비선형 Least Square 방법을 사용한 트레일러 System Identification
Car-Trailer System Identification Using Nonlinear Least Square Techniques

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1. Introduction

The subject of this research is to determine kinematic parameters, hitch and trailer lengths, experimentally using inexpensive sensors and nonlinear least square techniques. These parameters, L_1 and L_2, Fig. 1, are used to describe kinematics of a car-trailer system such that they are applied to a motion controller.

Robot parameters change from their nominal values for many reasons such as manufacturing and assembly errors, clearance, backlash, and wear. As a result, inaccuracy in parameters can weaken an established model such that performance of a controller will be deteriorated in a real system. In particular, trailer backing critically depends on car-trailer system parameters. Further, many commercial or customized trailers and hitches are available for different applications. Thus, identifying or calibrating system parameters is important as a preliminary work for fully autonomous control of the car-trailer system.

Least square techniques are widely used for system identification and calibration by using a set of observed parameters. In particular, their applications include manipulator and parallel robots' calibration and parameter identification [1, 2]. Much of this research has focused on more accurate least square algorithms for manipulator robots. In contrast, this research focuses on experimental parameter identification and calibration for the car-trailer system, which is a preliminary work for motion control.

In this research, a Dodge Grand Caravan is connected with a trailer through a trailer hitch and coupler. A closed form model is derived considering the system in forward motion. Using this closed form model, nonlinear least square techniques are then applied to determine trailer and hitch lengths. The proposed algorithm requires experimentally measuring a hitch angle and a curvature for parameter identification. The vehicle is thus driven for a short distance with a varying curvature to provide sufficient experimental data. Toward this goal, an angle sensor is mounted over the coupler to measure the trailer hitch angle. Further, path curvatures are estimated by using a steering map and wheel odometry at the rear axle, respectively.

A main contribution of this research is experimental system identification using the real car-trailer system and inexpensive sensors. The proposed algorithm can provide car-trailer system parameters for a motion controller. Further, the algorithm will allow using a variety of existing trailers and hitches without measuring car-trailer system parameters manually for different tasks.

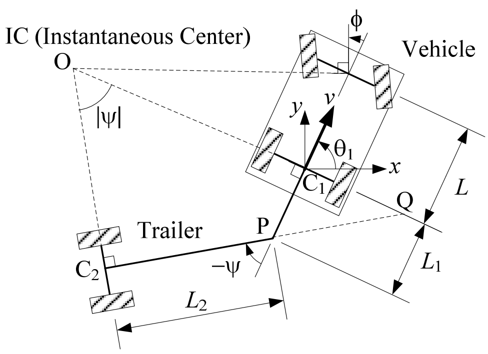


Fig. 1. Kinematics of a car-trailer system.

This paper is organized as follows: Sec. 2 presents steering kinematics for a car-trailer system and a closed form expression for system identification. A nonlinear least square method is described in Sec. 3. Experimental results are presented and discussed in Sec. 4. Conclusions are finally provided in Sec. 5.

2. Model

This section provides a kinematic model that will be used to identify car-trailer system parameters. First, a general steering model for a car-trailer system is presented. A closed form model in forward motion is then presented for simple implementation.

2.1. General Steering Model

Fig. 1 shows steering kinematics of a car-trailer system. The system state variables are the Cartesian coordinates, (x,y), the heading angle, θ_1, and the hitch angle, ψ. The state equations for this car-trailer system are then,

$$\begin{aligned} \dot{x} &= v\cos\theta_1 \\ \dot{y} &= v\sin\theta_1 \\ \dot{\theta}_1 &= v\frac{\tan\phi}{L} \\ \dot{\psi} &= \frac{v}{L_2}\left[\frac{\tan\phi}{L}\left(L_2 + L_1\cos\psi\right) - \sin\psi\right] \end{aligned} , \qquad (1)$$

where v is the linear velocity at the rear axle center, C_1, ϕ is the steering angle, L is the distance between front and rear axles, L_1 is the hitch length, and L_2 is the trailer length. Note that these state equations are highly nonlinear such that closed form solutions are not easily found. As a result, it is difficult to directly apply this general steering model to least square methods. Thus, a closed form model is discussed in the next section.

2.2. Closed Form Model in Forward Motion

This section provides a closed form model that can easily be applied to least square methods. Note that forward motion of the car-trailer system is exponentially stable whereas backing is naturally unstable [3]. Assuming forward motion, there exists an instantaneous center of rotation, O, for front, rear, and trailer axles as shown in Fig. 1, As a result, a closed form model can be found in forward motion such that this model is used to identify trailer parameters, L_1 and L_2.

Now consider the instantaneous center of rotation, O, and geometry of the quadrilateral, OC_1PC_2 to find a closed form expression for the hitch angle, ψ. Since $|\psi| = \angle C_1OP + \angle C_2OP$, the hitch angle can be formulated as a function of the path curvature, κ ($= \dot{\theta}_1/v$), at the point C_1 using trigonometric functions,

$$|\psi| = \tan^{-1}\left(\frac{L_1}{R}\right) + \sin^{-1}\left(\frac{L_2}{\sqrt{R^2 + L_1^2}}\right) = \tan^{-1}\left(|\kappa| L_1\right) + \sin^{-1}\left(\frac{|\kappa| L_2}{\sqrt{1 + |\kappa|^2 L_1^2}}\right),$$

(2)

where the radius of the curvature is $R=\overline{OC_1}=1/|\kappa|$. This equation indicates that L_1 and L_2 can be identified given the hitch angle and curvature data set by applying nonlinear least square techniques. Further, considering two triangles, QOC_2 and QC_1P similar to [4],

$$\sin|\psi| = |\kappa| L_1 \cos|\psi| + |\kappa| L_2 . \tag{3}$$

Assuming a small hitch angle, (3) can be linearized,

$$|\psi| \approx |\kappa|\left(L_1 + L_2\right) . \tag{4}$$

3. Nonlinear Least Square Techniques

Since the model (2) is nonlinear, a nonlinear least square method is applied to determine car-trailer system parameters, L_1 and L_2. Using (2), the Jacobian is then,

$$J = \begin{bmatrix} \dfrac{\partial|\psi|}{\partial L_1} & \dfrac{\partial|\psi|}{\partial L_2} \end{bmatrix} = \begin{bmatrix} \dfrac{|\kappa|}{A} - \dfrac{|\kappa|^3 L_1 L_2}{A\sqrt{A-(\kappa L_2)^2}} & \dfrac{|\kappa|}{\sqrt{A-(\kappa L_2)^2}} \end{bmatrix}, \tag{5}$$

where $A=1+(\kappa L_1)^2$. Car-trailer system parameters, $\beta=[L_1 \quad L_2]^T$, are iteratively estimated given a measured hitch angle and curvature data set, $[\psi_{measured}, \kappa_{measured}]$ by,

$$\beta^{k+1} = \beta^k + \Delta\beta , \tag{6}$$

where k indicates the number of iteration, and

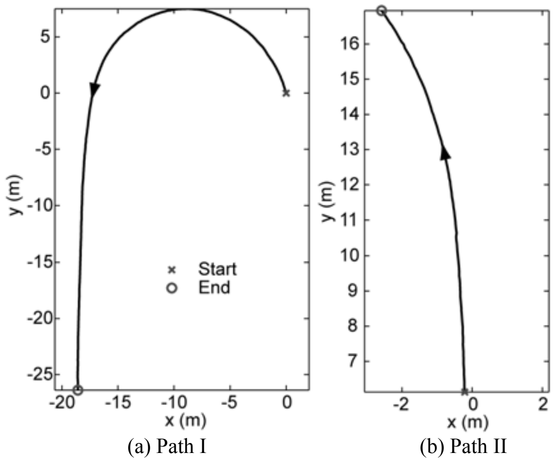
$$\begin{aligned} &\Delta\beta = \left(J^T J\right)^{-1} J^T \Delta\psi \\ &\Delta\psi = \psi_{measured} - \hat{\psi} \\ &\hat{\psi} = \tan^{-1}\left(|\kappa_{measured}| L_1\right) + \sin^{-1}\left(\frac{|\kappa_{measured}| L_2}{\sqrt{1+|\kappa_{measured}|^2 L_1^2}}\right) . \end{aligned} \tag{7}$$

The updates are continued until the changes, $\Delta\beta$, reach a properly chosen small criterion, ε,

$$\|\Delta\beta\| < \varepsilon . \tag{8}$$

4. Experimental Results and Discussion

The real car-trailer system is driven for short distances with varying curvatures (Paths I and II, Fig. 2) to identify trailer parameters. Path curvatures are estimated using a steering map and wheel odometry, respectively. Wheel odometry is based on quadrature encoders installed on the rear wheels. Further, the steering map is established to describe a relation between steering wheel rotation and path curvature through experiments. Hitch angles are measured using an angle sensor mounted on a hitch coupler.



(a) Path I (b) Path II

Fig. 2. Vehicle trajectory paths.

Table 1 summarizes parameter identification results applying 1) steering map, 2) raw odometry signals, and 3) filtered odometry signals, which illustrates performance of the algorithm. These results show that odometry based curvature estimation provides better performance. For Path I, unfiltered odometry data produces the best result whereas filtered odometry data provides the best result for Path II. Most importantly, better results are obtained when Path I is applied since it provides more sufficient data sets compared to Path II. These results also verify that system parameters can be identified using relatively inexpensive sensors with modest or higher accuracy. Further, future work will consider sensor noises and road disturbances to improve accuracy in system identification.

5. Conclusions

Car-trailer system parameters are experimentally determined applying nonlinear least square techniques. The proposed approach is simple and requires inexpensive sensors. Path curvatures are estimated by using odometry signals and a steering map, respectively. Experimental results from two different paths show that the proposed method can identify system parameters using relatively inexpensive sensors with modest or higher accuracy. These system parameters can thus be applied to a motion controller.

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References

[1] D. J. Bennett, and J. M. Hollerbach, "Identifying the kinematics of robots and their tasks," *Proceedings. 1989 IEEE International Conference on Robotics and Automation (Cat. No.89CH2750-8).* pp. 580-6.

[2] Y. Dayong, S. Xiwei, and W. Yu, "Kinematic calibration of parallel robots based on total least squares algorithm," *Proceedings of the 2007 IEEE International Conference on Mechatronics and Automation.* pp. 789-94.

[3] C. Pradalier, and K. Usher, "Robust trajectory tracking for a reversing tractor trailer," *Journal of Field Robotics,* vol. 25, no. 6-7, pp. 378-99, 2008.

[4] A. D. Jayakaran, "Enhanced trailer backing," Mechanical and Aerospace Engineering, M.S. Thesis, University of Florida, Gainesville, FL, 2004.

Table 1. Experimental results; Measured values: L_1=1.24 *m*, L_2=2.48 *m*

No	Path	Curvature estimation method	L_1 (m)	L_2 (m)	L_1 % error	L_2 % error
1		Steering map	1.296	2.563	3.7	3.3
2	Path I	Odometry w/ filter	1.307	2.580	4.5	4.0
3		Odometry w/o filter	1.242	2.450	-0.7	-1.2
4		Steering map	1.014	2.022	-18.9	-18.5
5	Path II	Odometry w/ filter	1.133	2.257	-9.4	-9.0
6		Odometry w/o filter	0.955	1.900	-23.6	-23.4