# 직선 개수로 횡단면에서의 소류사 이송과 지형 변화

## Bedload Sediment Transport and Morphological Change in Cross Sections of Straight Open-Channel


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Abstract

This study presents velocities of bedload sediment transport in both longitudinal and lateral directions and applied in considering morphological change of straight open channel. The velocities of particle motion have obtained by considering the forces balance acting on particles on the bed between the drag, tangential component of the immersed weight of the particle, and Coulomb's resistive forces. Numerical profiles of particle motion velocities reveals good agreement in comparison between this study and Kovacs and Parker (1994). The evaluated velocities components of particle transport are get used to estimate bedload transport rate in considering morphological change of straight open channel. For the application, numerical solution is applied to laboratory experiment which shows very close solution profiles between this study and observed data of a self-formed straight channel.



*Keywords*: Bedload sediment, Morphological Change, Numerical Solution, Velocities of particle motion, Sediment Transport.


## 1. INTRODUCTION

Sediment particles transported are frequently in contact with the bed in directly. The motion of particles plays an important role in erosion and deposition of channel both the lateral and longitudinal directions. It will affect the stability of bank and bed elevation, flow structure, water quality, water budgets, soil moisture as well as ecology. In addition, it also destroyed expensive infrastructure such as roads, bridges, buildings and infrastructure works near the bank line of rivers. Therefore, particle motion and bed elevation change have been becoming as an interesting and necessary topics in river engineering field that many researchers formulated of flow-bed evolution and proposed numerical modeling approaches.

## 2. PARTICLE MOTION AND BEDLOAD TRANSPORT RATE

### 2.1 Sediment particle motion

The motion of a particle is determined by the drag force, the component of immersed weight of the


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particle tangential to the bed surface and the dynamic Coulomb resistive force which represents the momentum loss due to collisions. It is assumed that particles motion has achieved local statistical equilibrium. This means that total forces acting on the particle at local statistical equilibrium is equal to zero.

$$\overrightarrow{F_D} + \overrightarrow{W_g} + \overrightarrow{F_C} = 0 \tag{1}$$

where $\overrightarrow{F_D}$ is the drag force, $\overrightarrow{W_g}$ is the component of the immersed weight of the particle tangential to the plane of the bed, and $\overrightarrow{F_C}$ is the dynamic Coulomb resistive force.

Using the expressions of drag force, immersed weight of particle and Coulomb resistance force with some derivative steps, the dimensionless form of equation (1) are illustrated as Eqs (2) and (3). The velocities of particle motion can estimate based on these equations and using Newton-Raphson method as the required iteration of numerical solution.

$$\frac{1}{a}|\overrightarrow{u_r^*}|(u_b^* - v_{ps}^*) = \tau_{co}^* \left( |k_3| t_{vp1} - \frac{k_1}{\mu_c} \right) \tag{2}$$

$$\frac{1}{a}|\overrightarrow{u_r^*}|(-v_{pp}^*) = \tau_{co}^* \left( |k_3| t_{vp2} - \frac{k_2}{\mu_c} \right) \tag{3}$$

where $t_{vp}$ is vector of particle motion, $\overrightarrow{u_r}$, $\overrightarrow{u_b}$, $\overrightarrow{v_p}$ are fluid velocity relative to the moving particle, bottom fluid velocity, and mean particle velocity, respectively, $\overrightarrow{u_r^*}$, $u_b^*$, $v_p^*$ and their components have been formed by dividing the respective dimensioned forms $\overrightarrow{u_r}$, $\overrightarrow{u_b}$, $\overrightarrow{v_p}$ by the velocity scale dividing $\sqrt{RgD}$, D is diameter of sediment particle, R is specific sediment gravity, and $\tau_{co}^* = \frac{4}{3}\frac{\mu_c}{a.c_D}$.

## 2.2 Bedload transport rate

The vectorial volume bedload transport rate of bed sediment per unit normal width $\overrightarrow{q_b}$ can be represented by according Kovacs and Parker’s model (1994). That is,

$$\overrightarrow{q_b} = \xi \overrightarrow{v_p} \tag{4}$$

This relation can be rendered dimensionless by dividing through by ($\sqrt{RgD}D$) such that $\overrightarrow{q_b^*}$ is defined equal to $\overrightarrow{q_b}/(\sqrt{RgD}D)$. The component $\overrightarrow{q_b^*}$ consists of two components in longitudinal and lateral directions. It can be expressed as following.

$$q_{bx}^* = \xi^* v_p^* \cos\psi \cos\alpha \tag{5}$$

$$q_{by}^* = \xi^* v_p^* \cos\omega \left( \cos\psi.\sin\alpha.\sin\omega + \sin\psi \sqrt{\sin^2\omega.\cos^2\alpha + \cos^2\omega} \right) \tag{6}$$

where $\xi^*$ is the dimensionless volume of particles participating in bedload transport, $\psi$ is the angle of the particle motion, $\alpha$ is angle of longitudinal channel slope, and $\omega$ is angle of lateral channel slope.

# 3. FLOW AND BED EVOLUTION OF CHANNEL

## 3.1 Flow equation

Due to consider morphological change of channel, lateral distribution of longitudinal depth-averaged velocity in channel is calculated based on momentum equation which was proposed by Wark et al. (1990).

$$ghI_x = \frac{B_g f}{8} U^2 - h\frac{\partial}{\partial y}\left[\overline{\varepsilon_y}\frac{\partial U}{\partial y}\right] \qquad (7)$$

where $g$ is acceleration of gravity, $h$ is the local flow depth, $I_x$ (= tan$\alpha$) is the longitudinal channel slope, $U$ is the longitudinal depth-averaged flow velocity, $y$ is the lateral coordinate, $f$ is the Darcy-Weisbach friction factor, $B_g$ is a geometrical factor ( $=\sqrt{1+\tan^2\alpha+\tan^2\omega}$ , where tan$\omega$ is lateral bed slope), and $\overline{\varepsilon_y}$ is depth-averaged eddy viscosity, $\rho$ is water density.

This equation illustrates that the driving force due to gravity is balanced by the bottom shear stress, and the lateral shear stress. And the boundary condition for numerical solution momentum equation is the free slip condition at the banks.

### 3.2 Mass conservation equation

Bed elevation of channel is obtained by solve the Exner equation which was based on the sediment mass conversation condition (Raudkivi, 1990).

$$\frac{\partial z_0}{\partial t} - \frac{1}{1-\lambda}\left[\frac{\partial q_{by}}{\partial y} + \frac{\partial q_{bx}}{\partial x}\right] = 0 \qquad (8)$$

where $z_0$ is the bed elevation, $t$ the time coordinates, $\lambda$ the porosity of the bottom sediment, $q_{bx}$ and $q_{by}$ are component of bedload vector of sediment in x and y direction, respectively.

It is assumed that the longitudinal variation in the cross- section is not significant and a simple explicit finite difference scheme is selected and used for computational simulation of Exner equation.

### 3.3 Sliding algorithm for marginal erosion

After applied Exner equation, the morphology of channel due to sediment transport by flow drag is considered through lateral local bed slope and angle of repose. Lateral slope between nodes of cross-section is checked based on bed elevation. If local bed slope is steeper than the angle of repose, the upper node height is lowered so that the slope angle becomes the angle of repose. And then the height of lower node is increased in order to accommodate the soil mass conservation. For each time step, this procedure is repeated iteratively until no slope angle above the angle of repose is found (Menendez et al. 2008).

## 4. APPLICATION

### 4.1 Calculate sediment particle velocities

Due to calculate sediment particle velocities, three important parameters as $a^{1/2}$, $\tau^*_{co}$, and $\mu_c$ are equal to 11.9, 0.035 and 0.84, respectively. The selected value of $\mu_c$ corresponds to angle of repose of 40°. For the purpose of numerical calculation, the streamswise angle of inclination $\alpha$ is fixed, and the lateral angle ω (local bed slope) is allowed to change continuously from zero to the value at which $\tau^*_c$ vanishes. The value of Shields stress $\tau^*$ is taken to be twice the critical value $\tau^*_{co}$ for a flat bed. The velocity of particle motion is calculated in two cases of α. In the first case $\tan\alpha$ is chosen to be vanishing, and in the second case it is set equal to $3\mu_c/4$. Results of calculation are shown from Figure 1 to Figure 3. In the first case, $\tau^*_c/\tau^*_{co}$ varies from unity at $\tan\omega=0.0$ to zero at $\tan\omega=\mu_c$ while this value takes only 0.21 at $\tan\omega=0.0$ and dropping to $0.0$ at $\tan\omega=0.553$ in second case. This means that the mobility of grains increases when streamwise or transverse slope is grown and it is clear that dimensionless velocities of particle motion in both longitudinal and lateral directions increase when lateral angle or channel slope rise. Larger erosions and depositions occur in increasing slope.

## 4.2 Morphological change

In order to consider particle motion in morphological change of straight channel, present study applies to experimental data performed by Izumi et al. (1991). The channel is straight, prismatic and symmetric. The initial cross-section is trapezoidal and unstable with longitudinal slope of 0.0024 and banks slope of 0.6. Initial flow depth and discharge are 0.0276 m and 0.00082 $m^3/s$. Both the banks and bottom channel are made of material which has mean diameter equals to 2.6 mm. The values adopted for computational solution were $a^{1/2}$ = 11.9, $\tau^*_{co}$=0.45 and $\mu_c$ =0.84. The initial cross-section was divided into 100 interval, resulting a spatial step $\Delta y$ = 0.00292 m, and the time step was chosen $\Delta t$ = 1 second. The results of simulation are illustrated from Figure 4 to Figure 6. Good agreements between numerical results and observed data are shown in comparison at 24 minutes and 120 minutes. Erosion occurs along upper banks and deposition in lower bank and around the central main channel. In addition, channel widens gradually with increasing of simulation time.

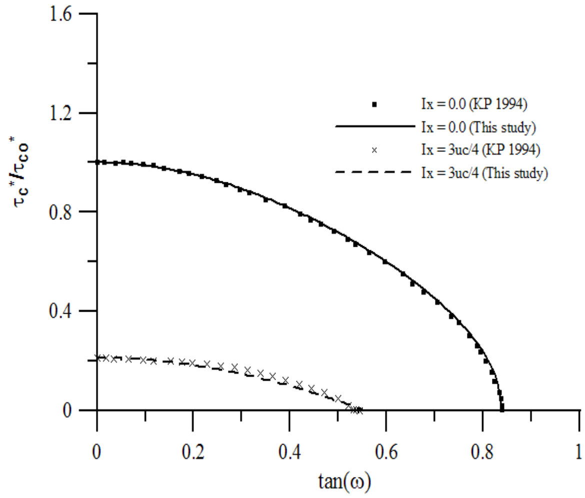


Figure 1. The ratio of critical Shields stress on a sloping bed ($I_x$ = 0.0)

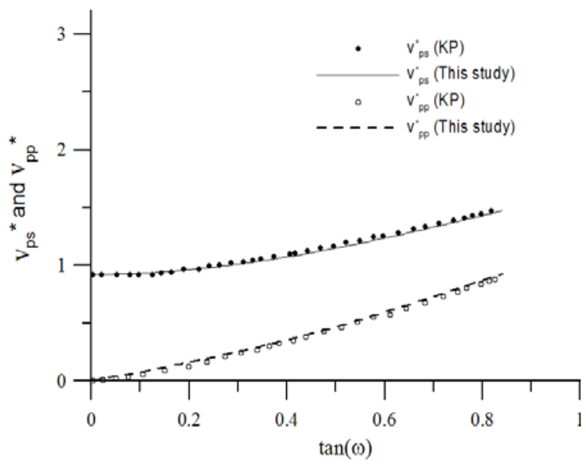


Figure 2. Dimensionless of particle sediment velocity ($I_x$ = 0.0)

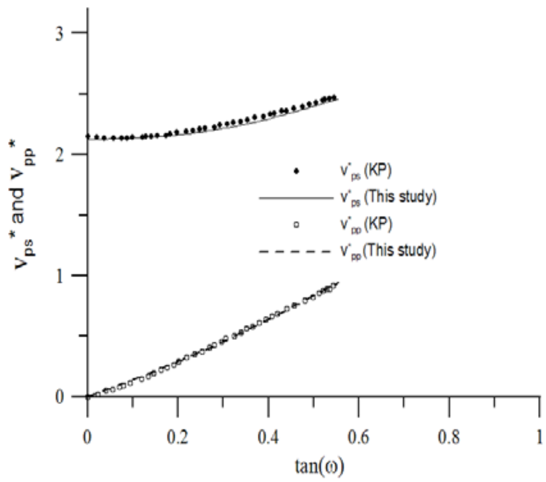


Figure 3. Dimensionless of particle sediment velocity ($I_x$ = $3\mu_c/4$)

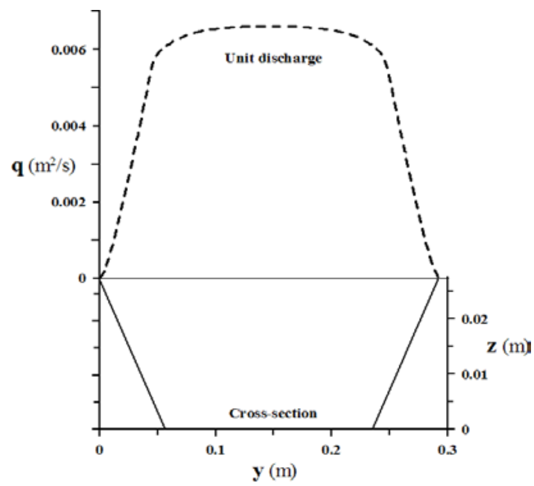


Figure 4. Lateral unit discharge at initial time

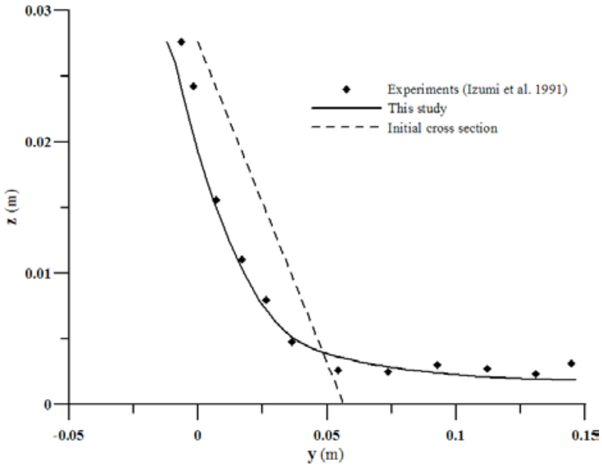


Figure 5. Change of cross section at 24 minutes

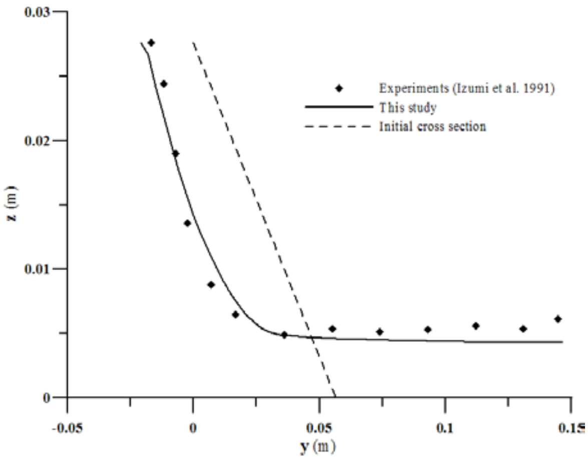


Figure 6. Change of cross section at 120 minutes

## 5. CONCLUSION

It is clear and well known that particle motion and morphological change in channel plays an important role in use and management of river. This research investigated effect of streamwise and transverse slope to dimensionless velocities of particle motion. The results show that particles will move faster when longitudinal and lateral slopes increase. Larger amount of erosions and depositions in increasing slope will occur. This is a good remark of widening channel in considering bank stability and inclined change. For consider morphological change, numerical solutions were simulated and compared with experimental data in very close. These results demonstrated that erosion often occurs at upper bank and deposition in lower and central main channel. More widening of channel width and flatter of bed at main channel will occur when simulation time increases.


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