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Wheel/Rail Noise due to Nonlinear Effects and Parametric Excitation

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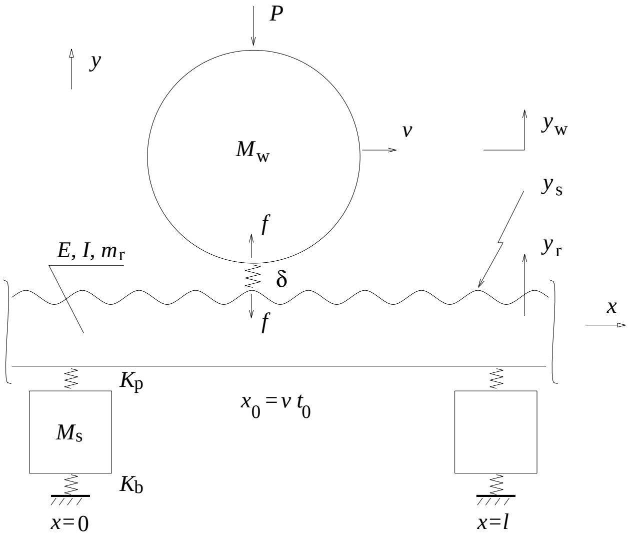
1. Introduction

The most comprehensive and widely used wheel/rail noise generation model is that developed by Remington and Thompson. It is linear, time-invariant, and performed in the frequency domain. They consider roughness only as an excitation mechanism. Alternative excitation mechanisms include nonlineareties and parametric excitation due to changing rail and contact receptance with position. In exciting the whole wheel/rail system, a broad insight into force generation would be most helpful in improving noise abatement.

2. Theory

A realistic rail model should include discrete supports. Two distinct properties are associated with the discrete supports: the sleeper-passing frequency f_s, and the pinned-pinned frequency f_{pp}. The wheels pass the sleepers with the sleeper-passing frequency $f_s = v/l$, where v is the speed and l the sleeper distances. At the pinned-pinned frequency f_{pp}, the bending wavelength of the rail is precisely two sleepers distant $\lambda_{pp} = 2l$, with nodes at the sleeper positions.

A wheel with mass M_w and preload P rolls with forward velocity v over the rail (see figure below). The rail has mass m_r per unit length, bending stiffness EI and a corrugation $y_s = r_0 \sin(2\pi/\lambda_0)x$ on the running surface. The coordinate axis for x points in the forward running direction, and for y upwards in the vertical direction. A perfectly smooth rail implies that $y_s = 0$ µm. The rail is periodically supported at discrete points with spacings l. Each support consists of a spring-mass-spring combination: K_p denotes pad stiffness with loss factor η_p, M_s sleeper mass and K_b ballast stiffness with loss factor η_b. The compression in the wheel-rail contact region, $\delta_{lin} = y_r + y_s - y_w$, is a function of vertical rail, y_r, and wheel, y_w, deflections plus the deviation from a perfect running surface, y_s. The contact force f, caused by the compression, excites the wheel (upwards) and the rail (downwards).



The vertical rail deflection, $y_r(x_0, t_0)$, at the moving point, $x_0 = vt_0$, (under the wheel) is a convolution of the rail Green's function, $g(x, x_0; t, t_0)$, and the contact force $f(x,t)$,

$$y_r(x_0, t_0) = \iint g(x, x_0; t, t_0)\, f(x,t)\, \mathrm{d}x\, \mathrm{d}t.$$

After discretisation, and using that the contact force moves forward, $f(x,t) = f(t)\delta(x - vt)$, the integration can be performed numerically. (See [1] for details.)

The force, known in the past, must be found by iterative improvements for the very last time step. The compression of the wheel/rail contact $\delta_{lin} = y_r + y_s - y_w$ a function of the contact force. Being unknown at the last time step, it can be approximated first with its most recent value. According to Hertz, the compression distance is a nonlinear function of the contact force f,

$$\delta_H = \left[\frac{2 f (1-\nu)}{G \sqrt{R_a}} \right]^{2/3} \alpha_\delta .$$

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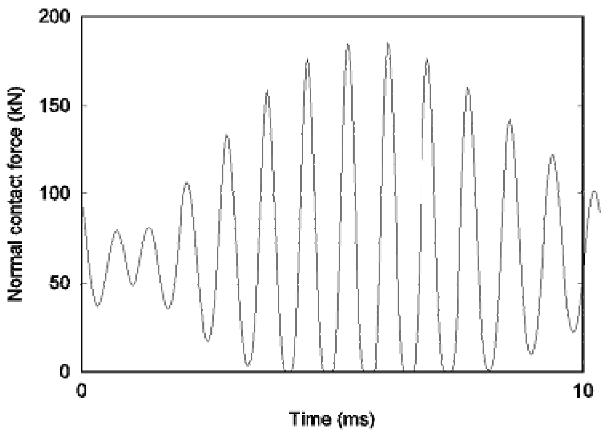
Of course, $f \geq 0$ always; $f = 0$ implies loss of wheel-rail contact, $\delta_{\text{lin}} \leq 0$. G is here the shear modulus, ν Poisson's ratio and R_{a} one of the radii of curvature at the contact point. The function α_{δ} depends on the elliptical shape of the contact area. Now, solving $\delta_{\text{lin}} - \delta_{\text{H}}(f) = 0$ iteratively, yields the Hertzian contact force f for the last time step.

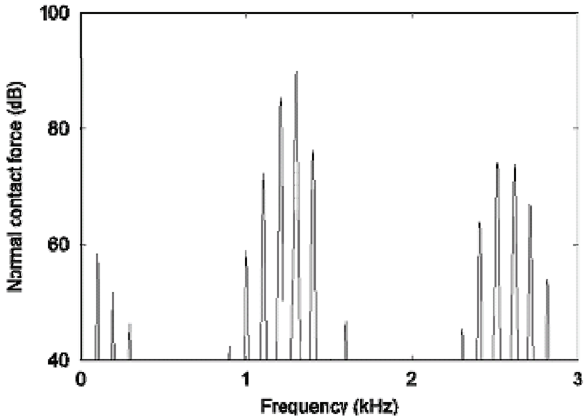
3. Numerical Results

The simulation assumes parameters according to a standard European high-speed wheel-rail system. (See [1] for details.) The speed $v = 60$ m/s. The corrugation wavelength $\lambda_0 = 4.60$ cm corresponds to the anti-resonance of the pinned-pinned mode, located at 1300 Hz, a "worst case", but not an unrealistic situation. Sleeper spacing $l = 0.6$ m. The corrugation amplitude $r_0 = 25$ μm.

Because of the discrete supports, the contact force amplitude varies extensively through the sleeper spans (see figure below). In the middle of a span, contact loss occurs, causing the contact force to vanish, allowed by the nonlinear contact model.



The contact force spectrum (see figure below) has three main regions. First, at low frequencies, the discrete sleeper supports cause an excitation at the sleeper-passing frequency $f_{\text{s}} = v / l = 100$ Hz, plus at its harmonics. Second, corrugation causes an excitation peak at the corrugation-passing frequency $f_0 = v / \lambda_0 = 1300$ Hz, accompanied by smaller peaks 100 Hz apart due to modulation by the sleeper-passing frequency. Third, contact loss and parametric excitation within the contact region, due to shape variations of the contact ellipse, cause an excitation of twice the corrugation-passing frequency $2 f_0 = 2600$ Hz, which is also modulated by sleeper passings.



4. Conclusions

Discrete rail sleeper supports cause the wheel to "see" a varying receptance downwards as it traverses a sleeper span. Alternative excitation mechanisms beside roughness include nonlineareties and parametric excitation due to changing rail and contact receptance with position. It is necessary to account for these effects to describe the response at low frequencies, determined by the sleeper-passing frequency, around 100 Hz, and around the pinned-pinned frequency, around 1 kHz— in particular if the rail is very smooth or has a corrugation with a wavelength corresponding to the pinned-pinned frequency. If the rail has a corrugation it may also be necessary to include the nonlinear contact spring, since loss of contact occurs for great corrugation amplitudes.

Acknowledgement

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References

[1] Anders Nordborg. Wheel/rail noise generation due to nonlinear effects and parametric excitation. Journal of the Acoustical Society of America, 111(4): 1772–1781, April 2002.