+\mu(\gamma-u)$, then the natural frequency of pendulum attaching with DVA decreases, leads to the decrease of optimal natural frequency ratio.
- As seen from Figs. 4-6, when location parameter $\gamma$ or the mass ratio $\mu$ or the initial angle $\theta_0$ increases, the effectiveness of DVA increases.

The plots of performance index versus the DVA' s damping ratio $\zeta$ for various parameters are shown in Figs. 7-10.

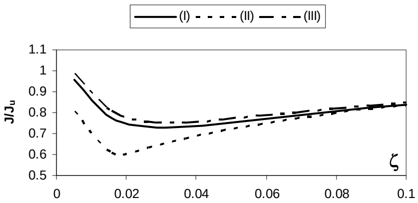



Fig. 7: $J$ as function of $\zeta$;
(I), (II), (III): $\alpha$=1.95, 2 and 2.05, respectively;
other parameters: $\theta_0$=20°, $\mu$=5%, $\gamma$=1,

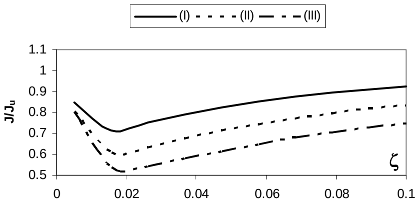



Fig. 8: $J$ as function of $\zeta$;
(I), (II), (III): $\theta_0$=15°, 20° and 25°, respectively;
other parameters: $\alpha$=2, $\mu$=5%, $\gamma$=1

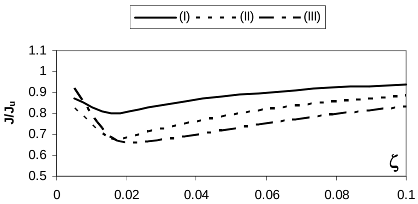



Fig. 9: $J$ as function of $\zeta$;
(I), (II), (III): $\gamma$=0.5, 1 and 1.5, respectively;
other parameters: $\alpha$=2, $\mu$=3%, $\theta_0$=20°

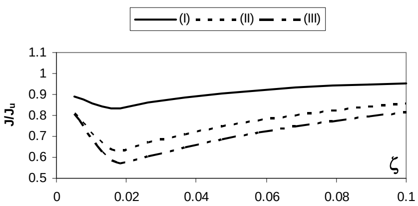



Fig. 10: $J$ as function of $\zeta$;
(I), (II), (III): $\mu$=1%, 4% and 6%, respectively;
other parameters: $\alpha$=2, $\gamma$=1, $\theta_0$=20°

Some remarks can be drawn:
- The optimal damping ratio is about 2%. When $\gamma$ or $\mu$ or $\theta_0$ decreases, the optimal damping ratio also slightly decreases.
- The shapes of curves are quite flat in the right side of optimal damping ratio. Therefore, a damping ratio larger than the optimal value will not reduce much the effectiveness.

## 4. Random vibration in 3D space

In practical situations, the random effect of vibration phase often breaks the requirement of 90° phase lag in the condition (14). Therefore, the DVA has good effect in random vibration case. Let consider the random vibration of the spherical pendulum under white noise excitations. The parameters are chosen as: total simulation time $T_f$=200s, pendulum damping ratio $\zeta_s$=0.5%, mass ratio $\mu$=3%, location parameter $\gamma$=1.5, natural frequency ratio of DVA $\alpha$=2, damping ratio of DVA $\zeta$=3%, the initial values are taken to be zero. The excitations $p_x$ and $p_y$ in equation (9) are taken as the white noises with the same intensity, which is denoted by $S_0$. This leads to the vibration amplitudes in two directions are nearly the same. However, the DVA still has good effect because of the random nature of vibration phase. The Monte Carlo simulation is used to obtain the mean values of quantities. The total number of sample is 1000. The ratios between performance indexes in cases of system without and with DVA are tabulated in Table 2

Table 2: Performance of DVA versus the white noise excitation intensity

| $S_0$ | 0.10 | 0.30 | 0.50 | 0.70 | 0.90 |
|---|---|---|---|---|---|
| $J/J_u$ | 0.93 | 0.87 | 0.83 | 0.80 | 0.78 |
| $S_0$ | 1.10 | 0.30 | 1.50 | 1.70 | 1.90 |
| $J/J_u$ | 0.76 | 0.74 | 0.73 | 0.72 | 0.71 |

We denote the non-dimensional total displacement as

$$d(t)=\sqrt{x_1^2(t)+y_1^2(t)}\Big/l_1=\sqrt{\cos^2\beta\sin^2\theta+\sin^2\beta} \quad (18)$$

The mean values of non-dimensional total pendulum displacement (18) and non-dimensional DVA' s displacement are plotted in Figs. 11-14, in which <•> denotes the mean value over the total number of samples in Monte Carlo simulation. The results in Table 2 show that the DVA performance improves with the increase of excitation intensity. From Figs. 11-14, we see that when the excitation intensity increases, the DVA displacement also increases resulting in the more energy dissipation to reduce pendulum vibration.

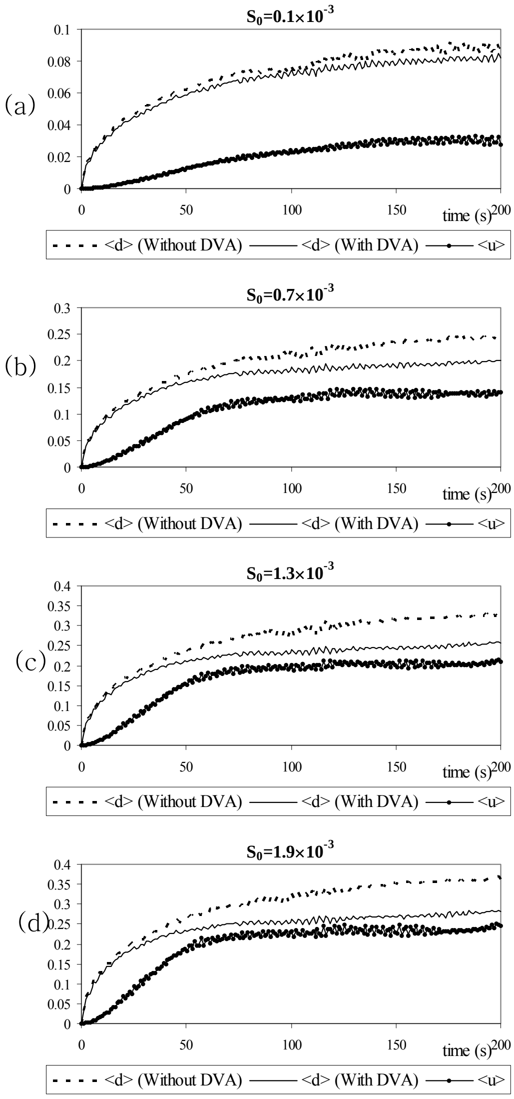



Fig. 11a-d: Time histories of mean values of displacement of pendulum and DVA in several case of non-dimensional white noise intensity

## 5. Conclusions

The DVA moving in radial direction has been considered to reduce vibration of the spherical pendulum structure. The advantage of this type of DVA is the capability of bidirectional vibration control by only one translation. The DVA has good performance in case of large vibration. Optimal parameters for the DVA are numerically calculated in order to reduce the integration of system's potential energy. In a special case, the spherical pendulum moves circularly in a horizontal plane, the DVA has quite little effect. However, in random vibrations, the random nature of vibration phase excludes the horizontal circular motion and then ensures the DVA effectiveness.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