Task Schedule Modeling using a Timed Marked Graph

Cheul Woo Ro, Yang Cao, Yun Xiang Ye, Wei Xu
Dep. of Computer & Info. Eng.
Silla University, Busan, KOREA

Abstract - Task scheduling is an integral part of parallel and distributed computing. Extensive research has been conducted in this area leading to significant theoretical and practical results. Stochastic reward nets (SRN) is an extension of stochastic Petri nets and provides compact modeling facilities for system analysis. In this paper, we address task scheduling model using extended timed marked graph, which is a special case of SRNs. And we analyze this model by giving reward measures in SRN.

1. Introduction

Marked graphs are special cases of Petri nets for modeling asynchronous concurrent systems. The model of a timed marked graph (TMG) is obtained from a marked graph by adding durations to the events in the system[1,2]. In complex man-made environments discrete event dynamic systems are frequently encountered, and a timed marked graph is widely accepted as a convenient tool to describe systems of this kind[3].

Task scheduling is an integrated component of computing. With the emergence of Grid and ubiquitous computing, new challenges appear in task scheduling based on properties such as security, quality of service, and lack of central control within distributed administrative domains[1]. The design of a scheduling mechanism that can adaptive to different types of tasks and adjust the behavior and response of a system to meet certain performance requirements is a tedious and challenging problem[4].

In this paper, we consider two task scheduling models with different task processing sequences in discrete event dynamic system represented as extended timed marked graphs. And we analyze these two models by giving reward measures.

2. System Modeling

2.1 Timed Marked Graph

Time may be associated either with the Petri net places or transitions, or with both. The deterministic time association is a Petri net model extension enabling performance analysis using time relations. The uncertainty about the continuation of the transition firing being in a conflict needs to model time behavior in a stochastic way[5,6]. The model of TMG is obtained from a marked graph by adding durations to the events in the system. The timed marked graphs are delimited by the following definition. A timed marked graph is given by $\boldsymbol{TMG}=(\boldsymbol{P},\boldsymbol{T},\boldsymbol{F},\boldsymbol{W},\boldsymbol{M_0},\boldsymbol{\pi},\boldsymbol{\tau})$ [6] *where* $\boldsymbol{P}$ *is a set of places,* $\boldsymbol{T}$ *is a set of transitions,* $\boldsymbol{F} \subseteq (\boldsymbol{P} \times \boldsymbol{T}) \cup (\boldsymbol{T} \times \boldsymbol{P})$ *is a flow relation,* $\boldsymbol{W}: \boldsymbol{F} \rightarrow \{1, 2, ...\}$ is a weight function, $\boldsymbol{M_0}$ is an initial marking, and $\boldsymbol{\pi}$ is the place delay function $:\boldsymbol{P} \rightarrow \boldsymbol{R}^+$ (the set of non-negative real numbers), $\boldsymbol{\tau}$ is the transition firing time function $\boldsymbol{\tau}$: $\boldsymbol{T} \rightarrow \boldsymbol{R}^+$.

Intuitively, the dynamic operation of the TMG is as follows. When a transition receives a token on all its incoming links, it performs some internal computation and then sends one token along each of its outgoing links. It takes time that a token travel along a link[1].

Normally, the timed marked graphs have all arc weights equal to one and each place has one token. In this paper, we use an extended timed marked graph that some special places can have multiple tokens. The meaning of these places will be introduced in the next part.

2.2 SRN

SRN is an extension of stochastic Petri nets and provides compact modeling facilities for system analysis. SRN has the ability to allow extensive marking dependency. It also has one important feature of expressing complex enabling/disabling conditions through guard functions. To get the performance and reliability/availability measures of a system, appropriate reward rates associated with the markings are assigned to its SRN[7,8].

2.3 System Modeling

We built two task scheduling models using extended timed marked graph. The main parts of these two models are same except that task processing sequences are different with some control.

Task *i* has number of operations (i=1~3): $\boldsymbol{T_1}$ has three operations which are executed through transition $\boldsymbol{t_{11}}$, $\boldsymbol{t_{12}}$ and $\boldsymbol{t_{13}}$ with server $\boldsymbol{S_1}$ and $\boldsymbol{S_3}$. λ is the job arriving rate. For task 2-3, the interpretation is the same. Server *j* includes several task operations (j=1~3): S_1 has process parts operation of Task$_1$ and Task$_2$ which are executed through transition t_{11} and t_{21}. For server 2-3, the interpretation is the same. We can depict the activities of tasks and servers as the cycles:

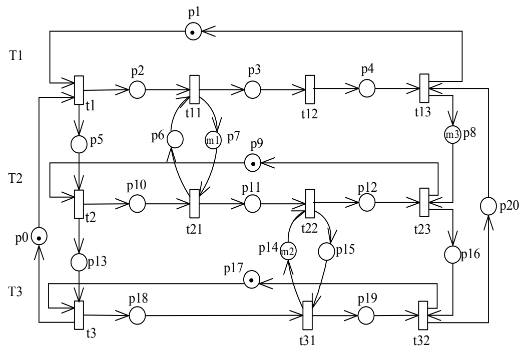
T_1: $p_1 \rightarrow p_2 \rightarrow p_3 \rightarrow p_4 \rightarrow p_1$
T_2: $p_9 \rightarrow p_{10} \rightarrow p_{11} \rightarrow p_{12} \rightarrow p_9$
T_3:$p_{17} \rightarrow p_{18} \rightarrow p_{19} \rightarrow p_{17}$
S_1: $p_6 \rightarrow p_7 \rightarrow p_6$
S_2: $p_{14} \rightarrow p_{15} \rightarrow p_{14}$
S_3: $p_8 \rightarrow p_{16} \rightarrow p_{20} \rightarrow p_8$

We extended the timed marked graph by putting multiple tokens of m1, m2, and m3 in p_7, p_{14}, and p_8 respectively, which means that each server has the capability to handle multiple task operations.

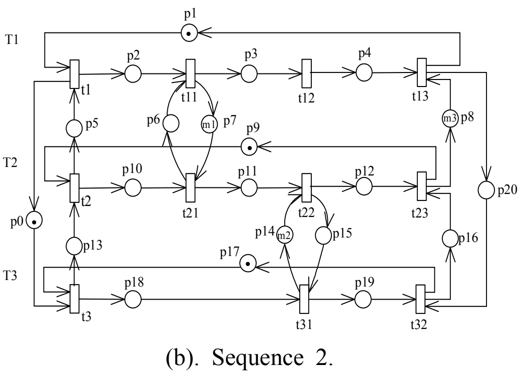
Figure 1 shows two extended timed marked graph that describes task scheduling system consisting of three servers (M_1,M_2,M_3) and three tasks (J_1,J_2,J_3).

The cycle $p_0 \rightarrow p_5 \rightarrow p_{13} \rightarrow p_0$ in Figure 1(a) indicates that jobs executing sequence is $T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_1$.

The cycle $p_0 \rightarrow p_5 \rightarrow p_{13} \rightarrow p_0$ in Figure 1(b) indicates that jobs executing sequence is $T_2 \rightarrow T_1 \rightarrow T_3 \rightarrow T_2$.



(a). Sequence 1.



(b). Sequence 2.

Figure 1. TMG with Multiple Servers of sequence 1 &2.

3. Model Analysis

3.1 Measures of Interest

In order to obtain the interested measures numerically from the SRN model, underlying CTMC is generated and solved through the use of the software package SPNP. We assume that all transition firing rates in our SRN models are exponentially distributed, so we perform the steady-state analysis of the model we have constructed[7].

Task Processing Time (TPT)

We get the numerical results by giving some different input parameters f(λ, m1, m2, m3) to these two models. To calculate the processing time of task1, we use the following formula:

TPT$_1$=1/rate(t_{11})+1/rate(t_{12})+1/rate(t_{13}).

The formula for calculating TPT$_2$ and JPT$_3$ are given as the same way.

3.2 Numerical Results

Table 1 shows the tasks processing time on each server with different tasks arriving rateλ and different task processing sequences.

Table 1. Different TPTs of two models

T1→T2→T3→T1				
λ	m1-3	TPT-1	TPT-2	TPT-3
0.2	m1=3 m2=2 m3=4	118.82	118.82	79.21
0.4		101.69	101.69	67.80
0.6		96.35	96.35	64.24
0.8		93.75	93.75	62.50
1.0		92.21	92.21	61.48
2.0		89.17	89.17	59.45
4.0		87.67	87.67	58.44
6.0		87.17	87.17	58.11
T2→T1→T3→T2				
λ	m1-3	TPT-1	TPT-2	TPT-3
0.2	m1=3 m2=2 m3=4	119.55	119.55	79.70
0.4		102.98	102.98	68.65
0.6		98.17	98.17	65.45
0.8		95.99	95.99	63.99
1.0		94.76	94.76	63.17
2.0		92.56	92.56	61.70
4.0		91.60	91.60	61.07
6.0		91.31	91.31	60.88

4. Conclusion

In this paper, two task scheduling models have been constructed to get performance analysis using timed marked graph. We only focus on the task processing part of the model, and manually give the scheduling method.

There are still lots of work to do with that system, such as applying different scheduling algorithms and system performance optimization, etc. We will tackle on these situations in our future work and get more deep and complicated analysis.

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