EFFECTS OF AVERAGING AND COMPLIANCE ON NEWMARK-TYPE DEFORMATION ANALYSIS

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ABSTRACT

The performance of slopes during earthquake is often accessed in terms of permanent deformation. In the assessment of permanent deformation, Newmark-type rigid block analysis is widely used. Original Newmark-type block approach, however, assumes the potential sliding mass to be rigid, and has been criticized to be potentially unconservative. The paper reviews analytically the impact of this noncompliance assumption on computed permanent deformations. The results indicate that there is a simple criterion that can be used to determine the level of conservativeness of the rigid block approach in cases of gently-sloping slip surfaces and retaining walls.

Key Words: Seismic Slope Deformation, Newmark-Type Analysis, Compliance, Spatial Averaging

1 INTRODUCTION

The serviceability of a slope after an earthquake can be assessed with seismically-induced permanent deformations rather than the factor of safety given by a pseudo-static analysis. Newmark (1965) and Seed and Goodman (1964) used the analogy of a rigid block resting on an inclined plane, to propose a simple way for estimating a permanent displacement of the sliding mass due to earthquake shaking

The original Newmark method assumed the potential sliding mass to be rigid and the effect of compliance and spatial averaging were not taken into account. This assumption, however, has been criticized to be potentially unconservative. In this paper, the effect of spatial averaging and compliance in the decoupled analysis is analytically reviewed. The paper proposes a relatively simple criterion that can be used to determine the level of conservativeness of the rigid block approach.

2 SIMPLIFIED DECOUPLED PROCEDURE

Newmark (1965) used the analogy of a block resting on an inclined plane, to propose a convenient way for estimating a permanent displacement of the sliding mass due to earthquake shaking. It is assumed that the whole sliding mass moves as a single rigid body with resistance mobilized along the sliding surface as shown in Figure 1.

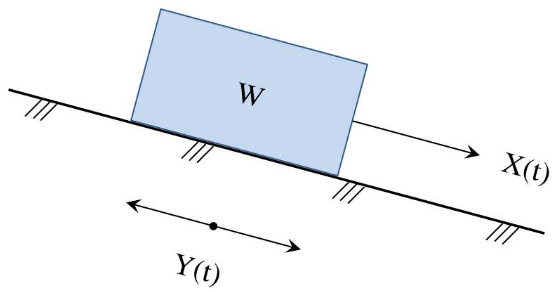

Figure 1. Estimation of Displacement: Rigid Block on a Moving Support

2.1 Average Seismic Coefficients

Earthquake-induced inertia forces alternate in direction many times in soil masses within a slope. It is this pulsating force, superimposed on the initial self-weight of soil masses, which disturbs the stability of a slope. It has been a serious interest among earthquake engineers to estimate the gross (overall) effects of earthquake shaking on the potential sliding masses of slopes (e.g., Seed and Martin 1966, Chopra 1967, Matasovic et al. 1998). The total lateral force acting on the potential sliding mass bounded by the slip surface at any particular instant is given by (e.g., Seed and Martin, 1966):

$$F = \sum m(y) \cdot \ddot{u}(y) \qquad (1)$$

where $m(y)$ is the mass of a slice and $\ddot{u}(y)$ is the corresponding horizontal acceleration of the slice at a depth of y. The lateral force may alternatively be expressed by a product of an average seismic

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coefficient k_{ave} and the total weight W of the sliding mass as:

$$F = k_{ave} \cdot W \qquad (2)$$

In this way, the average seismic coefficient at any instant is given by:

$$k_{ave} = \frac{F}{W} = \frac{1}{W}\sum m(y) \cdot \ddot{u}(y) \qquad (3)$$

The time history of the average seismic coefficient k_{ave} can thus be obtained by evaluating k_{ave} at each instant of time during the earthquake.

Some engineers prefer to use a horizontal equivalent acceleration (HEA) time history, which is given as a product of the seismic coefficient and gravitational acceleration as:

$$HEA(t) = k_{ave}(t) \cdot g \qquad (4)$$

in which g is the gravitational acceleration.

The average seismic coefficient k_{ave} for a potential sliding mass can be estimated by performing dynamic response analyses of a slope cross section (e.g., Chopra 1966, Makdisi and Seed 1978, Idriss et al. 1973).

2.2 Limitations of the decoupled approach

A number of investigations (e.g., Lin and Whitman 1983, Gazetas and Uddin 1994, Kramer and Smith 1997, Rathje 1997, Matasovic et al. 1998) have explored the effectiveness of the decoupled approach and other simplifying assumptions. It was reported that the decoupled approach in general provides slightly conservative results compared to those of the coupled approach. It was however reported (e.g., Bray et al. 1998) that the direct use of input acceleration time history without considering system compliance can be significantly unconservative (i.e., can produces significant small displacements).

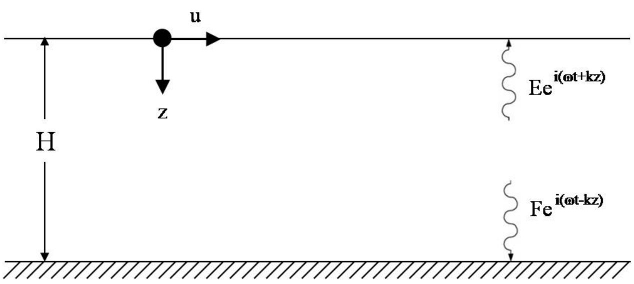

Figure 2. Wave Propagation in Uniform Elastic Soil Layer overlying a Halfspace of Rock

3 SPATIAL AVERAGE OF MOTION ACTING ON HORIZONTAL GROUND

Figure 2 shows the case of a uniform horizontal elastic soil layer overlying a halfspace of rock. The vertical propagation of horizontal shear waves through visco-elastic medium (Kelvin-Voigt solid) is given as:

$$\ddot{U}(z,t) = -E\omega^2(e^{ikz} + e^{-ikz})e^{i\omega t} = -2E\omega^2 \cos kz\, e^{i\omega t} \qquad (5)$$

The gross (overall) effects of earthquake shaking on the potential sliding masses are of our ultimate interest. The overall effects of earthquake shaking on the potential sliding masses may be assessed by integrating all the acceleration time history along the height H of the soil mass above a potential sliding surface in case of gently-sloping slip surfaces and retaining walls on hard underlying ground, given as:

$$\begin{aligned} HEA(t) &= \frac{1}{H}\int_0^H \ddot{U}(z,t)\, dz \\ &= \frac{1}{H}\int_0^H -2E\omega^2 \cos kz\, e^{i\omega t}\, dz \\ &= \frac{-2E\omega^2 e^{i\omega t}}{H}\int_0^H \cos kz\, dz \\ &= \frac{-2E\omega^2 e^{i\omega t}\sin kH}{kH} \end{aligned} \qquad (6)$$

The frequency response (or transfer) function for the HEA (i.e., average acceleration) and the motion at the bottom of the soil layer (i.e., input acceleration time history) can thus be given as:

$$\begin{aligned} H(\omega) &= \frac{HEA(t)}{\ddot{U}(z = H, t)} \\ &= \frac{\sin kH}{kH \cos kH} \\ &= \frac{\tan kH}{kH} \end{aligned} \qquad (7)$$

Equations 6 and 7 are valid for the case of rigid bedrock. The above results can thus be taken as an upper bound of slope response that overlies on elastic bedrock.

4 EFFECTS OF SPATIAL AVERAGING

Figure 3 shows the ratio between the amplitudes of the HEA and bottom acceleration (i.e., amplification function ($|H(\omega)|$) along with that between the amplitudes of the free surface and bottom acceleration. the average motion is amplified at small *kH* up to around the fundamental frequency of the soil deposit and the motion is de-amplified as *kH* increases, except around natural frequencies. These findings generally support the previous landfill investigations and their finding that the direct use of an input acceleration time history in the Newmark rigid block analysis without considering system compliance can be significantly unconservative (i.e., produces smaller displacement). That is mainly because the computed slope displacement is influenced mainly by low frequency motion that amplifies and is not sensitive to high frequency motion (e.g., beyond 5-10 *rad/s*) that de-amplifies due to the averaging. On the other hand, the direct use of rock outcrop motion as input motion in the Newmark deformation analysis may be justified, if the fundamental period of a slope is sufficiently short (i.e., shallow soil deposit with high shear velocity) or is sufficiently long (i.e., deep soil deposit with low shear velocity). The upper and lower bounds of this fundamental period are of our special interest.

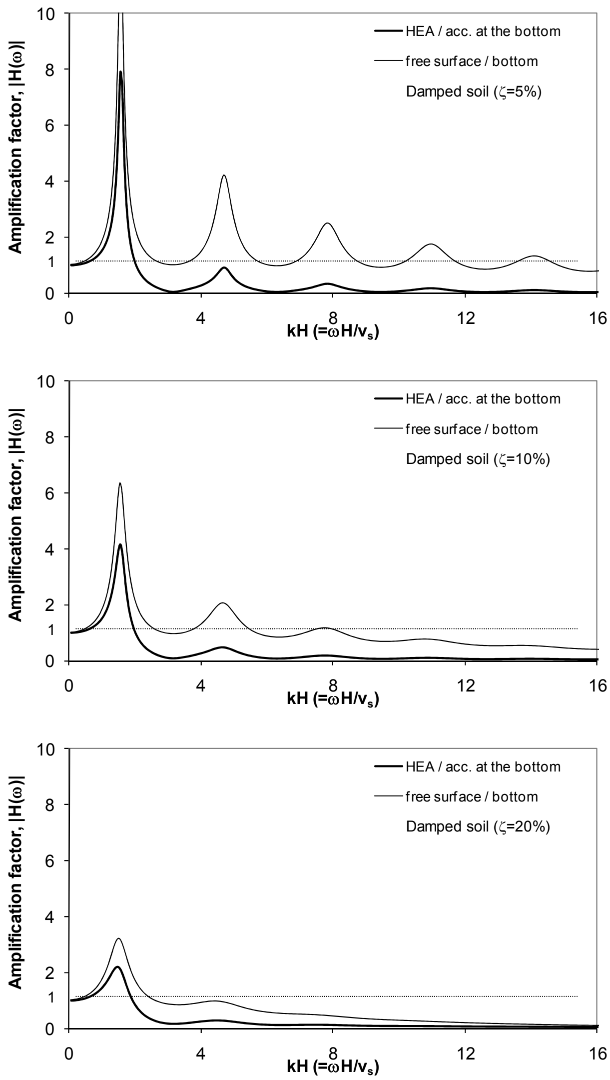

Figure 3. Ratio Between the Amplitudes of the HEA and Bottom Acceleration (i.e., Amplification Function $|H(\omega)|$) Along with That Between the Amplitudes of the Free Surface and Bottom Accelerations.

In this purpose, we need to find the boundary values of the period that satisfies:

$$|H(\omega)| = \left|\frac{\tan kH}{kH}\right| = \left|\frac{\sin kH}{kH * \cos kH}\right| \approx 1.0 \tag{8}$$

Where k is a complex number such that $k = \frac{\omega}{v_s^*} = \frac{\omega}{v_s}(1 - i\zeta)$.

Equation 8 needs to be manipulated further into a real valued equation such that:

$$|H(\omega)| = \left|\frac{\sin kH}{\cos kH * kH}\right| = \frac{|\sin kH|}{|\cos kH| * |kH|}$$
$$= \frac{\sqrt{\sin^2(\omega H / v_s) + \sinh^2(\omega H \zeta / v_s)}}{\sqrt{\cos^2(\omega H / v_s) + \sinh^2(\omega H \zeta / v_s)} * (\omega H / v_s) * \sqrt{1+\zeta^2}} \tag{9}$$

Where $|\sin kH| = \sqrt{\sin^2(\omega H / v_s) + \sinh^2(\omega H\zeta / v_s)}$, $|\cos kH| = \sqrt{\cos^2(\omega H / v_s) + \sinh^2(\omega H\zeta / v_s)}$, and $|kH| = (\omega H / v_s)\sqrt{1+\zeta^2}$

It is found from Equation 9 that $|kH| \le 0.52$, $|kH| \ge 2.0$ satisfies $|H(\omega)| \le 1.1$ (i.e., less than 10 % difference). Therefore, the boundary values of the period of the slope that separate the range of period where average motion is amplified or deamplified can be estimated as:

$$|kH| = \frac{\omega}{v_s} H\sqrt{1+\zeta^2} = \frac{\omega}{4}\frac{4H}{v_s}\sqrt{1+\zeta^2}$$
$$= \frac{\omega T}{4}\sqrt{1+\zeta^2} \le 0.52 \quad or \quad \ge 2.0 \tag{10}$$

Because the computed displacement is not sensitive to high frequency motion (beyond 5-10 *rad/s*) and damping ratio is usually less than 0.2, the maximum value of the site period can be determined as:

$$T \le \frac{2.2}{\omega} = \frac{2.2}{10} \approx 0.2\ s \quad T \ge \frac{8.0}{\omega} = \frac{8.0}{10} \approx 0.8\ s \tag{11}$$

Therefore, it can be generally concluded that the direct use of rock outcrop motion as input motion in the Newmark deformation analyses is justified, if the fundamental period of the slope is less than 0.2 second and/or more than 0.8 second.

For the period of 0.2 second to 0.8 second, however, direct use of rock outcrop motion as input motion in the Newmark deformation analyses can be unconservative and may need to be analyzed further. These tentative conclusions may need to be verified with the results from previous empirical research because these conclusions are based on the analytical analyses that assume the soil to have constant shear modulus and damping. This assumption is necessary for the analytical derivation and may be problematic for strong ground motion. The effect of nonlinearity, however, can be inferable from the numerical analysis-based previous research. In order to compare with the previous research, Figure 4 is redrawn in terms of *Ts/Teq* from Figure 3. *Ts* is a fundamental period of the slope and *Teq* is a period of ground motion. It is found from Figure 5 that $T_s/T_{eq} \le 0.33$, $T_s/T_{eq} \ge 1.26$ satisfies $|H(\omega)| \le 1.1$ (i.e., less than 10 % difference). Therefore $T_s/T_{eq} \ge 1.26$ and corresponding $T_s \ge 0.8\ s$ determined from the analytical derivation can be considered as an upper bound. The lower bound that accounts for nonlinearity, however, is difficult to be determined because the figure does not provide any clear boundary since the resonance period is shifted due to modulus reduction. It is therefore suggested that the lower bound, $T_s = 0.2\ s$ determined from the analytical derivation should be used with caution only for low intensity input ground motion.

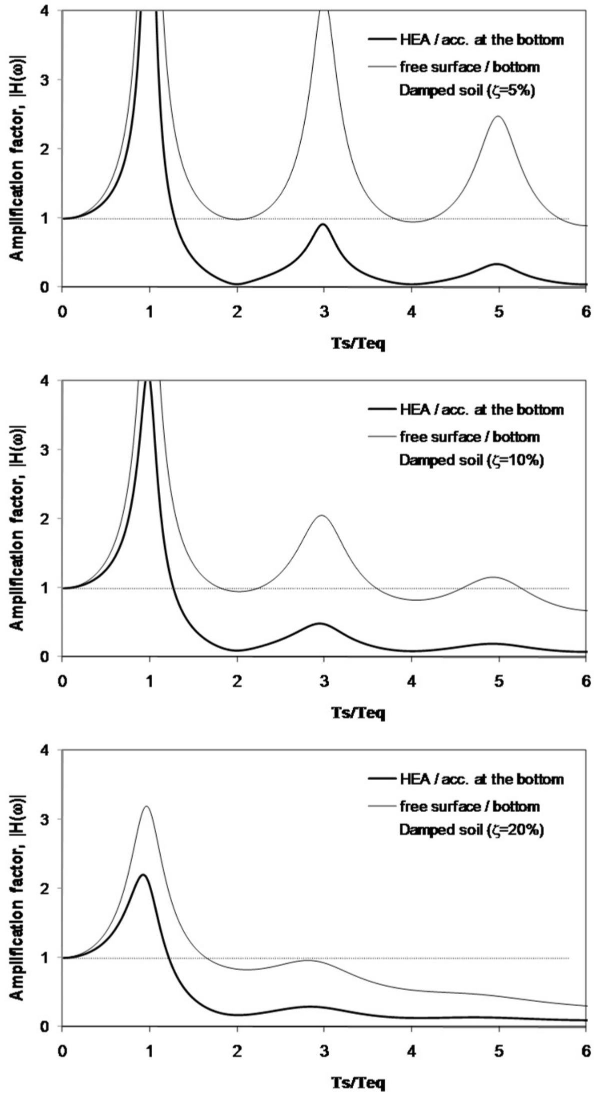

Figure 4. Ratio Between the Amplitudes of the HEA and Bottom Acceleration (i.e., Amplification Function $|H(\omega)|$) Along With That Between the Amplitudes of the Free Surface and Bottom Accelerations.

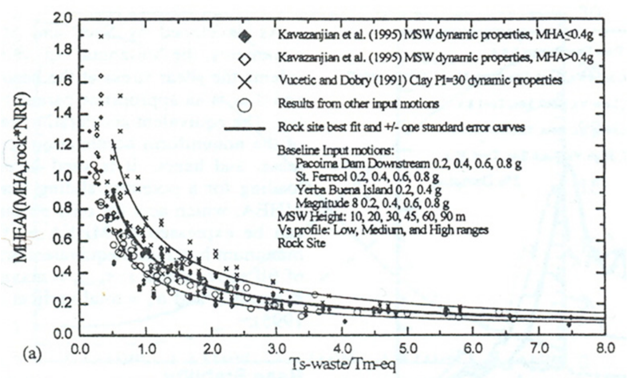

Figure 5. Normalized Maximum Equivalent Acceleration versus Normalized Fundamental Period of Waste Fill at Rock Sites (Bray and Rathje, 1998)

5 SUMMARY AND CONCLUSIONS

The paper reviewed the impact of noncompliance assumption on the computed permanent deformation of the Newmark-type rigid block deformation method in the analysis of earthquake-induced deformation of a slope. Conclusions are drawn as follows.

The results generally support the previous finding that the direct use of input acceleration time history in the Newmark-type block analysis without considering system compliance can be significantly unconservative. However, the results also suggest that the direct use of rock outcrop motion as input motion in the Newmark-type deformation analyses can be justified, if the fundamental period of the slope is sufficiently short (i.e., shallow soil deposit with high shear velocity) or relatively long(i.e., deep soil deposit with low shear velocity)

Those results lead to a relatively simple criterion that can be used to determine the level of conservativeness that may be useful when selecting input motion for deformation analyses. It is concluded that the direct use of rock outcrop motion as input motion in the Newmark-type rigid block deformation analysis can be justified, if the fundamental period of the potential sliding mass is far less than 0.2 second and more than 0.8 second. If the period is between 0.2 second and 0.8 second, the direct use of rock outcrop motion can be significantly unconservative (i.e., produces smaller displacement) and therefore, it may be necessary to perform ground response analyses to determine the average inertia force acting on the sliding mass.

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