Tail dependence of Bivariate Copulas for Drought Severity and Duration

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Abstract

Drought is a natural hazard with different properties that are usually dependent to each other. Therefore, a multivariate model is often used for drought frequency analysis. The Copula based bivariate drought severity and duration frequency analysis is applied in the current study in order to show the effect of tail behavior of drought severity and duration on the selection of a copula function for drought bivariate frequency analysis. Four copula functions, namely Clayton, Gumbel, Frank and Gaussian, were fitted to drought data of four stations in Iran and Canada in different climate regions. The drought data are calculated based on standardized precipitation index time series. The performance of different copula functions is evaluated by estimating drought bivariate return periods in two cases, [$D \ge d$ and $S \ge s$] and [$D \ge d$ or $S \ge s$]. The bivariate return period analysis indicates the behavior of the tail of the copula functions on the selection of the best bivariate model for drought analysis.

Key words: Drought, Copula, Bivariate frequency analysis, tail dependence

1. Introduction

Drought is a special natural hazard which is observed in both humid and arid regions. The disaster impacts of drought events have been reported in different countries and regions of the world (Frick et al., 1990; Panu and Sharma, 2002; IPCC, 2007; Panu and Sharma, 2009). Many investigators have been trying to characterize different elements of drought such as intensity, duration and recurrence intervals and their effects on agriculture and water supply resources (Frick et al., 1990; Sharma, 2000; Shiau and Shen, 2001; Bonaccorso et al., 2003; Cancelliere and Salas, 2004; Shabbar and Skinner, 2004; Pielke et al., 2005; Arena et al., 2006; Modarres, 2007; Ouarda et al., 2008; Karamouz et al., 2009; Khaliq et al., 2009; Panu and Sharma, 2009).

Among various categories of drought characterizations, the study of drought probability and risk is the most controversial and interesting category among scientists. The reason is that drought is a stochastic phenomenon in space and time. In addition, the probabilistic and statistical approaches are appropriate for assessing drought characteristics. A number of probabilistic approaches have been applied to model drought properties such as severity, duration, and return period. Many studies have applied univariate models and have ignored the relationship between drought characteristics. However, in recent years the bivariate probabilistic approaches which take into account the relationship between drought properties have been applied for modeling multi-attributable nature of drought events (Salas et al., 2001; Panu and Sharma, 2002; Cancelliere and Salas, 2004; Laux et al., 2009; Nadarajah, 2009; Shiau and Modarres, 2009; Kao and Govindaraju, 2010; Song and Singh, 2010). One way of the bivariate drought analysis is to employ bivariate distributions such as bivariate gamma distribution (Nadarajah, 2007) and bivariate Pareto distribution (Nadarajah, 2009). The complexity and the limitation of the marginal distribution hinder applying this approach.

Another alternative is to use a copula function (Modarres, 2007; Laux et al., 2009; Shiau and Modarres, 2009; Kao and Govindaraju, 2010; Song and Singh, 2010). The copula functions have been applied in different hydrological frequency analysis such as rainfall and flood (De

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Michele and Salvadori, 2003; Favre et al., 2004; Salvadori and De Michele, 2004; Chebana and Ouarda, 2007; De Michele et al., 2007; Kao and Govindaraju, 2008; Chebana and Ouarda, 2010). For bivariate drought frequency analysis, the joint distribution of drought characteristics, e.g. duration and severity, has been modeled through different copula functions and different marginal distributions. Shiau (2006) applied different types of copulas for joint distribution of drought duration and severity. He also fitted exponential and gamma distributions to drought duration and severity as marginals, respectively.

The aim of the present study is to perform a bivariate drought frequency analysis using copula functions.

2. Drought Definition and Copula

For copula-based severity-duration-frequency analysis, drought characteristics are defined based on popular standardized precipitation index (SPI) developed by (McKee et al., 1993). The SPI = 0 is selected as the threshold of a drought event. Drought duration, D, is defines as the consecutive events with negative SPI and drought severity, S, is then the cumulative values of SPI within the duration of drought. For convenience, drought severity is multiplied by -1 to make it a positive value. Therefore, we have:

$$S = -\sum_{i=1}^{D} SPI_i \qquad (1)$$

The bivariate relationship between two variables is usually based on the Sklar's theorem (Sklar, 1959) which states that if $F_{X,Y}(x,y)$ is a bivariate Cumulative Density Function (CDF) of X and Y variables with respective marginal CDFs and , then there exists a copula C such that

$$F_{X,Y}(x, y) = C(F_X(x), F_Y(y))$$

3. Study area

For bivariate drought frequency analysis in this study, four SPI time series of two stations in Iran called Anzali and Zahedan stations and two stations in Canada, called Grande Prairie and La Tuque stations are selected.

Although Canada is usually considered to be a country with abundant freshwater and ranked among the top five countries in terms of per-capita water supply, drought is now becoming a major problem in Canada, especially in western provinces and for the agriculture sectors of Canada and a topic of different studies as well (i.e. Shabbar and Skinner, 2004; Schindler and Donahue, 2006; among others). The stations selected for this study are located in western Canada as a region at risk (Grande Prairie station, Alberta province) and one of the major agricultural region of Canada (La Tuque station, Quebec province).

3. Results

As drought is a bivariate event characterized by drought duration and severity, the frequency analysis of drought should consider the joint and conditional properties of return periods of drought events. The joint return period, is defined in two cases for drought risk: return period of drought events with [$D \geq d$ and $S \geq s$] and [$D \geq d$ or $S \geq s$]. The copula-based drought joint return period can be defined as the following (Shiau, 2006):

$$T_{DS} = \frac{E(L)}{P(D \geq d, S \geq s)} = \frac{E(L)}{1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))} \qquad (2)$$

$$T'_{DS} = \frac{E(L)}{P(D \geq d \text{ or } S \geq s)} = \frac{E(L)}{1 - C(F_D(d), F_S(s))} \qquad (3)$$

where T_{DS} denotes the joint return period for [$D \geq d$ and $S \geq s$] and T'_{DS} denotes the joint return period for [$D \geq d$ or $S \geq s$] and *E(L)* is the expected drought interarrival time. The interarrival time is defined as the time between consecutive arrivals. The bivariete return periods of drought duration and severity for Anzali and Grande Prairie Stations have been given in Figures 1 and 2 for the two cases, T_{DS} and T'_{DS}. From these figures, it can be seen that the shape and variation of the return period curves of both T_{DS} (left panels) and T'_{DS}(right panels in Figure 1) highly rely on the type of the copula function. In Figure 1, T_{DS} and T'_{DS} of the

Clayton copula (b-1 and b-2) are similar to the independent case (a-1 and a-2). In other words, for the Clayton function of drought, the relationship between drought duration and severity seems to be insignificant. As an example, for $P(D \geq 5 \text{ and } S \geq 5)$, the return period of drought events in the independent case in Figure 1(a-1) is about 14 years while it is 7 years for the Clayton copula and about 4 years for other three copulas. However, for the case of T_{DS}, the effect for different copula is less significant than those for T_{DS} as shown in right panels in Figure 1. For $P(D \geq 5 \text{ or } S \geq 5)$, the return period of drought is between 1 and 2 years according to the independent and the Clayton copula function, while it is more than two years based on other copula functions.

Finally, the return period plotsof four stations with the selected copula functions are givenin Figure 2. It reveals that the three stations (Anzali, La Tuque, and Grande Prairie) provide the similar return period while the Zahedan station has higher return period for the same events implying that a certain event is less frequent than the other stations. Therefore, the selection of suitable copula function is a critical task forwater resources management. The underestimation of the return period for a drought event might increase the risk of the failure of water resources management practices, both for at-site and regional scales.

4. Conclusions

We have observed that Clayton copula which is selected for drought bivariate frequency analysis in literature without elaborate selection processes is not capable of modeling the association between the drought duration and severity. The empirical return period curves of DS and DS' (Eqs. (2) and (3)) are significantly different from those of Clayton copula model. By employing Clayton copula, a considerable overestimation of the return period for a certain drought event is induced especially in the case of DS (Eq. (2). Therefore, high-frequent drought events in reality may be estimated as low-frequent drought events according to Clayton copula function. Eventually, this leads to the failure of the drought risk management practices and disaster mitigation.

From these results, we conclude that the Clayton copula is not an appropriate choice for drought bivariate frequency analysis, at least for our examples. This Clayton copula function does not give much more information than when the drought duration and severity are assumed to be independent. Alternatively, the Frank or Gumbel copula can be selected for the bivariate drought analysis since these copulas relatively well mimic the upper tail dependence between drought duration and severity. The presented results are limited only when the definition of a drought event is with Eq. (1). In other words, if one defines a drought event differently (e.g. Salas et al., 2005), different aspects can be observed, e.g. Clayton copula might be preferable.

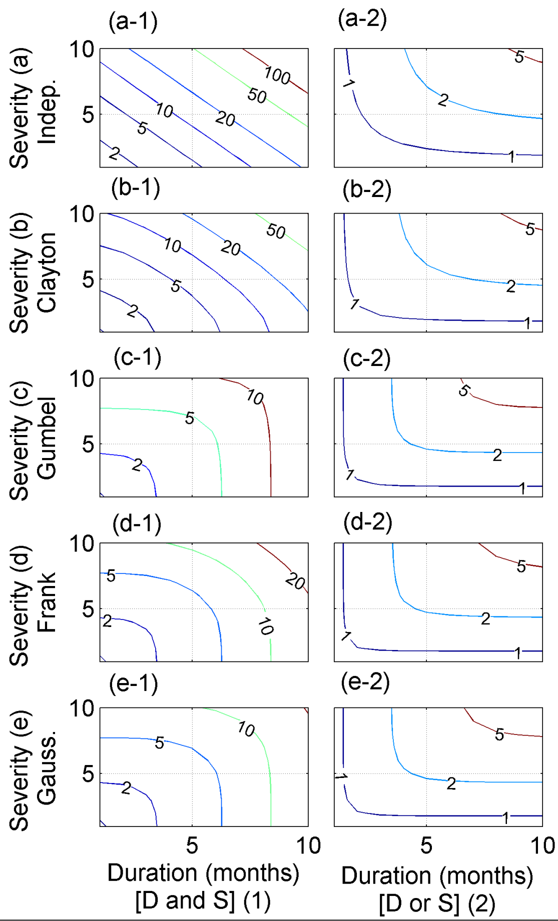

Figure 1 Return period (years) of (1) DS in Eq.(2, left column and (2) DS' in Eq. (3) right column with (a) independent, (b) Clayton, (c) Gumbel, (d) Frank, (e) Gaussian for Anzali station.

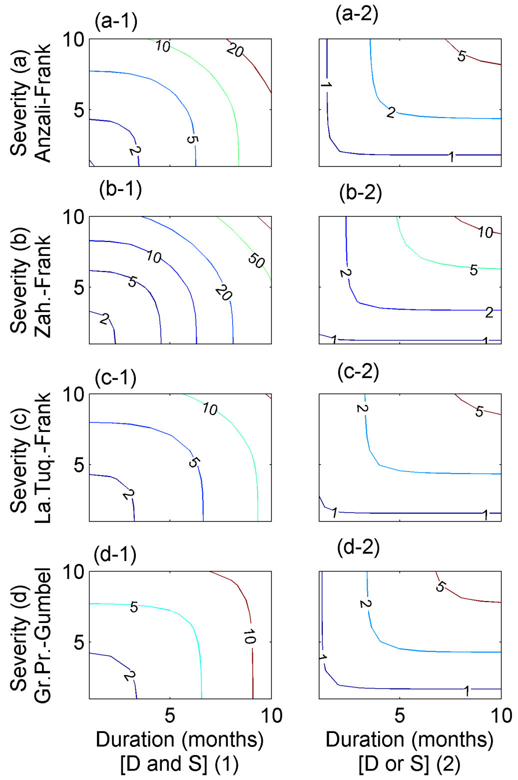

Figure 2 Return period of (1) DS in Eq.(2, left column and (2) DS' in Eq. (3,right column for (a)Frank-Anzali, (b)Frank - Zahedan, (c) Frank - LaTuque, and (d)Gumbel -Grande Prairie stations.

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