Network based Control Systems with Uncertain Time Delay using Model Matching and Pade Approximation

Hyun Cheol Cho[1], Kwan Hyung Kim [2], Young Jin Lee [3], and Kwon Soon Lee[4]

[1] Assistant Professor, School of Electrical & Electronic Eng., Ulsan College, Ulsan
[2] Assistant Professor, Department of Computer Eng., Tongmyong Univ., Busan
[3] Associate Professor, Dept of Electrical Eng., Sachon Ploytechnic College, Sachon
[4] Professor, Department of Electrical Eng., Dong-A Univ., Busan

Abstract

This paper presents a control design for networked control systems (NCS) with uncertain time delay using model matching. The dynamics of the time delay are approximated through the Pade linearization and the uncertain delay term is recursively estimated by the recursive least square (LS) algorithm. Computer simulation illustrates that the proposed control compares favorably with a recently published control approach.

1. Introduction

Networked control systems (NCS) are feedback control systems where the plant is connected to the controller through a communication network. NCS are popular in industry because they can eliminate unnecessary wiring and provide flexible system configurations [1], [1]. However, a challenging issue in NCS is the effect of the delay which usually occurs between the remote controller and the targeted plant. Time delay often results in deterioration in control performance and can even cause instability. Thus, it is necessary to consider the time delay in the design of NCSs to guarantee satisfactory performance. Recently, engineers have actively investigated complicated control solutions to this problem [2].

Recent publications include multiple tutorial articles that discuss the stability and control of linear time-invariant NCS. The simplest design approach is the use of an augmented model to determine the control parameters. In [3], the authors proposed a control approach using an augmented discrete-time system model for periodic delay and extended their approach to NCS with non-identical time delay in [4].

Several stochastic approaches have been proposed for the more realistic case of a randomly varying time delay. In [5] and [6], a first-in-first-out queuing scheme was applied to construct a stochastic predictor of the future state. The authors used a predictor to statistically predict the arrival rate for a queue model with predefined probability density. Optimal stochastic control was applied in [7] to overcome the adverse effects of a deterministic time delay on NCS with random output. In [8], the authors used stochastic control of random NCS with two time delays (sensor and actuator) modeled as homogeneous Markov chains. They used the simplifying assumption of statistically stationary time delays, which is rarely valid in practice.

Proportional-Integral (PI) control was designed in [9] for time-delay systems with time-varying delay whose parameters are adaptively updated. However, the controller design against uncertain time delay is difficult to obtain explicitly for practical application. More recently, complicated NCSs have been considered using sophisticated system theory such as robust control, system perturbation theory, etc [10], [11]. In [12], the authors treated a nonlinear MIMO NCS for which Lyapunov perturbation stability was applied to design nonlinear control system. State feedback control was utilized for linear continuous NCS in [13] and [14], respectively. They expressed the NCS with state-space representation and utilized continuous linear system theory for deriving the control system. Alternatively, unlike typical control design, sampling time scheduling where the sampling interval is arbitrarily changed online to maintain a stable NCS was proposed in [15] and applied to multi-dimensional NCSs in [16].

In much of the published NCS research, authors usually dealt with NCSs characterized by linear time-invariant models, fixed time delay, or deterministic behavior. While the proposed methodologies can be successfully implemented in simulated experiments, significant errors are unavoidable in practice due to the nonlinear and random nature of NCSs. Thus, it is essential to change the controller framework to cope with a more realistic system environment. This often requires robustifying NCSs with respect to modeling errors and disturbances [1], [1]. However, it is hard to analytically model NCS dynamics because changes in the NCS environment are not completely predictable.

In this paper, we consider uncertain time delay which varies around a nominal value. This assumption is reasonable in practice. We first approximate the time delay by a rational first order function via the Pade method. We then use model matching to construct a controller for the NCS. To estimate the perturbed time delay, we apply a recursive LS algorithm. The proposed control design is explicitly applicable in practical implementation. We demonstrate the advantages of our control approach compared with the control of [1].

This paper is organized as follows. In Section 2, we describe a NCS with uncertain time delays. We propose model matching based control design for the NCS in Section 3. We derive the recursive LS algorithm to estimate the perturbed time delays in Section 4. A simulation example is given in Section 5 and conclusions are given in Section 6.

2. NCS with time delays

We consider a SISO NCS with two time delays: a control delay τ_1 and an observation delay τ_2 (see Fig. 1). The transfer function of the closed-loop system is expressed by

$$G(s)=\frac{Y(s)}{R(s)}=\frac{G_0(s)C(s)e^{-\tau_1 s}}{1+G_0(s)C(s)e^{-\tau_1 s}e^{-\tau_2 s}} \qquad (1)$$

where $R(s)$ is the reference input, $Y(s)$ is the system output, and $G_0(s) = N_0(s)/D_0(s)$ and $C(s) = N_C(s)/D_C(s)$ are transfer functions of the nominal plant and the controller, respectively. We assume that the transfer function of the plant is strictly proper. The time delays are expressed using a first order Pade approximation [17] as

$$e^{-\tau_1 s}\approx G_1(s)=\frac{N_1(s)}{D_1(s)}=\frac{s-2T_1^{-1}}{s+2T_1^{-1}} \qquad (2)$$

$$e^{-\tau_2 s}\approx G_2(s)=\frac{N_2(s)}{D_2(s)}=\frac{s-2T_2^{-1}}{s+2T_2^{-1}} \qquad (3)$$

where T_1, $T_2 > 0$. Substituting (2) and (3) into (1), we obtain

$$\begin{aligned}G(s)&=\frac{G_0(s)G_1(s)C(s)}{1+G_0(s)G_1(s)G_2(s)C(s)}\\&=\frac{D_2(s)N_0(s)N_1(s)N_C(s)}{D_0(s)D_1(s)D_2(s)D_C(s)+N_0(s)N_1(s)N_2(s)N_C(s)}\end{aligned} \qquad (4)$$

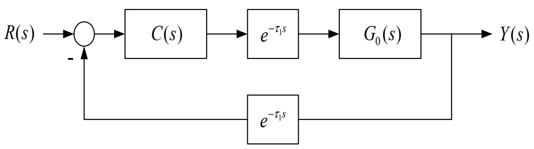

Fig. 1. A block diagram of time-delayed control systems.

3. Model matching based control design for an uncertain NCS

We first derive a control design for the nominal NCS using model matching. The control objective is to construct a controller transfer function $C(s)$ such that the dynamics of the overall system $G(s)$ follow a specified model:

$$G_M(s)=\frac{N_M(s)}{D_M(s)} \qquad (5)$$

that is, $G(s) \to G_M(s)$. Equating to the transfer function of (4) yields the controller

$$C(s)=\frac{N_C(s)}{D_C(s)}=\frac{G_M(s)}{G_0(s)G_1(s)\left(1-G_2(s)G_M(s)\right)} \qquad (6)$$

In terms of the numerator and denominator polynomials, we have the controller transfer function

$$C(s)=\frac{N_C(s)}{D_C(s)}=\frac{N_M(s)D_0(s)D_1(s)D_2(s)}{N_0(s)N_1(s)\left(D_M(s)D_2(s)-N_2(s)N_M(s)\right)} \qquad (7)$$

Next, we design the NCS control with randomly varying but bounded uncertain time delays. This occurs in practical implementation due to change of network environment, system uncertainty, etc [7]. We express the time delays as the sum of nominal and perturbed terms as

$$\tau_1(t)=\tau_1^*+\Delta\tau_1(t)\,,\quad \tau_2(t)=\tau_2^*+\Delta\tau_2(t) \qquad (8)$$

where * and Δ denote nominal and perturbed variables respectively. We similarly express each approximated transfer function for the perturbation as

$$G_1(s)=\frac{N_1(s)}{D_1(s)}=\frac{N_1^*(s)+\Delta N_1(s)}{D_1^*(s)+\Delta D_1(s)} \qquad (9)$$

$$G_2(s)=\frac{N_2(s)}{D_2(s)}=\frac{N_2^*(s)+\Delta N_2(s)}{D_2^*(s)+\Delta D_2(s)} \qquad (10)$$

Substituting (9) and (10) into (4), we obtain the perturbed transfer function of the overall system

$$G_\Delta(s)=\frac{\left(D_2^*+\Delta D_2\right)N_0\left(N_1^*+\Delta N_1\right)N_C}{D_0\left(D_1^*+\Delta D_1\right)\left(D_2^*+\Delta D_2\right)D_C+N_0\left(N_1^*+\Delta N_1\right)\left(N_2^*+\Delta N_2\right)N_C} \qquad (11)$$

Similarly, we obtain the transfer function of the controller for the perturbed plant

$$C_\Delta(s)=\frac{N_\Delta(s)}{D_\Delta(s)}=\frac{D_0\,N_M\left(D_1^*+\Delta D_1\right)\left(D_2^*+\Delta D_2\right)}{N_0^*\left(N_1^*+\Delta N_1\right)\left[D_M\left(D_2^*+\Delta D_2\right)-N_M\left(N_2^*+\Delta N_2\right)\right]} \qquad (12)$$

We rewrite (12) in terms of nominal and perturbation terms, i.e.

$$C_\Delta(s)=\frac{N_{C_\Delta}^*(s)+N_{C_\Delta}^p(s)}{D_{C_\Delta}^*(s)+D_{C_\Delta}^p(s)} \qquad (13)$$

where

$$N_{C_\Delta}^*(s)=D_0\,N_M D_1^* D_2^* \qquad (14)$$

$$N_{C_\Delta}^p(s)=D_0\,N_M\left(D_1^*\Delta D_2+D_2^*\Delta D_1+\Delta D_1\Delta D_2\right) \qquad (15)$$

and

$$D_{C_\Delta}^*=\left(D_M-N_M\right)\left(N_0^*N_1^*D_2^*\right) \qquad (16)$$

$$\begin{aligned}D_{C_\Delta}^p&=D_M\left(N_0^*N_1^*\Delta D_2+N_0^*\Delta N_1 D_2^*+N_0^*\Delta N_1\Delta D_2\right)\\&\quad-N_M\left(N_0^*N_1^*\Delta N_2+N_0^*\Delta N_1 N_2^*+N_0^*\Delta N_1\Delta N_2\right)\end{aligned} \qquad (17)$$

The nominal terms in (13) are given as fixed values in the design procedure, but the perturbed values are estimated online in real-time implementation.

4. Online parameter estimation using recursive LS estimation

We apply recursive LS estimation for the perturbation parameters in (13). First, we alternatively express the input-output model in Fig. 1 as

$$A(s)Y(s)=B(s)U(s) \qquad (18)$$

where

$$A(s) = D_0(s)D_C(s)\left(D_1^*(s) + \Delta D_1\right)\left(D_2^*(s) + \Delta D_2\right) + N_0(s)N_C(s)\left(N_1^*(s) + \Delta N_1\right)\left(N_2^*(s) + \Delta N_2\right) \quad (19)$$

and

$$B(s) = D_C(s)N_0(s)\left(D_2^*(s) + \Delta D_2\right)\left(N_1^*(s) + \Delta N_1\right) \quad (20)$$

Similarly, the two polynomials are expressed with separate nominal and perturbation terms as

$$A(s) = A^*(s) + A^p(s), \quad B(s) = B^*(s) + B^p(s) \quad (21)$$

where

$$A^*(s) = D_C(s)N_0(s)D_2^*(s)N_1^*(s) \quad (22)$$

$$A^p(s) = D_C(s)N_0(s)D_2^*(s)\Delta N_1(s) + N_1^*(s)\Delta D_2(s) + \Delta D_2(s)\Delta N_C(s) \quad (23)$$

$$B^*(s) = D_C(s)N_0(s)D_1^*(s)D_2^*(s) + N_0(s)N_C(s)N_1^*(s)N_1^*(s) \quad (24)$$

$$B^p(s) = D_C(s)D_0(s)\left(D_1^*(s)\Delta D_2(s) + \Delta D_1(s)D_2^*(s) + \Delta D_1(s)\Delta D_2(s)\right) + N_0(s)N_C(s)\left(N_1^*(s)\Delta N_2(s) + \Delta N_1(s)N_2^*(s) + \Delta N_1(s)\Delta N_2(s)\right) \quad (25)$$

For simplicity, let the two polynomials of (21) be

$$A(s) = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0 \quad (26)$$

$$B(s) = \beta_{n-1}s^{n-1} + \beta_{n-2}s^{n-2} + \cdots + \beta_1 s + \beta_0 \quad (27)$$

where

$$\alpha_i = \alpha_i^* + \Delta\alpha_i, \quad \beta_i = \beta_i^* + \Delta\beta_i, \quad i = 0, \cdots, n-1$$

Based on (26) and (27) we have

$$Y(s)\left(s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0\right) = U(s)\left(\beta_{n-1}s^{n-1} + \beta_{n-2}s^{n-2} + \cdots + \beta_1 s^1 + \beta_0\right) \quad (28)$$

We express a current output from (28) as

$$y = -\frac{1}{\alpha_0}\left\{\begin{array}{l}\left(\alpha_1 Dy + \alpha_2 D^2 y + \cdots + \alpha_{n-1}D^{n-1} + D^n y\right) \\ +\left(\beta_{n-1}D^{n-1}u + \cdots + \beta_1 Du + \beta_0 u\right)\end{array}\right\} \quad (29)$$

where $y = y(t)$, $u = u(t)$, and $D = d/dt$. In vector form, equation (29) is written as

$$y = \theta^T w \quad (30)$$

where

$$\theta = \left[\theta_{\alpha_n} \ \theta_{\alpha_{n-1}} \ \cdots \ \theta_{\alpha_1} \ \theta_{\beta_{n-1}} \ \theta_{\beta_{n-2}} \ \cdots \ \theta_{\beta_1} \ \theta_{\beta_0}\right]^T = \frac{1}{\alpha_0}\left[-1 \ -\alpha_{n-1} \ \cdots \ -\alpha_1 \ \beta_{n-1} \ \beta_{n-2} \ \cdots \ \beta_1 \ 1\right]^T \quad (31)$$

and

$$w = \left[D^n y \ D^{n-1}y \ \cdots \ Dy \ D^{n-1}u \ D^{n-2}u \ \cdots \ Du \ u\right]^T \quad (32)$$

For recursive estimation generally and digital control implementation, we need the discretized model

$$y(k) = h^T(k)\theta + v(k) \quad (33)$$

where v is a zero mean Gaussian random variable and the discrete measurement vector is given by

$$h = \left[y(k-n), y(k-n+1), \cdots, y(k-1), u(k-n+1), u(k-n+2), \cdots, u(k)\right]^T \quad (34)$$

The parameter estimate $\hat{\theta}$ is recursively updated by

$$\hat{\theta}(k+1) = \hat{\theta}(k) + K(k)\left[y(k) - h^T(k)\hat{\theta}(k)\right] \quad (35)$$

where

$$K(k+1) = P(k)h(k)\left[h^T(k)P(k)h(k) + \frac{1}{w(k)}\right]^{-1} \quad (36)$$

and $P(k)$ is the error covariance matrix. The error covariance is updated with the rule

$$P(k+1) = \left[I - K(k)h^T(k)\right]P(k) \quad (37)$$

5. Numerical example

We adapt the time-delayed system of [2] for our simulation example. The nominal plant model with no delay is given by

$$G_0 = \frac{0.5}{s(s+0.1)} \quad (38)$$

The system is a second-order model with marginally stable dynamics. The nominal time delays are $\tau_1 = \tau_2 = 2.7\times10^{-4}$ sec, thus we have

$$G_1(s) = G_2(s) = \frac{s - 0.74\times10^4}{s + 0.74\times10^4} \quad (39)$$

and the controller transfer function

$$C(s) = \frac{N_m s(s+0.1)\left(s+0.74\times10^4\right)^2}{0.1\left(s-0.74\times10^4\right)\left[D_m\left(s+0.74\times10^4\right) - N_m\left(s-0.74\times10^4\right)\right]} \quad (40)$$

We select the desired model transfer function as

$$G_M(s) = \frac{N_m}{D_m} = \frac{1}{s+10} \quad (41)$$

Substituting (41) into (40), we finally obtain the nominal controller

$$C(s) = \frac{s(s+0.1)\left(s+0.74\times10^4\right)^2}{0.1\left(s-0.74\times10^4\right)\left[(s+10)\left(s+0.74\times10^4\right) - \left(s-0.74\times10^4\right)\right]} \quad (42)$$

Next, we derive the controller for the perturbed plant with uncertain time delays given by

$$G_1(s) = \frac{s - 0.74\times10^4 + \Delta\tau_1}{s + 0.74\times10^4 + \Delta\tau_1} \quad (43)$$

and

$$G_2(s) = \frac{s - 0.74\times10^4 + \Delta\tau_2}{s + 0.74\times10^4 + \Delta\tau_2} \quad (44)$$

respectively where $\Delta\tau_1$ and $\Delta\tau_2$ are zero-mean Gaussian random variables with time-varying variances. Similarly, substituting (43) and (44) to (42), we have a controller for the perturbed model as

$$C(s) = \frac{s(s+0.1)\left(s+0.74\times10^4+\Delta\tau_1\right)\left(s+0.74\times10^4+\Delta\tau_2\right)^2}{0.1\left(s+0.74\times10^4+\Delta\tau_2\right)\left(s-0.74\times10^4+\Delta\tau_1\right)\left[(s+10)\left(s+0.74\times10^4+\Delta\tau_2\right) - \left(s-0.74\times10^4+\Delta\tau_2\right)\right]} \quad (45)$$

The transfer function of the plant with this controller is expressed as

$$\frac{Y(s)}{U(s)}=\frac{0.1(s-0.74\times10^4+\Delta\tau_1)[(s+10)(s+0.74\times10^4+\Delta\tau_2)-(s-0.74\times10^4+\Delta\tau_2)]}{s(s+1)(s+0.74\times10^4+\Delta\tau_1)[(s+10)(s+0.74\times10^4+\Delta\tau_2)-(s-0.74\times10^4+\Delta\tau_2)+(s-0.74\times10^4+\Delta\tau_2)]} \quad (46)$$

Using this formula, we estimate the uncertain parameters $\Delta\tau_1$ and $\Delta\tau_2$ using the recursive LS algorithm of Section 4. For numerical simulation, the perturbed time delays are realized as nonstationary Gaussian random distributions, i.e. $\Delta\tau_1 \sim N(0,\sigma_1)$ and $\Delta\tau_2 \sim N(0,\sigma_2)$ where random variances σ_1 and $\sigma_2 \in (0,3]$. We simulated the system using MATLAB© to evaluate our control method.

Fig. 2 gives the step response of the proposed control with that of the PID control of [1]. For the design given in [1], PID control gives a large overshoot and a highly oscillatory step response with a large settling time. By contrast, our control gives a smaller overshoot with a settling time of about 25 sec. Based on our computer simulations, we conclude that our proposed control achieves its design objectives and outperforms the traditional PID approach.

6. Conclusion

We propose a novel control design for NCS with uncertain time-delays by using model matching. Time delays are approximated by the Pade linearization and uncertain delays are estimated online via a LS algorithm. Computer simulation shows that the proposed control outperforms PID control. Future work includes extension of the proposed control design to multi-variable systems.

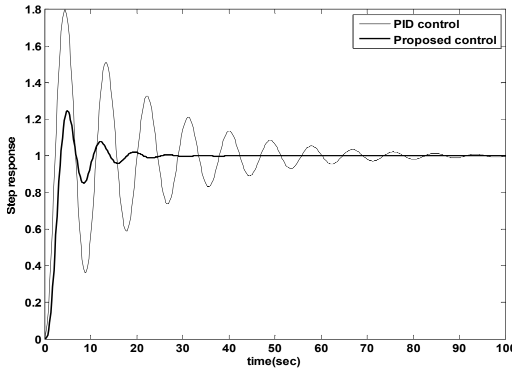

Fig. 2. System responses for PID and proposed controls.

References

[1] T. C. Yang, "Networked control systems: a brief survey," *IEEE Proc.-Control Theory and Applications.*, vol. 153, no. 4, pp. 403-412, 2006.

[2] Y. Jianyong, Y. Shimin, and W. Haiqing, "Survey on the performance analysis of networked control systems," *IEEE Int. Conf. on System, Man and Cybernetics*, pp. 5068-5073, 2004.

[3] Y. Tipsuwan and M.-Y. Chow, "Control methodologies in networked control systems," *Control Engineering Practice*, vol. 11, pp. 1099-1111, 2003.

[4] Y. Halevi and A. Ray, "Integrated communication and control systems: Part I - analysis," *J. of Dynamic Systems, Measurement and Control*, vol. 110, pp. 367-373, 1988.

[5] L.-W. Liou and A. Ray, "Integrated communication and control systems: Part III - nonidentical sensor and controller sampling," *J. of Dynamic Systems, Measurement and Control*, vol. 112, pp. 357-364, 1990.

[6] R. Luck and A. Ray, "Experimental verification of a delay compensation algorithm for integrated communication and control systems," *Int. J. of Control*, vol. 59, no. 6, pp. 1357-1372, 1994.

[7] H. Chan and U. Ozguner, "Closed-loop control of systems over a communications network with queues," *Int. J. of Control*, vol. 62, no. 3, pp. 493-510, 1995.

[8] J. Nilsson, *Real-time control systems with delays*, Ph.D. dissertation, Lund Institute of Technology, 1998.

[9] L. Zhang, Y. Shi, T. Chen, and B. Huang, A new method for stabilization of networked control systems with random delays," *IEEE Trans. on Automatic Control*, vol. 50, no. 8, pp. 1177-1181, 2005.

[10] N. B. Almutairi, M.Y. Chow, and Y. Tipsuwan, "Network-based controlled DC motor with fuzzy compensation," *The 27th annual conf. of the IEEE Industrial Electronics society*, vol. 3, pp. 1844-1849, 2001.

[11] G. C. Walsh, O. Beldiman, and L. Bushnell, "Error encoding algorithms for networked control systems," *Proc. of the IEEE Conf. on Decision and Control*, vol. 5, pp. 4933-4938, 1996.

[12] F. Goktas, *Distributed control of systems over communication networks*, Ph.D. dissertation, University of Pennsylvania, 2000.

[13] G. C. Walsh, O. Beldiman, and L. Bushnell, "Asymptotic behavior of nonlinear networked control systems," *IEEE Trans. on Automatic Control*, vol. 46, no. 7, pp. 1093-1097, 2001.

[14] L. A. Montestruque and P. J. Antsaklis, "On the model-based control of networked systems," *Automatica*, vol. 39, pp. 1837-1843, 2003.

[15] D. Yue, Q.-L. Han, and C. Peng, "State feedback controller design of networked control systems," *IEEE Trans. on Circuits and Systems*, vol. 51, no. 11, pp. 640-644, 2004.

[16] S. H. Hong, "Scheduling algorithm of data sampling times in the integrated communication and control systems," *IEEE Trans. on Control Systems Technology*, vol. 3, no. 2, pp. 225-230, 1995.

[17] S. H. Hong and W.-H. Kim, "Bandwidth allocation scheme in CAN protocol," *IEEE Proc.-Control Theory and Applications*, vol. 147, no. 1, pp. 37-44, 2000.

[18] J. G. Truxal, *Automatic feedback control systems synthesis*, McGraw-Hill, New York, 1955.