P1 비순응요소를 이용한 판재보강의 위상최적화
Topology Optimization of Plate Reinforcement by using P1-Nonconforming Element

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Key words : Topology Optimization, P1-nonconforming Element, Mindlin Plate, Plate reinforcement

1. Introduction

Topology optimization technique could help engineers solve structure problems by providing the optimal distribution of a limited amount of material in a design space. Here, a partially clamped plate under distributed load is analyzed to get optimal shape for the given type of loading.

To avoid shear locking problem, P1-nonconforming element and selective reduced integration are employed. For P1-nonconforming element, displacement are continuous at the mid-side points of each element[1]. In other words, the displacements of vertex common to different element could be different from each other.

2. Theory Formulation

By principle of minimum potential energy, the relationship of bending moment and displacement can be expressed as [2, 3]

$$\boldsymbol{KU} = \boldsymbol{F} \tag{1}$$

where U is displacement vector, F is load vector. K is the global stiffness matrix. Let's introduce relative density ρ to elastic modulus E [4]

$$E = E_0 \rho^n \tag{2}$$

where E_0 is original elastic modulus of material and n is penalization power. Usually, for basic plate, relative density ρ_b will always be 1. For reinforced plate, relative density ρ_r will range from 0 to 1.

A mean compliance minimization problem by using the P1-nonconforming element is formulated as

$$\underset{\rho \in \Re^{N_D}}{\text{Minimize}}: c(\rho) = F^T U \tag{3a}$$

$$\text{Subject to}: \sum_{e=1}^{N_D} \rho_e v_e \le V \tag{3b}$$

$$0 \le \rho_e \le 1, \; e=1, 2, 3, \ldots ND$$

where v_e is the volume of each element, V is the total volume of the plate.

Objective and constraint sensitivity analysis can be calculated by

$$\frac{\partial c}{\partial \rho_e} = -U^T \left(n\rho_e^{n-1} \int_\Omega B^T D_0 B dA \right) U \tag{4a}$$

$$\frac{\partial g}{\partial \rho_e} = \frac{\partial}{\partial \rho_e} \left(\sum_{e=1}^{N_D} \rho_e v_e - V \right) = v_e \tag{4b}$$

3. Numerical Examples

3.1 Partial Clamped Plate Problem

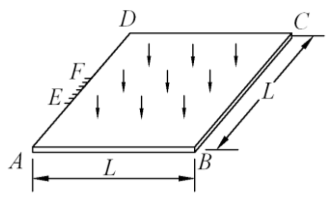

Fig. 1 Partial clamped plate under pressure

As shown in Fig. 1, the steel plate is partially clamped and subjected to uniform load P =-0.75 *psi*. The length of each edge is 20 *inch*. The plate can be divided into basic and reinforced parts. The thicknesses of them are 0.03 *inch* and 0.07 *inch*. Elastic modulus E is 30×10^6 *psi* and passion's ratio v is 0.3. *EF* edge is partial clamped along *AD* line and its length is 2 *inch*.

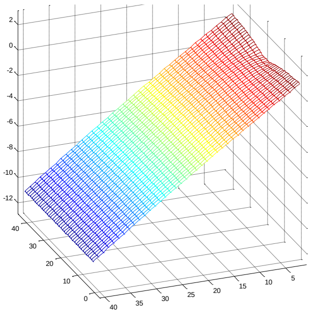

Fig. 2 Deformation of partial clamped plate under distributed load

The calculated maximum displacement at the edge *BC* is 13.2 *inch*. The deformation of the plate is plotted as shown in Fig. 2 by using Matlab software.

3.2 Optimal Shape of Clamped Plate

The clamped plate is discretized with 160*160 P1-nonconforming elements as shown in Fig. 3. Here, the volume fraction is 20% and penalization power is 3. By using 20% mass of plate, the deflection of this shape is smaller than that of any other shapes under same load condition.

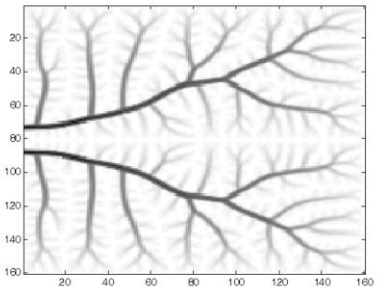

Fig. 3 Optimal shape of partial clamped plate under distributed load

The optimal shape is calculated by the Method of Moving Asymptotes(MMA) [5]. This method could avoid local minimum value problem.

4. Discuss and Conclusions

This paper presents a topology optimization of plate under bending moment by using reinforced plate model. This model is more reasonable than the model made up by void and solid elements because that load cannot be applied on void element. Based on displacement and strain energy, theoretical formulation of topology optimization is developed with P1-nonconforming elements.

Due to the locking-free and discontinuity property of the P1- nonconforming elements, the shearing locking problem in thin plate caused by Reissner-Mindlin plate model is solved with the help of selective reduced integration. It provides a new idea to apply this kind of discontinuous element to solve shearing locking problem and volumetric locking problem.

Acknowledgement

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (Grant number: 2010-0021248)

REFERENCES

1. Park, C, Sheen, D., "P1-nonconforming quadrilateral finite element method for second-order elliptic problems," *SIAM J. on Numerical Analysis*, 624-640. 2003.
2. Backlund, J., "Mixed finite element analysis of plates in bending," *Chalmers Univ. of Technology, Gutenborg, Sweden*, publication 71:4. 1972.
3. Timoshenko and Woinowsky-Krieger S., "Theory of plates and shells," *McGraw-Hill*, New York, 1959.
4. Bendsφe, M. P. "Optimal shape design as a material distribution problem," *Struct. Optim. 1*, 193-202, 1989.
5. Svanberg, K. "The method of moving asymptotes - a new method for structural optimization," *Int. J. Number. Meth. Engrg. 24*, 359-373. 1987.